D. Shuman 7/5/2010

Quartz tube theoretical buckling pressures; quick estimate. Formulas from table 35 Roark's Formulas for Stress and Strain, 6th ed. These values are not conservative, as slight imperfections will reduce buckling pressure, but realistic buckling values can be within +/-20% of theoretical. Required Factor of safety on buckling is not clear, it is probably not 8, as required in PUB3000 on strength for brittle materials used in pressure systems. These tubes are not part of the pressure containment system, so a lower factor of safety, based on failure consequence to experiment (only) is warranted.

from http://www.insaco.com/MatPages/mat_display.asp?M=Quartz

modulus of elasticity $E := 10.5 \cdot 10^6 psi$

Poisson's ratio $\nu := 0.17$

try several thicknesses in parallel calculation $t := \begin{pmatrix} 1 \\ 1.5 \\ 2 \end{pmatrix} mm$

tube nominal radius $r := 0.75in$

very long tube ($l > l_{cr}$):

$$l_{cr} := 4.9r \cdot \sqrt{\frac{r}{t}} \qquad l_{cr} = \begin{pmatrix} 0.407 \\ 0.333 \\ 0.288 \end{pmatrix} m$$

buckling pressure:

$$p_{cr\_lt} := \frac{1}{4} \cdot \frac{E}{1 - \nu^2} \cdot \frac{t^3}{r^3} \qquad p_{cr\_lt} = \begin{pmatrix} 26.6 \\ 89.8 \\ 212.8 \end{pmatrix} bar$$

for short tube, length l, or long tube constrained circular at lengths l: $l := 0.5m$

$$p_{cr\_st} := \overrightarrow{\left[0.807 \cdot \frac{E \cdot t^2}{l \cdot r} \cdot \sqrt[4]{\left(\frac{1}{1 - \nu^2} \right)^3 \cdot \frac{t^2}{r^2}} \right]} \qquad \text{(approx. formula)}$$

$$p_{cr\_st} = \begin{pmatrix} 14.2 \\ 39.1 \\ 80.2 \end{pmatrix} bar$$

check compressive strength, (no factor of safety here)

$S_{comp} := 1150MPa$ Suprasil CG (no data for Suprasil 310)

$$p_{comp} := S_{comp} \cdot \frac{t}{r} \qquad p_{comp} = \begin{pmatrix} 596 \\ 893 \\ 1 \times 10^3 \end{pmatrix} bar$$