Mesh tension required:

$\varepsilon_0 := 8.85 \cdot 10^{-12} \frac{F}{m}$

Mesh modulus: assume no wire crossing

wire dia	wire spacing	wire elastic modulus	Voltage	gap
$d_{mw} := .0012in$	$s_{mw} := 0.5mm$	$E_{ss} := 28 \cdot 10^6 psi$	$V_{el} := 9kV$	$d_{el} := 3mm$

$E_{el} := \frac{V_{el}}{d_{el}}$ $\qquad E_{el} = 30 \frac{kV}{cm}$

mesh modulus will be material modulus * cross section areal fill ratio

$N_a := \frac{\frac{\pi}{4} d_{mw}^2}{s_{mw} \cdot d_{mw}}$ $\qquad N_a = 0.048$

$E_{mesh} := E_{ss} N_a$ $\qquad E_{mesh} = 9.243 \times 10^9 \, Pa$

Pressure, from electric field

$P_{el} := \varepsilon_0 \frac{V_{el}^2}{2 d_{el}^2}$ $\qquad P_{el} := 0.5 \varepsilon_0 \cdot E_{el}^2$ $\qquad P_{el} = 39.8 \, Pa$

We desire a 1% uniformity of EL gradient:

$\delta_{max} := .01 d_{el}$ $\qquad \delta_{max} = 0.03 \, mm$

frame radius, (circular approximation)

$R_{mp} := 7cm$

For thin membrane, with no plate stiffness, deflection from pressure is a function of initial stress only (up to three thicknesses of deflection)

<u>A New Analytical Solution for Diaphragm Deflection and its Application to a Surface Micromachined Pressure Sensor</u>. W.P.Eaton, et. al

$\delta_{mem} := \frac{P_{el} \cdot R_{mp}^2}{4 \sigma_i \cdot d_{mw}}$ $\qquad \sigma_i := \frac{P_{el} \cdot R_{mp}^2}{4 \delta_{max} \cdot d_{mw}}$ $\qquad \sigma_i = 7738 \, psi$ Uh oh! ref. eq. 2

actual wire stress:

$S_{y\_ss} := 75000psi$

$\sigma_{mw} := \sigma_i \cdot \frac{s_{mw} \cdot d_{mw}}{\frac{\pi}{4} d_{mw}^2}$ $\qquad \sigma_{mw} = 1.616 \times 10^5 \, psi$

way too high to achieve 1% EL gradient uniformity (10% just possible at yield)

Large Deflection Formula for Circular Membrane with initial stress under pressure loading (also from Eaton, et. al.)

let: $\sigma_{ps} := 0.75 S_{y\_ss} \cdot N_a$ $\qquad \sigma_{ps} = 2.693 \times 10^3 \, psi$

Poisson's ratio is given only for free transverse boundary, so for mesh it will depend only on wire dia and spacing (probably ν = θ):

$h := d_{mw}$ $\qquad a := R_{mp}$ $\qquad \varepsilon_i := \frac{\sigma_{ps}}{E_{mesh}}$ $\qquad \varepsilon_i = 0.201\%$ $\qquad E := E_{mesh}$ $\qquad \nu := \frac{d_{mw}}{s_{mw}}$ $\qquad \nu = 0.061$

$P := P_{el}$ $\qquad \eta := 1$

$$D := \frac{\eta \cdot E \cdot h^3}{12\left(1 - \nu^2\right)} \qquad D = 2.189 \times 10^{-5}\, N \cdot m$$ ref. eq 3

$$\alpha := 14 \cdot \frac{(2 \cdot h)^2 + 3a^2 \cdot \varepsilon_i \cdot (1 + \nu)}{(1 + \nu) \cdot (23 - 9\nu)}$$ ref. eq 17

$$\beta := -7P \cdot a^4 \frac{h^2}{8D \cdot (1 + \nu) \cdot (23 - 9\nu)}$$

$$\gamma := \sqrt{\frac{\alpha^3}{27} + \frac{\beta^2}{4}}$$

$$f := \sqrt[3]{\frac{-\beta}{2} + \gamma} + \sqrt[3]{\frac{-\beta}{2} - \gamma} \qquad f = 0.081\, mm \qquad \frac{f}{d_{el}} = 2.7\,\%$$

Additional (assumed) Stress from deflection:

Let:

$$w(r) := f\left[1 - \left(\frac{r}{a}\right)^2\right]^2$$ and $$\phi(r) := \frac{d}{dr}\left(2 \cdot B_1 \cdot r - \frac{r^3}{a^4} + \frac{2}{3} \cdot \frac{r^5}{a^6} - \frac{1}{6} \cdot \frac{r^7}{a^8}\right)$$ ref. eqs 12,13

define some constants

$$B_1 := \frac{5 - 3\nu}{1 - \nu} \cdot \frac{1}{a^2} \qquad A_\phi := \frac{f^2 E}{12}$$

then for :

$r := .01a$ (stresses are maximum near center)

$$\frac{d}{dr}\left[1 - \left(\frac{r}{a}\right)^2\right]^2 = -4 \cdot \left(1 - \frac{r^2}{a^2}\right) \cdot \frac{r}{a^2} \qquad dwdr := f \cdot -4 \cdot \left(1 - \frac{r^2}{a^2}\right) \cdot \frac{r}{a^2} \qquad dwdr = -4.623 \times 10^{-5}$$

$$\frac{d}{dr}\left[-4 \cdot \left(1 - \frac{r^2}{a^2}\right) \cdot \frac{r}{a^2}\right] = 8 \cdot \frac{r^2}{a^2 \cdot a^2} + \frac{-4 + 4 \cdot \frac{r^2}{a^2}}{a^2} \qquad d2wdr2 := f \cdot \left(8 \cdot \frac{r^2}{a^4} + \frac{-4 + 4 \cdot \frac{r^2}{a^2}}{a^2}\right) \qquad d2wdr2 = -0.066\, \frac{1}{m}$$

$$\frac{d}{dr}\left[B_1 \cdot r^2 - \frac{1}{4}\left(\frac{r}{a}\right)^4 + \frac{1}{9}\left(\frac{r}{a}\right)^6 - \frac{1}{48}\left(\frac{r}{a}\right)^8\right] = 2 \cdot B_1 \cdot r - \frac{r^3}{a^4} + \frac{2}{3} \cdot \frac{r^5}{a^6} - \frac{1}{6} \cdot \frac{r^7}{a^8}$$

$$d\phi dr := A_\phi \cdot \left(2 \cdot B_1 \cdot r - \frac{r^3}{a^4} + \frac{2}{3} \cdot \frac{r^5}{a^6} - \frac{1}{6} \cdot \frac{r^7}{a^8}\right) \qquad d\phi dr = 7.39\, \frac{N}{m}$$

$$\frac{d}{dr}\left(2 \cdot B_1 \cdot r - \frac{r^3}{a^4} + \frac{2}{3} \cdot \frac{r^5}{a^6} - \frac{1}{6} \cdot \frac{r^7}{a^8}\right) = 2 \cdot B_1 - 3 \cdot \frac{r^2}{a^4} + \frac{10}{3} \cdot \frac{r^4}{a^6} - \frac{7}{6} \cdot \frac{r^6}{a^8}$$

$$d2\phi dr2 := A_\phi \cdot \left(2 \cdot B_1 - 3 \cdot \frac{r^2}{a^4} + \frac{10}{3} \cdot \frac{r^4}{a^6} - \frac{7}{6} \cdot \frac{r^6}{a^8}\right) \quad d2\phi dr2 = 1.056 \times 10^4\,\mathrm{Pa}$$

Stress is given by:

$$\sigma_r := \frac{-6D}{h^2}\left(d2wdr2 - \frac{\nu}{r} \cdot dwdr\right) + \frac{1}{r} \cdot d\phi dr \qquad \sigma_r = 2.803\,\mathrm{psi}$$

ref. eqs 18,19

$$\sigma_\theta := \frac{-6D}{h^2}\left(\frac{1}{r} dwdr - \nu \cdot d2wdr2\right) + d2\phi dr2 \qquad \sigma_\theta = 2.803\,\mathrm{psi}$$

(note: formula in ref. must be incorrect)