Mesh Deflection D. Shuman 3/21/11

Operating parameters frame radius

$p := 15\text{bar}$ $\quad EovrP := 4\dfrac{\text{kV}}{\text{cm}\cdot\text{bar}}$ $\quad E_{EL} := EovrP\cdot p$ $\quad E_{EL} = 60\dfrac{\text{kV}}{\text{cm}}$ $\quad E_{dr} := 1\dfrac{\text{kV}}{\text{cm}}$ $\quad R_{mp} := 51\text{cm}$

gap Voltage

$d_{EL} := 10\text{mm}$ $\quad V_{EL} := d_{EL}\cdot E_{EL}$ $\quad V_{EL} = 6\times 10^{4}\,\text{V}$

Mesh modulus: assume no wire crossing; isotropic properties

mesh wire dia wire spacing wire elastic modulus open area fraction

$d_{mw} := .009\text{in}$ $\quad s_{mw} := \dfrac{1}{14}\text{in}$ $\quad E_{ss} := 28\cdot 10^{6}\text{psi}$ $\quad E_{ss} = 193\,\text{GPa}$ $\quad \eta_a := \dfrac{\left(s_{mw}-d_{mw}\right)^2}{s_{mw}^{\,2}}$ $\quad \eta_a = 76\,\%$

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equivalent mesh elastic modulus will be material modulus * cross section areal fill ratio

$$N_a := \frac{\frac{\pi}{4}d_{mw}^{\,2}}{s_{mw}\cdot d_{mw}} \qquad N_a = 0.099$$

$$E_{mesh} := E_{ss}N_a \qquad E_{mesh} = 19.1\,\text{GPa}$$

Pressure, from electric field (EL -drift field) (ignore gravity for dual mesh planes)

$$P_{EL} := 0.5\varepsilon_0\cdot\left(E_{EL}-E_{dr}\right)^2 \qquad P_{EL} = 154\,\text{Pa}$$

Large Deflection Formula for Circular Membrane with initial stress under pressure loading:

deflection is primarily a function of initial stress condition (prestress)

reference: A New Analytical Solution for Diaphragm Deflection and its Application to a Surface Micromachined Pressure Sensor. W.P.Eaton, et. al:

let equivalent mesh prestress be:

$$\sigma_{ps} := 0.75S_{u\_ss}\cdot N_a \qquad \sigma_{ps} = 38.38\,\text{MPa}$$

let:

$$h := d_{mw} \quad a := R_{mp} \quad E := E_{mesh} \quad P := P_{EL}$$

Initial strain:

$$\varepsilon_i := \frac{\sigma_{ps}}{E_{mesh}} \qquad \varepsilon_i = 0.201\,\%$$

Plate constant:

$$D := \frac{E\cdot h^3}{12\left(1-\nu^2\right)} \qquad D = 0.019\,\text{N}\cdot\text{m}$$

S.S Ultimate Strength

$$S_{u\_ss} := 75000\text{psi} \qquad S_{u\_ss} = 517.1\,\text{MPa}$$

Poisson's ratio

$$\nu := \frac{d_{mw}}{s_{mw}} \qquad \nu = 0.126$$

Poisson's ratio is given only for free transverse boundary, so for mesh it will depend only on wire dia and spacing (probably $\nu = \theta$):

$$d_{mw} = 2.286\times 10^{-4}\,\text{m}$$

ref. eqs 3

compute the following:

$$\alpha := 14 \cdot \frac{(2 \cdot h)^2 + 3a^2 \cdot \varepsilon_i \cdot (1+\nu)}{(1+\nu) \cdot (23 - 9\nu)} \qquad \alpha = 1.004 \times 10^{-3}\,m^2$$ ref. eqs 17

$$\beta := -7P \cdot a^4 \frac{h^2}{8D \cdot (1+\nu) \cdot (23 - 9\nu)} \qquad \beta = -1.001 \times 10^{-6}\,m^3$$ ref. eqs 17

$$\gamma := \sqrt{\frac{\alpha^3}{27} + \frac{\beta^2}{4}} \qquad \gamma = 6.141 \times 10^{-6}\,m^3$$ ref. eqs 17

deflection is:

percentage gap decrease

$$f := \sqrt[3]{\frac{-\beta}{2} + \gamma} + \sqrt[3]{\frac{-\beta}{2} - \gamma} \qquad f = 0.997\,mm \qquad \frac{2f}{d_{EL}} = 19.9\,\%$$ ref. eqs 17