Accelerated Life Testing
Tutorial with NASA and DOD Applications

NASA Statistical Engineering Symposium

Dr. Laura Freeman

Institute for Defense Analyses

May 5, 2011

Outline

- **Introduction**
  - Lifetime data & reliability analysis
  - Weibull distribution
  - Life Tests
  - Accelerated Life Tests (ALTs)
  - Censoring
- **Designing Accelerated Life Tests**
  - Guidelines
  - Monte Carlo Methods
- **Applications**
  - NASA COPV Example
  - Air Force Transponder Mounting Bracket Example

Introduction

- **Reliability: ability of a system to perform a required function**

- **Lifetime data: a quantity of paramount importance to product reliability**
  - Life Tests
  - Accelerated Life Tests

- **Popular distributions for modeling lifetime data**
  - Weibull*
  - Lognormal
  - Exponential
  - Gamma

Weibull Distribution

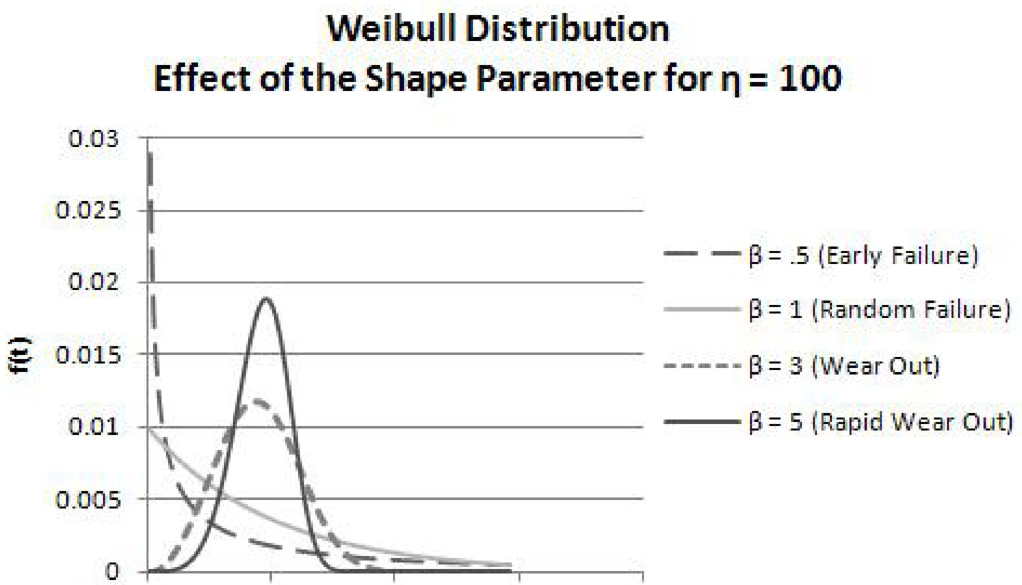
- **Probability density function:**

$$f(t, \beta, \eta) = \frac{\beta}{\eta}\left(\frac{t}{\eta}\right)^{\beta-1} e^{-\left(\frac{t}{\eta}\right)^{\beta}}$$

- **Hazard Function:**

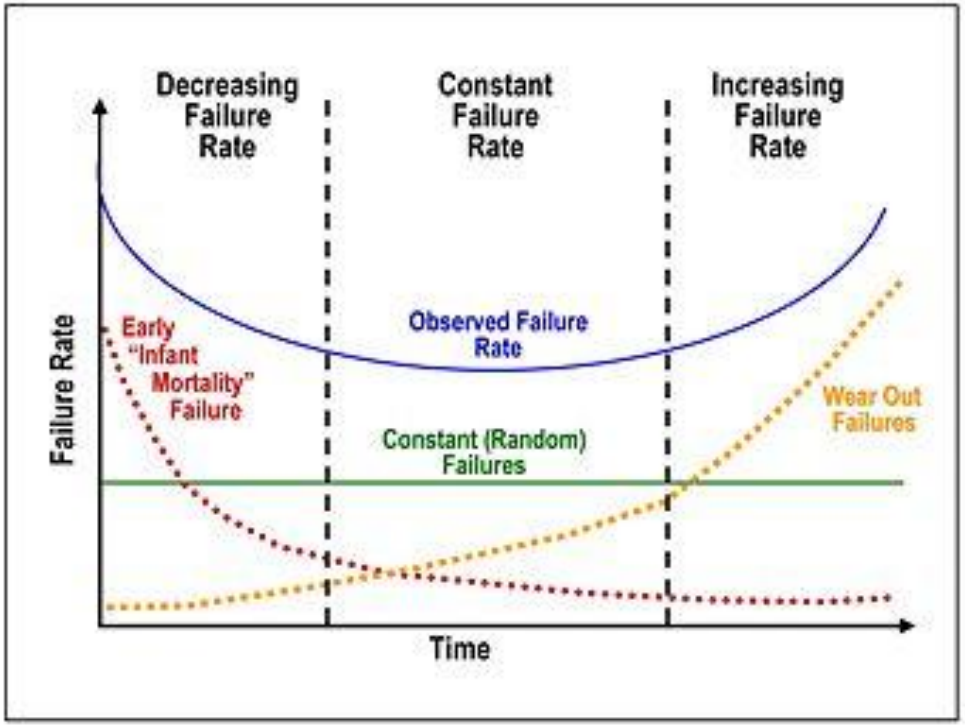
$$h(t) = \frac{\beta}{\eta}\left(\frac{t}{\eta}\right)^{\beta-1}$$

- **Popular distribution because of its flexibility to model different failure mechanisms**



Weibull Distribution

- **Bathtub Hazard Function**
  - Can be modeled as the mixing of three Weibull distributions.



Designed Experiments & Reliability Testing

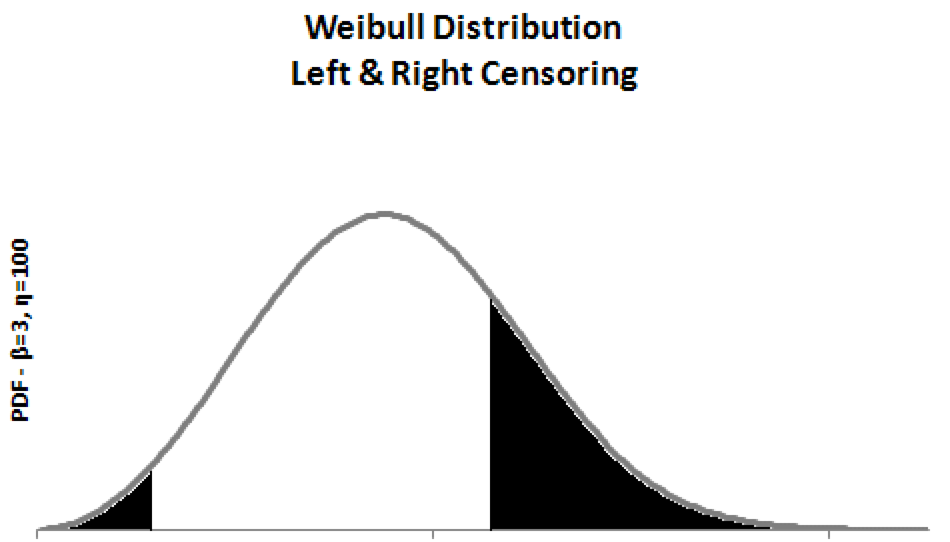
- **Life Tests (LTs)**
  - Goal: model product lifetimes a use conditions
- **Accelerated Life Tests (ALTs)**
  - Goal: Increase the probability of failure by modeling product lifetimes at accelerated conditions
    - » Accelerated in temperature, voltage, humidity, stress, etc.
    - » Project back to use conditions through linearizing relationship
- **Common DOEs for LTs and ALTs**
  - Completely randomized
  - Optimal
  - Designs focus on:
    - » How many units should we use?
    - » How long should we run the test?
- **Complicating issues**
  - Censoring
  - Prediction beyond design space

Censoring

- **Maximum likelihood estimation easily incorporates censoring**

- **Censoring – what is it?**
  - When we are unable to observe a failure time exactly
  - We do know that the unit in question will fail in a certain range

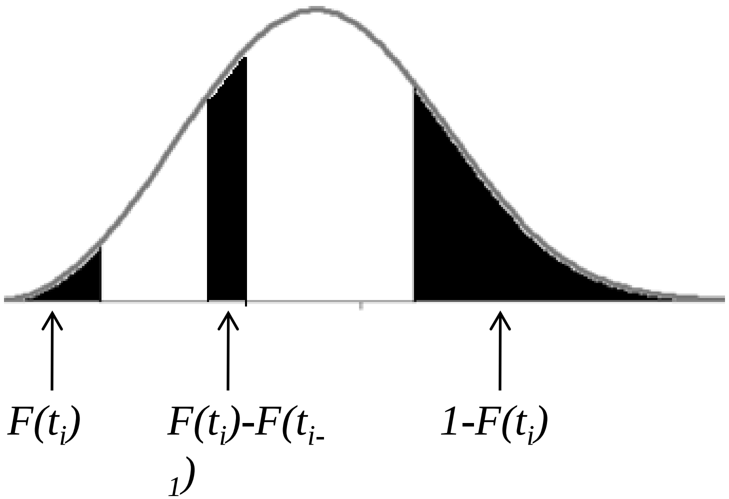
- **Types of Censoring**
  - Left
  - Right (Type I & Type II)
  - Interval



Censoring

- **Contributions to Likelihood**
  - An exact failure time is not observed for a unit
  - Instead we have a range in which the failure occurs
  - Where $F(t_i)$ is the cumulative distribution function at a given time

Censoring Type	Range for Failure Time, T	Likelihood Contribution
Left	$T \leq t_i$	$F(t_i)$
Right	$T \geq t_i$	$1-F(t_i)$
Interval	$t_{i-1} \leq T \leq t_i$	$[F(t_i)-F(t_{i-1})]$
None (Exact Failure)	N/A	$f(t_i)$



Censoring

- **Total Likelihood – product of all likelihood contributions:**

$$L(\theta|y_1,\dots,y_n) = \mathcal{C}\prod_{i=1}^{n}[F(t_i)]^{l_i}\,[F(t_i)-F(t_{i-1})]^{d_i}\,[f(t_i)]^{\delta_i}\,[1-F(t_i)]^{r_i}$$

Left Censoring Contribution

Interval Censoring Contribution

Exact Failure

Right Censoring Contribution

$$\delta_i = \begin{cases} 1 & \text{if the observation is exact} \\ 0 & \text{if the observation is censored} \end{cases}$$

$$l_i = \begin{cases} 1 & \text{if the observation is left censored} \\ 0 & \text{otherwise} \end{cases}$$

$$r_i = \begin{cases} 1 & \text{if the observation is right censored} \\ 0 & \text{otherwise} \end{cases}$$

$$d_i = \begin{cases} 1 & \text{if the observation interval censored} \\ 0 & \text{otherwise} \end{cases}$$

Life Tests

- **Designed to measure product lifetime under typical use conditions.**
- **Weibull Model:** $L(\mu, \sigma; \mathrm{t}) = \mathcal{C} \prod_{i=1}^{n} \left\{ \frac{\beta}{t_i} \phi \left[\beta \left(\log(t_i) - \mu \right) \right] \right\}^{\delta_i} \left\{ 1 - \Phi \left[\beta \left(\log(t_i) - \mu \right) \right] \right\}^{1-\delta_i}$

$$\delta_i = \begin{cases} 1 & \text{if the observation is exact} \\ 0 & \text{if the observation is censored} \end{cases}$$

$$\log[f(t_i)] = \log\left(\frac{\beta}{t_i}\right) + z_i - \exp(z_i)$$

$$\log[1 - F(t_i)] = -\exp(z_i)$$

$$z_i = \beta \left[\log(t_i) - \mu_i \right] \qquad \mu_i = x_i^T \boldsymbol{\theta} + \epsilon_i$$

- **Limitation**
  - Reliable products may not fail in a reasonable timeframe

Accelerated Life Tests

- **Accelerate the number of failures observed during the test by using one or more accelerating factor**

- **Common methods:**
  - Temperature
  - Stress
  - Humidity

- **Linearizing relationship between model parameters and accelerating variable must be understood.**
  - Engineering knowledge of the relationship is of paramount importance otherwise, model fit will be wrong and projections to use conditions will be nonsensical.

- **Common linearizing relationships:**
  - Arrhenius relationship (temperature)
  - Inverse power law (stress, voltage, pressure acceleration)
  - Generalized Eyring (one or more non-thermal accelerating variables)

Designing Accelerated Life Tests

- **Experimental designs to date focus on:**
  - How many units should we use?
  - How long should we run the test?
  - Under what conditions should I accelerate the units?
- **Prior knowledge of the model parameters is key for planning ALTs**
- **Monte Carlo simulations can be used to construct optimum designs**
  - Minimizing standard error
  - Minimizing the determinant of the Fisher Information matrix
- **Meeker & Escobar recommendations**
  - Caution about using optimum designs without augmentation
  - Use insurance units at use conditions
  - Use 3-4 levels of the accelerating variable
  - Minimize extrapolation (use the lowest level of acceleration possible)
  - Allocate more units to lower levels of the accelerating variable and fewer units to higher levels of the accelerating variable

Applications of Accelerated Life Testing

- NASA Carbon Fiber Strands for encasing the Composite Overwrapped Pressure Vessel (COPV)
- Air Force Transponder Mounting Bracket

Composite Overwrapped Pressure Vessel (COPV)

- **Problem Statement: Bursting carbon fiber strands is a failure mode that has been observed in the lab but never under use conditions. We need to understand this failure mechanism.**

- **Goal: to develop a model that predicts time to failure for carbon fiber strands at use conditions.**

- **Historical Data:**
  - Kevlar Fiber Strand Testing

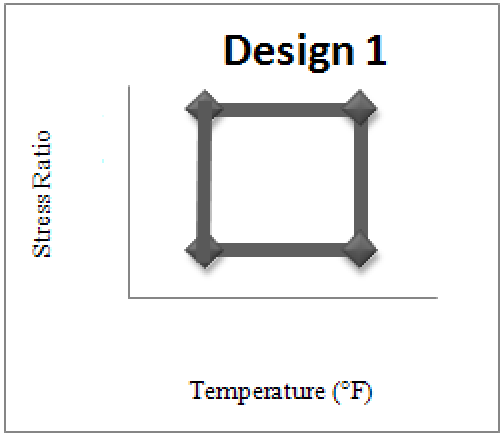
- **Test Approach**
  - Previous data for Kevlar strands focuses on stress ratio acceleration
  - Add temperature acceleration
  - Modified Factorial Design to accommodate ALT specific concerns.

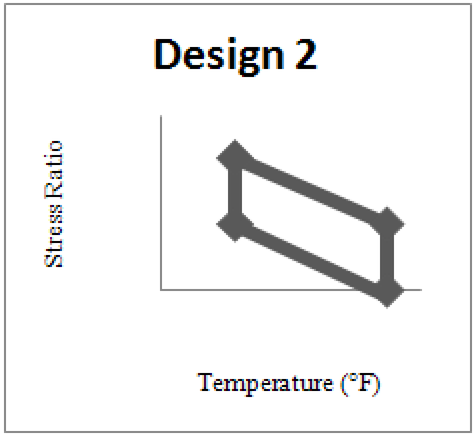
Composite Overwrapped Pressure Vessel (COPV)

- **Classic Power Law model:** $$F(t_i) = 1 - \exp\left[-\left[\frac{t_i}{t_{ref}} SR^{\rho}\right]^{\beta}\right]$$

$$F(t_i) = 1 - \exp\left[-\left[\frac{t_i}{\exp(\gamma_0)} SR^{-\gamma_1}\right]^{\beta}\right]$$

- **Weibull Model:** $$F(t_i) = 1 - \exp\left[-\left[\frac{t_i}{\exp(\gamma_0 + \gamma_1 \log(SR))}\right]^{\beta}\right]$$

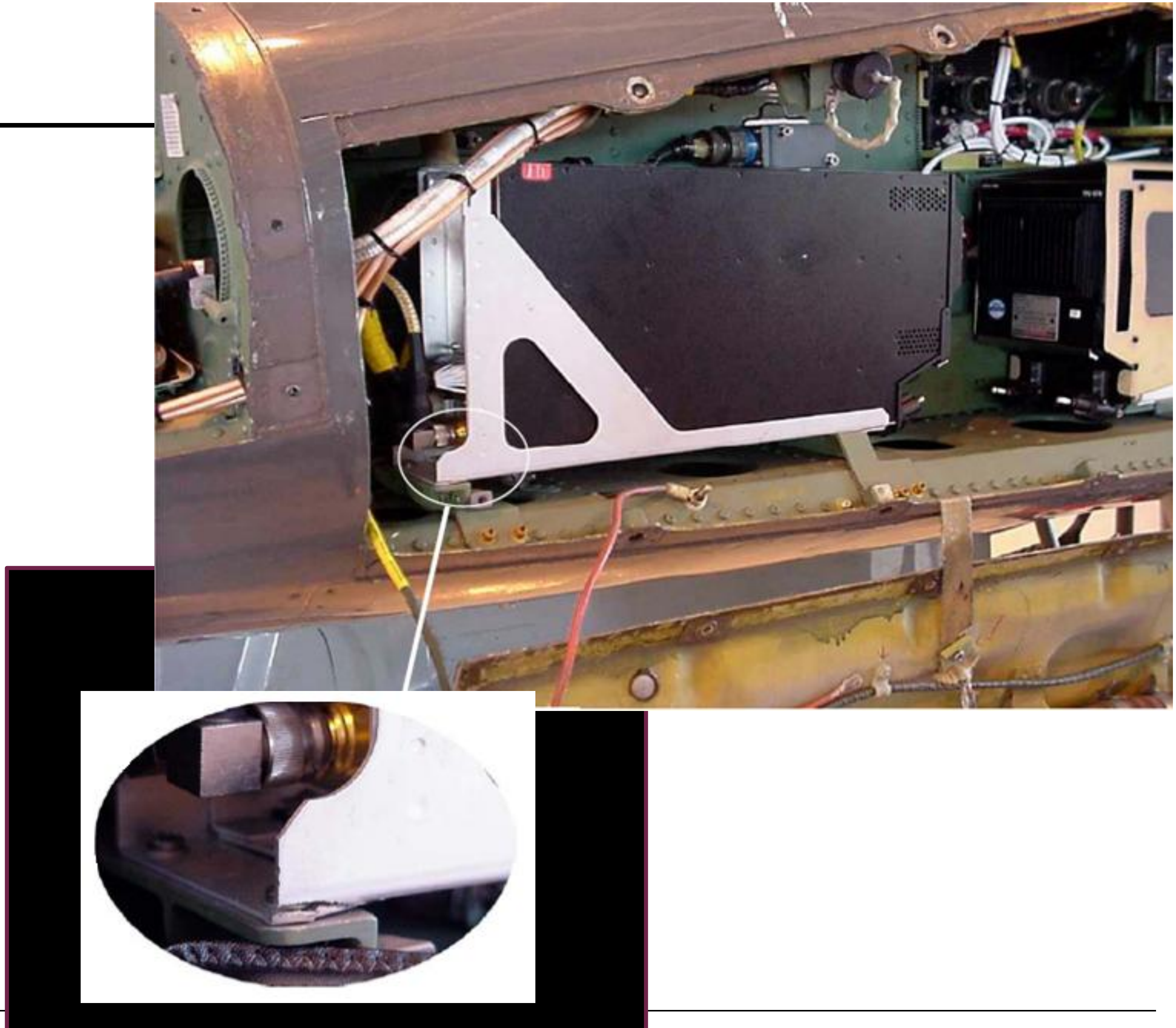




Stress Ratio (SR)	Temp (°F)	Number of Strands	Expected Number of Failures at One Week
Low	High	25	4.49
Medium	Low	25	11.72
Medium	High	15	5.11
High	Low	15	9.25
Total Number		80	

Mounting Bracket for Aircraft Transponder Tray

- **Problem Statement:**
  - The mounting bracket that holds the transponder tray in place on military aircraft are cracking. They were designed to be used on commercial aircraft. To fix the problem the Air Force has proposed an updated mounting tray with an extra stabilizer. However, there is concern that this additional stabilizer may induce a new failure mechanism.
- **Goal: to develop a model that predicts time to failure for the new mounting bracket.**
- **Historical Data:**
  - Time to failure for historical mounting bracket.
  - Times are interval censored.
- **Test Approach**
  - Vibration Acceleration
  - For operational realism, mounting bracket needs to be tested with actual aircraft and transponder tray.



Applications & Challenges in DoD Testing

- **Need for ALT Application in DoD Testing**
  - Nearly all military systems have reliability requirements that are not achievable in the typical test period.
  - Increased emphasis on reliability.
  - Upgrades to existing systems.

- **Challenges**
  - General caution about statistical models, they have not been differentiated from modeling and simulation.
  - Projection beyond the test design space caries increased risk.
  - Limited capabilities to implement these types of statistical methodologies in DoD testing.

References

- **Textbooks:**
  - Lawless, J. F. (2002). *Statistical Models and Methods for Lifetime Data.* Hoboken, New Jersey: John Wiley & Sons Inc.
  - Meeker, W. Q., & Escobar, L. (1998). *Statistical Methods for Reliability Data.* New York: John Wiley & Sons Inc.
  - Nelson, W. (1990). *Accelerated Testing: Statistical Models, Test Plans and Data Analysis.* New York: John Wiley and Sons.

- **Kevlar Fiber Strand Papers:**
  - Feiveson, A., and Kulkarni, P. Reliability of Space-Shuttle Pressure Vessels with Random Batch Effect. Technometrics 42, 4 (2000), 332{344.
  - Leon, R., Ramachandran, R., Ashby, A., and Thyagarajan, J. Bayesian Modeling of Accelerated Life Tests with Random Effect. Journal of Quality Technology39, 1 (2009), 3-16.