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Viscosity of the Noble Gases in the Temperature Range 25–700°C*

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New relative measurements of the viscosity of xenon at a pressure $P=1$ atm and in the temperature range $T=25$–700°C are reported. The viscosity of argon has also been remeasured. A careful analysis of the long-term reproducibility of the oscillating-disk viscometer employed in the work is presented together with best estimates of the viscosity of all the noble gases. Standard reference values of the viscosity of the gases at atmospheric pressure and at zero density at 25°C are given. The data for all the monatomic gases are represented by means of a single empirical correlation.

I. INTRODUCTION

The paper presents new relative measurements of the viscosity of xenon at a pressure $P=1$ atm and in the temperature range $T=25$–700°C. The viscosity of argon has been remeasured to provide a continuous check on the operation of the instrument.

The use of the Chapman–Enskog theory for monatomic gases allows us to compute tables of the thermal conductivity of the two gases which, we contend, enjoy a higher level of confidence than data obtained by direct measurement.

We have previously performed a number of independent measurements of the viscosity of the pure monatomic gases and the paper includes a careful assessment of the long-term reproducibility of our oscillating-disk instrument. Standard reference values for the viscosity of He, Ne, Ar, Kr, and Xe at 25°C are given.

The experimental technique and the general layout of the instrument were the same as in a previous publication. Therefore, in order to conserve space, we describe experimental detail only where it differs from that of our previous work, and concentrate on a presentation of the experimental results.

II. EXPERIMENTAL METHOD

The same instrument was employed for the measurements as in Ref. 1, but the suspension system was modified by the use of a different strand and a smaller separation between the disk and the fixed plates. The entire instrument was dismantled, rebuilt and aligned specially for these measurements. The characteristics of the suspension system employed are given in Table I.

The conditions imposed by the absolute theory of our instrument[2,3] were not satisfied within a sufficient margin for the present experimental arrangement; therefore, we have evaluated our results on the relative basis suggested by Kestin and Wang.[4] Figure 1 represents the calibration curve for the edge-correction factor C as a function of the dimensionless boundary layer thickness (δ/b). The standard values for the viscosity of the various gases used in the calibration were taken from Ref. 3. The calibration data were fitted by the method of least squares to the expression

$$C=1.245326-\frac{0.063024}{(\delta/b+0.3352)}+\frac{0.000371}{(\delta/b+0.3352)^2}-\frac{0.002287}{(\delta/b+0.3352)^3}. \tag{1}$$

The deviation from this correlation did not exceed ±0.06%, the standard deviation being 0.04%.

In our previous work[1,5–7] we have made use of the data given by Whitelaw[8] when applying the small correction necessary to account for the internal damping of the quartz suspension strand. However, in this series of measurements we have been able to measure the logarithmic decrement *in vacuo*, Δ_0, directly with the suspension strand mounted in the instrument. The decrement was measured in our customary fashion,[1] and the pressure in the instrument was computed from the observed pressure in the vacuum pumping line. It was estimated to be less than 5×10^{-5} mm Hg. The measurements were restricted to temperatures above 500°C, that is to the temperature range where the correction for internal damping contributes more than 0.05% to the viscosity. Table II lists our results for Δ_0 and compares them with the values given by Whitelaw.[8] The uncertainty associated with each set of results is

Table I. Characteristics of the suspension system. Suspension strand: fused quartz. Length 23.4 cm, diameter 0.001 in.

Total separation between the fixed plates	$D=0.70027\pm0.0006$ cm
Upper separation	$b_1=0.29829\pm0.00025$ cm
Lower separation	$b_2=0.29829\pm0.00025$ cm
Radius of disk	$R=3.25081\pm0.0006$ cm
Thickness of disk	$d=0.10369\pm0.00008$ cm
Moment of inertia of the suspension system	$I=40.1370\pm0.0009$ g·cm^2
Period of oscillation at 25°C *in vacuo*	$T_0=41.408\pm0.003$ sec

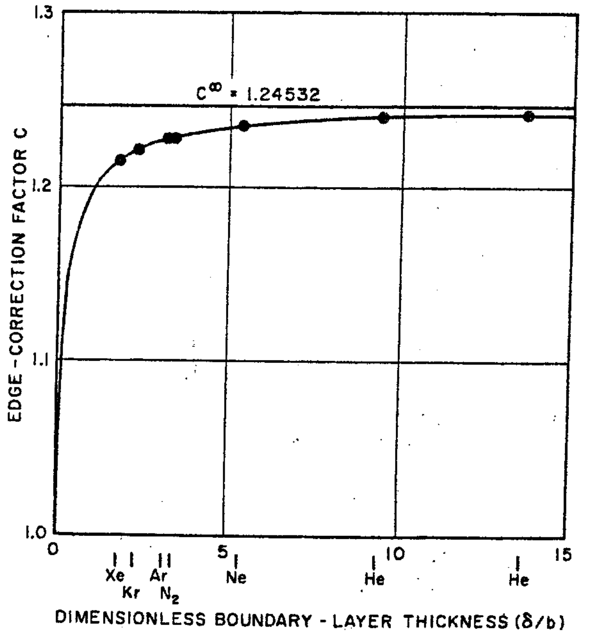

FIG. 1. Calibration curve. Edge-correction factor $C=C(\delta/b)$, after Ref. 4.

±20% and discrepancies between the data beyond this limit are attributed to differences in the characteristics of the individual strands used in the two determinations. In computing the viscosities reported here, we have employed the present values for Δ_0 above 500°C and those of Whitelaw[8] below this temperature.

The small temperature corrections necessary to obtain the viscosity at nominal temperatures were applied as in Ref. 1. The suppliers and purity of the gases used are specified in Table III.

Measurements of the viscosity of argon made during the cooling of the instrument differed from the values obtained during the heating cycle by less than ±0.1%. Throughout the entire course of the measurements the zero-point did not move by more than ±4 mm on the scale, and upon completion of the work, the instrument was dismantled and the disk found to be free of any cracks or distortions.

TABLE II. The logarithmic decrement *in vacuo*, Δ_0.

Temperature (°C)	Logarithmic decrement *in vacuo*, Δ_0 Present measurements	Whitelaw[a]
25	...	0.3×10^{-5}
100	...	0.3×10^{-5}
200	...	0.3×10^{-5}
300	...	0.5×10^{-5}
400	...	0.35×10^{-4}
500	0.18×10^{-3}	0.11×10^{-3}
600	0.42×10^{-3}	0.33×10^{-3}
700	0.96×10^{-3}	0.74×10^{-3}

[a] See Ref. 8.

TABLE III. Suppliers and purity of the gases.

Gas	Supplier	Purity
Argon, Ar	Matheson Co., Inc.	>99.9995%
Xenon, Xe	Union Carbide, Linde, Rare Gas Department	>99.9975%

III. EXPERIMENTAL RESULTS

The experimental viscosities for argon and xenon, at nominal temperatures, are listed in Table IV.

According to the Chapman–Enskog theory, the viscosity of a pure monatomic gas is given by the relation[9]

$$\mu=[26.693(MT)^{1/2}/\sigma^2\Omega^{(2,2)*}(T^*)]f_\mu^{(3)}(T^*), \quad (2)$$

where $T^*=\mathbf{k}T/\epsilon$, and where μ is measured in micropoise. The factor $f_\mu^{(3)}$ denotes the third approximation, and both it and the collision integral $\Omega^{(2,2)*}(T^*)$ are functionals of the intermolecular potential.

In previous publications,[6,7,10] it has been shown that the (11-6-8) potential model suggested by Hanley and Klein[11] provides a satisfactory basis for the correlation of our experimental viscosity data for Ne, Ar, and Kr. [Throughout the paper the parameter γ of the (11-6-8) potential is assigned the value $\gamma=3.0$.] Therefore, for the sake of completeness, we have correlated the present data in terms of the functionals of this potential model with the aid of Eq. (2). The numerical values of the scaling parameters ϵ and σ, securing an optimum fit to the present viscosity data for Ar and Xe, are listed in Table V.

Figure 2 displays the deviations of the present viscosity data from this correlation for both gases. It is at once clear that while the data for Ar are satisfactorily represented by the correlation (max. dev. ±0.4%), the same cannot be said about the results for Xe (max. dev. 0.9%).

TABLE IV. The viscosity of argon and xenon.

Temperature (°C)	Viscosity, μP Ar	Xe
25	226.1_6	230.8_5
100	273.2_8	288.4_4
200	329.2_2	359.0_9
300	378.9_5	423.7_6
400	425.1_2	483.1_6
500	467.6_8	538.4_1
600	505.8_1	587.8_2
700	544.0_4	637.6_9

TABLE V. The optimum parameters for the (11-6-8) potential.

Gas	ϵ/k (K)	σ (Å)
Argon, Ar	153.97	3.297
Xenon, Xe	287.72	3.853

A more satisfactory correlation for the xenon data is afforded by the use of the empirical functional of the intermolecular potential Ω_{22}.[10] This is defined by the relation,

$$\Omega_{22}(T^*)=26.693(MT)^{1/2}/\sigma^2\mu. \tag{3}$$

This functional has been shown to be universal among the monatomic gases.[10]

We have correlated the present experimental data for xenon together with our best estimates of the viscosity of the other monatomic gases (Sec. VI) by means of the single polynomial expression,

$$\ln\Omega_{22}=0.45667-0.53955(\ln T^*)+0.18265(\ln T^*)^2 -0.03629(\ln T^*)^3+0.00241(\ln T^*)^4. \tag{4}$$

A brief discussion of the universal nature of this functional is given in Sec. VII. It is sufficient to state here that the scaling factors ϵ and σ for Xe are assigned the values,

$$\epsilon/\mathrm{k}=285.27K,$$
$$\sigma=3.848\ \text{Å}, \tag{5}$$

and that Eqs. (3) and (4) may be used with confidence above our experimental temperature range for Xe in order to permit us to make a comparison with experimental data already available in the literature.

Figure 3 displays the deviations of our present viscosity data for Xe from this correlation. The deviation does not exceed ±0.3%. The diagram also includes a comparison with the data of Dawe and Smith[12] and with the results of Goldblatt and Wageman.[13] The former data lie systematically below the correlation by about 1.3%, whereas the results reported in Ref. 13 deviate upwards from the correlation by as much as 1.6%. These trends are essentially the same as those that have been observed in comparisons of our results with the results of the same two groups for the other monatomic gases.[1,5–7]

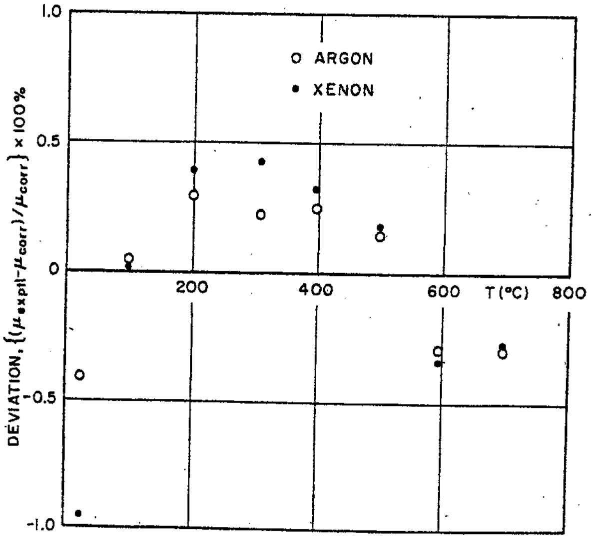

FIG. 2. The deviation of the present data for Ar and Xe from the optimum fit provided by the (11-6-8) potential.

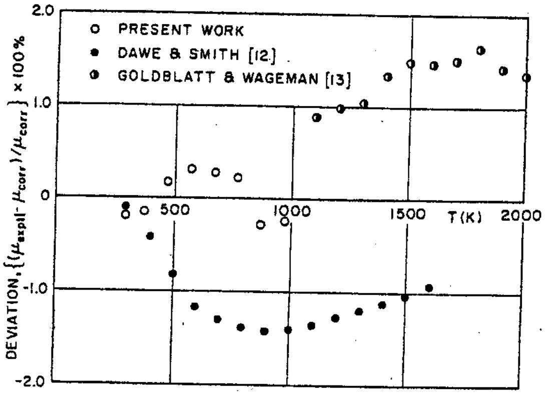

FIG. 3. The deviation of the present data for Xe from the empirical correlation, Eqs. (3) and (4), and a comparison with previous work.

It is worthwhile to record that the present data for argon do not deviate by more than ±0.5% from those previously reported by this laboratory.[1,5,6,14]

IV. THE THERMAL CONDUCTIVITY

In previous publications,[10,15] we have examined the thermodynamic consistency of the available data on the thermal conductivity of He, Ne, Ar, and Kr, and our own viscosity measurements, by means of the relation,[9]

$$\tfrac{2}{5}(k/\mu c_v)=\xi(T^*)=f_k^{(3)}/f_\mu^{(3)}. \tag{6}$$

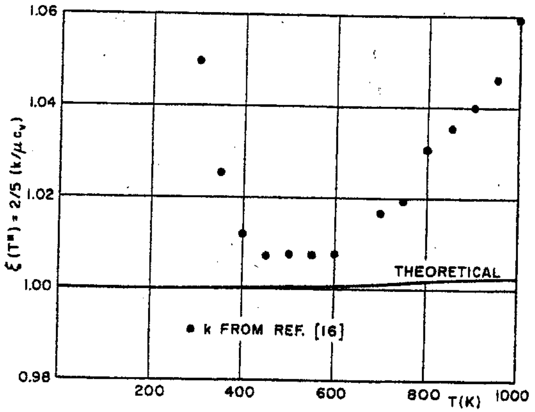

FIG. 4. A comparison between the experimental ratio $\xi(T^*)=2k/5\mu c_v$ for Xe, and the values computed for the (11-6-8) potential model.

TABLE VI. The thermal conductivity of argon and xenon.

Temperature (°C)	Thermal conductivity, $10^6 k$ (cal cm^{-1}·sec^{-1}·K^{-1}) Ar	Xe
25	42.19	13.09
100	51.01	16.36
200	61.50	20.38
300	70.83	24.06
400	79.48	27.44
500	87.46	30.59
600	94.62	33.41
700	101.8	36.26

The present viscosity results for xenon enable us to complete this investigation for the monatomic gases. Figure 4 shows the experimental quantity $\xi(T^*)$ as a function of temperature. It has been computed using our viscosity results and the thermal conductivity data given in a recent compendium.[16] A comparison is provided with the curve computed for the (11-6-8) potential. The theoretical value of $\xi(T^*)$ is, however, only weakly dependent on the model used for its computation,[15] so that Fig. 4 illustrates that the experimental data deviate by as much as 6% from the thermodynamic consistency required by Eq. (6). The situation is thus exactly analogous to that observed for the other monatomic gases.[10,15] We omit a similar plot for argon since it is essentially identical with that given earlier.[10,15]

We attribute these discrepancies to uncertainties in the published thermal conductivity data, and contend that more reliable values may be computed from our viscosity with the aid of Eq. (6). Table VI lists the computed thermal conductivities for Ar and Xe. The uncertainty in the quoted values is the same as that in the viscosities.

V. LONG-TERM REPRODUCIBILITY OF THE INSTRUMENT

Upon completion of our measurements of the viscosity of the monatomic gases at a pressure $P=1$ atm and in the temperature range $T=25$–700°C, it is relevant to examine the long-term reproducibility of our results.

The measurements to be included in the analysis have been performed over a period of four years in the same oscillating-disk instrument.[1,5–7,14] Each determination of the viscosity of a particular gas has been independent, since prior to each series of measurements, the entire instrument has been dismantled, rebuilt, and realigned. In addition, a different suspension strand has been employed for each series, and in most cases, a different separation between the disk and the fixed plates was used; it ranged from 0.1 to 0.4 cm.

First, the reproducibility for each gas has been assessed in terms of the standard deviation of the results from their mean value at each temperature. Since there is no apparent correlation between the deviation and any property of the gases, we have averaged the standard deviations for all the gases to yield a final value for the long-term reproducibility σ_ϕ at each temperature. The latter is listed as a function of temperature in Table VII. We propose to adopt the nominal values as the over-all uncertainty in our reported viscosities.

TABLE VII. The long-term reproducibility of our measurements.

Temperature (°C)	Long-term reproducibility, σ_ϕ(%)	Nominal values of reproducibility, %
25	±0.06	±0.1
100	±0.14	±0.1
200	±0.18	±0.15
300	±0.12	±0.15
400	±0.11	±0.15
500	±0.20	±0.2
600	±0.18	±0.2
700	±0.32	±0.3

TABLE VIII. Best estimates for the viscosity of the monatomic gases at atmospheric pressure.

Temperature (°C)	Viscosity, μP He	Ne	Ar	Kr	Xe
25	198.5_9	317.5_6	226.1_9	253.8_9	230.8_5
100	231.6_2	370.6_0	273.2_4	312.1_9	288.4_4
200	273.4_6	434.6_9	328.4_5	380.5_5	359.0_9
300	312.7_6	494.9_5	378.3_0	442.8_0	423.7_6
400	350.4_2	550.0_4	423.4_8	499.9_5	483.1_6
500	386.0_3	601.9_8	466.2_8	553.3_6	538.4_1
600	421.2_6	651.4_5	505.7_8	603.7_5	587.8_2
700	456.1_5	698.5_8	545.2_3	651.9_5	637.6_9

VI. STANDARD REFERENCE VISCOSITIES FOR THE MONATOMIC GASES

The foregoing analysis enables us to establish best estimates for the viscosity of all the monatomic gases. These are listed in Table VIII; they are the mean values of all our determinations for a particular gas, each of which has been assigned an equal weight. The viscosities refer to a pressure $P=1$ atm, their associated uncertainties are given in Table VII.

Most tabulations of viscosity are based on measurements performed with capillary instruments. The first step in a tabulation is usually a careful assessment of the viscosity of a substance at atmospheric pressure or at zero density and at a standard temperature, say at 25°C. Since the accuracy of a tabular entry can only be judged if measurements performed by different methods are available, it is useful to provide precise reference

TABLE IX. Standard reference values of the viscosity of the monatomic gases, the linear temperature correction and the density correlation.

Gas	Viscosity at 1 atm and 25°C, $\mu(\mu P)$	Zero-density viscosity at 25°C, $\mu_0(\mu P)$		Temperature correction, $(d\mu/dT)_{25°C}$ $(\mu P/°C)$	First viscosity virial, $b_1(\mu P\cdot cm^6/g)$	Second viscosity virial, $b_2(\mu P\cdot cm^6/g^2)$	Reference
		Present work	After Ref. 17				
Helium, He	198.5_9	$198.6_1\pm0.2$	198.82 ± 0.02	0.449	−106.44	···	17
Neon, Ne	317.5_6	$317.5_2\pm0.3$	···	0.720	42.82	···	3
Argon, Ar	226.1_9	$226.0_1\pm0.2$	$226.0_9\pm0.04$	0.640	110.99	···	17
Krypton, Kr	253.8_9	$253.6_0\pm0.3$	···	0.789	81.48	251.1	3
Xenon, Xe	230.8_5	$230.5_1\pm0.2$	···	0.793	61.67	167.1	3

values obtained in an instrument which differs radically from the one most commonly employed. It seems to us that the oscillating-disk viscometer developed in our laboratory has now reached a sufficiently high degree of reliability to merit consideration as a good source of reference data.

Consequently, we present in Table IX our best estimates of the viscosity of the five monatomic gases, at atmospheric pressure and at zero density at 25°C.

For helium and argon the correction to zero-density has been effected with the aid of the correlations provided in Ref. 17, whereas for the remaining gases the viscosity virial coefficients reported in Ref. 3 were used. In the cases of neon and krypton it was necessary to make use of the fact that the excess viscosity,

$$\mu-\mu_0=b_1'\rho+b_2'\rho^2, \tag{7}$$

is independent of temperature over the small temperature range 20–25°C.[3,18] The density correlations employed are included in the tabulation for the sake of completeness. All densities have been computed with the aid of the virial coefficients provided by Dymond and Smith.[19]

The uncertainties in both the viscosity at atmospheric pressure and the viscosity at zero-density are estimated as ±0.1%, whereas the uncertainty in the linear temperature correction factor, $(d\mu/dT)_{25°}$, is one of ±2.5%. Table IX includes the best estimates of the zero-density viscosity of helium and argon deduced from careful measurements performed with a different oscillating disk viscometer in our laboratory.[17] The two sets of values are equivalent within their mutual uncertainty.

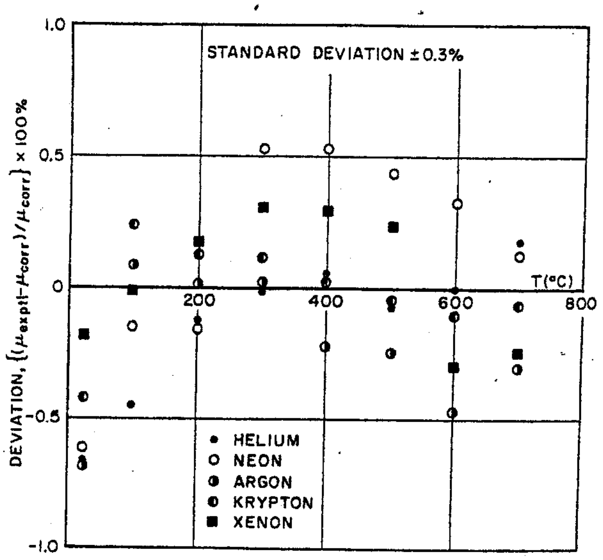

FIG. 5. The deviation of our best estimates of the viscosity of the noble gases from the universal correlation, Eqs. (3) and (4).

VII. UNIVERSAL CORRELATION

The extended law of corresponding states described in Ref. 10 showed that the functional Ω_{22}, defined in Eq. (3), is universal among the monatomic gases. The correlation for viscosity presented in that paper extended over the reduced temperature range

$$T^*=0.7 \text{ to } 190. \tag{8}$$

In order to cover this immense range, it was necessary to include data which have an inferior precision to those from our own measurements.

The results for xenon presented in this paper extend the reduced temperature range of our own measurements. It is now

$$T^*=1.0 \text{ to } 90. \tag{9}$$

Consequently, we have been able to refine the correlation provided in Ref. 10, in the light of our new data, in such a way that it more precisely reproduces our best estimates of the viscosity of the monatomic gases (Table VIII).

We have followed the procedure of Ref. 10 and have been able to correlate the entire body of data listed in Table VIII with the aid of Eqs. (3) and (4). The scaling parameters ϵ and σ for the gases have been based on the values for krypton given in Ref. 10, and their numerical values are listed in Table X together with those reported for the earlier correlation. The small differences apparent between the two sets are an indication of the

TABLE X. Scaling parameters of the corresponding states correlation.

	ϵ/k (K)		σ (Å)	
Gas	Present correlation, Eq. (4)	Correlation of Ref. 10	Present correlation, Eq. (4)	Correlation of Ref. 10
Helium, He	11.29	11.29	2.556	2.556
Neon, Ne	45.58	45.58	2.707	2.707
Argon, Ar	153.61	152.08	3.291	3.291
Krypton, Kr	211.33	211.33	3.518	3.518
Xenon, Xe	285.27	285.27	3.848	3.858

refinement of the present correlation due to the use of only the most precise data. It must be emphasized that these small changes in the parameters do not alter the conclusions of Ref. 10 regarding the establishment of the law of corresponding states for other transport and equilibrium properties of the monatomic gases.

The present correlation, represented by Eqs. (3) and (4) and the parameters of Table X, reproduces our best estimates of the viscosity with a standard deviation of ±0.3%, the maximum deviation being ±0.6%. The deviations from the correlation are displayed in Fig. 5.

The use of the more precise viscosity data at low reduced temperatures (for xenon) causes the correlation presented here to depart from the earlier one[10] by as much as 0.8%; this deviation is, however, commensurate with the mutual uncertainty of the two correlations. The present correlation is naturally to be preferred in its limited reduced temperature range.

The universality of the functional Ω_{22} at once allows us to compute tables of the viscosity and thermal conductivity of the monatomic gases over ranges of temperature inaccessible to direct experiment. We omit such tabulations here since the data can readily be generated by the use of Eqs. (3), (4), and (6) and the parameters of Table X.

APPENDIX

b	Spacing between the disk and the fixed plates
b_1', b_2'	Virial coefficients of viscosity [Eq. (7)]
c_v	Specific heat at constant volume
d	Thickness of the disk
D	Total separation between the fixed plates
$f_k^{(3)}$	Third-order correction factor for the thermal conductivity
$f_\mu^{(3)}$	Third-order correction factor for the viscosity
I	Moment of inertia of suspension system
$\mathbf{k}$	Boltzmann's constant
k	Thermal conductivity
M	Molar mass
P	Pressure
R	Radius of the disk
T	Absolute temperature
T^*	Reduced temperature
T_0	Period of oscillation *in vacuo*
γ	A parameter of the (11-6-8) potential
δ	Boundary layer thickness
Δ_0	Logarithmic decrement *in vacuo*
ϵ	Scaling factor for energy
μ	Viscosity
μ_0	Zero-density viscosity
$\xi(T^*)$	A functional of the intermolecular potential [Eq. (6)]
ρ	Density
σ	Length scaling factor
σ_ϕ	Standard deviation
$\Omega^{(2,2)*}$	First-order collision integral
Ω_{22}	Empirical collision integral

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[1] A. S. Kalelkar and J. Kestin, J. Chem. Phys. 52, 4248 (1970).
[2] G. F. Newell, Z. Angew. Math. Phys. 10, 160 (1959).
[3] J. Kestin and W. Leidenfrost, Physica 25, 1033 (1959).
[4] J. Kestin and H. E. Wang, J. Appl. Mech. 24, 197 (1957).
[5] J. Kestin, W. Wakeham, K. Watanabe, J. Chem. Phys. 53, 3773 (1970).
[6] J. Kestin, S. T. Ro and W. Wakeham, J. Chem. Phys. 56, 4086 (1972), this issue.
[7] J. Kestin, S. T. Ro and W. Wakeham, J. Chem. Phys. 56, 4036 (1972), this issue.
[8] J. H. Whitelaw, J. Sci. Instr. 41, 215 (1964).
[9] J. O. Hirschfelder, C. F. Curtiss, and R. B. Bird, *The Molecular Theory of Gases and Liquids* (Wiley, New York, 1964).
[10] J. Kestin, S. T. Ro, and W. Wakeham, "Extended Law of Corresponding States for the Equilibrium and Transport Properties of the Noble Gases," Physica (to be published).
[11] H. J. M. Hanley and M. Klein, J. Chem. Phys. 53, 4722 (1970).
[12] R. A. Dawe and E. B. Smith, J. Chem. Phys. 52, 693 (1970).
[13] M. Goldblatt and W. E. Wageman, Phys. Fluids 14, 1024 (1971).
[14] R. DiPippo and J. Kestin, *Proceedings of the 4th Symposium on Thermophysical Properties*, edited by J. R. Moszynski, (A.S.M.E., New York, 1968), p. 304.
[15] J. Kestin and W. Wakeham, *Proceedings of the 5th Symposium on Thermophysical Properties*, C. F. Bonilla, Ed., (A.S.M.E., New York, 1970), p. 55.
[16] N. B. Vargaftik, L. I. Filippov, A. A. Tarzimanov and R. P. Yurchak, *Teploprovodnost' gazov i Zhidkostei* (Thermal Conductivity of Gases and Liquids) (Izd. Kom. Standartov, Moscow, 1970).
[17] J. Kestin, E. Paykoç, and J. V. Sengers, Physica, 54, 1 (1971).
[18] J. G. Giddings, J. T. F. Kao and R. Kobayashi, J. Chem. Phys. 45, 578 (1966).
[19] J. H. Dymond and E. B. Smith, *The Virial Coefficients of Gases* (Oxford U. P., Oxford, England, 1969).