Cryogenic recovery panel area required for fast recovery. **DRAFT** D. Shuman 6/17/2011

rev A 9/10/2011 - added cryo recovery time, Xe solid density

Thermal conductivities and heat capacities; from: Lanzanou, arxiv1103.1841v3.pdf, fig 3

$K_{T\_Xe\_80K} := .005 W\cdot cm^{-1}\cdot K^{-1}$

$K_{T\_Xe\_160K} := .002 W\cdot cm^{-1}\cdot K^{-1}$

$C_{p\_Xe\_80K} := 200 J\cdot kg^{-1}\cdot K^{-1}$

$C_{p\_Xe\_160K} := 250 J\cdot kg^{-1}\cdot K^{-1}$ all these values are fairly linear in this range

Heat removal required, assuming we cool a copper panel with LN2:

We need to bring (most of the) Xe from RT to an average temperature of

$T_{c\_avg} := 0.5(77K + 161K)$ $T_{c\_avg} = 119\,K$

$K_{T\_Xe\_avg\_s} := 0.5\left(K_{T\_Xe\_80K} + K_{T\_Xe\_160K}\right)$ $K_{T\_Xe\_avg\_s} = 0.35\,W\cdot m^{-1}\,K^{-1}$

$C_{p\_Xe\_avg\_s} := 0.5\left(C_{p\_Xe\_80K} + C_{p\_Xe\_160K}\right)$ $C_{p\_Xe\_avg\_s} = 225\,J\cdot kg^{-1}\,K^{-1}$

Heat of fusion $H_f := 2.26 kJ\cdot mol^{-1}$ $C_{fus} := \frac{H_f}{M_{a\_Xe}}$ $C_{fus} = 16.618\,kJ\cdot kg^{-1}$

Heat of vaporization $H_v := 12 kJ\cdot mol^{-1}$ $C_{vap} := \frac{H_v}{M_{a\_Xe}}$ $C_{vap} = 88.235\,kJ\cdot kg^{-1}$

Heat capacity, at constant volume, in mass units: $M_{a\_Xe} = 0.136\,kg\,mol^{-1}$

$C_{v\_Xe} := \frac{1.5R}{M_{a\_Xe}}$ $C_{v\_Xe} = 0.092\,kJ\cdot kg^{-1}\cdot K^{-1}$

Total heat removal required (assume 14/15 of all gas needs to be solidified and brought to avg. solid temp to bring vapor press. to 1 atm)

Total Xenon mass $M_{Xe} := 150kg$ $\rho_{Xe\_l} := 3.1 gm\cdot cm^{-3}$ $\rho_{Xe\_s} := 3.64\frac{gm}{cm^3}$ Wikipedia

Total Xenon volume $V_{Xe\_s} := \frac{M_{Xe}}{\rho_{Xe\_s}}$ $V_{Xe\_s} = 0.041\,m^3$

Total heat removal required

$$Q_T := \frac{14}{15}M_{Xe}\cdot\left[C_{p\_Xe\_avg\_s}\cdot\left(161K - T_{c\_avg}\right) + C_{fus} + C_{vap} + C_{v\_Xe}\cdot(293K - 161K)\right]$$

$Q_T = 17.7\,MJ$

Heat flux equation:

$$q := K\cdot A\cdot\left(\frac{d}{dx}T\right)$$

Since dx is equal to thickness, which builds up over time, flux is an inverse function of Xenon thickness. We assume linearity across thickness, x, then, time derivative of total heat transfered, Q is

$\frac{d}{dt}Q := \frac{K\cdot A\cdot \Delta T_s}{x}$ ΔT_s is temperature difference across ice layer

$x := \frac{V}{A}$ $x := \frac{M}{\rho\cdot A}$ $x := \frac{Q}{C\cdot\Delta T_{gs}}\cdot\frac{1}{\rho\cdot A}$ ΔT_{gs} is temp drop from RT to avg ice temp

$$\frac{d}{dt}Q := A^2 \cdot \rho \cdot C \cdot K \cdot \Delta T_s \cdot \Delta T_{gs} \cdot Q^{-1}$$

separating variables:

$$Q \cdot dQ := A^2 \cdot \rho \cdot C \cdot K \cdot \Delta T_s \cdot \Delta T_{gs} \cdot dt$$

integrating both sides:

$$0.5Q^2 + C_1 := A^2 \cdot \rho \cdot C \cdot K \cdot \left(\Delta T_{gs} \cdot \Delta T_s\right) \cdot t + C_2$$

Q=0 for t=0, so constants of integration are zero; then area required to freeze all xenon in time τ:

$$A := \sqrt{\frac{2 \cdot Q^2}{\rho \cdot C \cdot K \cdot \Delta T_s \cdot \Delta T_{gs} \cdot \tau}} \qquad \text{a}$$

Using real values, let average thermal conductivity, and density of solid layer be:

$K_T := K_{T\_Xe\_avg\_s}$ $\quad K_T = 0.35\, W \cdot m^{-1} \cdot K^{-1}$ $\quad \rho_{Xe\_s} = 3.64 \times 10^3 \frac{kg}{m^3}$

Temperature change, from RT to avg ice temp.

$$\Delta T_{gs} := [293 - 0.5 \cdot (161 - 77)]K$$

Average Ice temperature, (assuming cold panel held at 77K)

$$\Delta T_s := 161K - 77K$$

average heat capacity, from RT gas to avg. final state:

$C_T := \frac{Q_T}{M_{Xe} \cdot \Delta T_{gs}}$ $\quad C_T = 470\, J \cdot kg^{-1} \cdot K^{-1}$

If we want to lose no more than 10% of gas through a broken feedthrough (see above), we need to bring pressure down in time:

$\tau := 0.1 t_e$ $\quad \tau = 9.304\, s$

Cold panel area required at 77K (throughout entire heat transfer of 17MJ)

$$A_{xe} := \sqrt{\frac{2 \cdot Q_T^{\,2}}{\rho_{Xe\_s} \cdot C_T \cdot K_T \cdot \Delta T_{gs}^{\,2} \cdot \tau}}$$

$$A_{xe} = 42.244\, m^2$$

This is clearly infeasible for a panel, but maybe feasible using cooled charcoal or molecular sieve. However, we need a given heat capacity of Q_T for a small temp rise in the cold panel or some way to conduct the heat

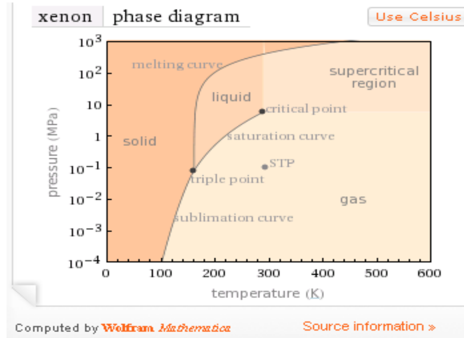
Recovery time for cold panel for given area

Total Xe in system; solid volume:

$M_{Xe} = 150\, kg$ $\quad V_{Xe\_s} = 0.041\, m^3$

If we chill all sides of a recovery vessel, ice may, at some point, build from the (inside) surface inward, choking off the deposition area. So, to maintain 1D ice buildup, as the formula above assumes, we imagine a vessel with a chilled bottom surface and sides insulated (heat flow only in vertical direction). This assumes that ice will build up at its nominal density of 3 gms/cm^3, going through a liquid phase first. This appears to be the case for the majority of Xenon (from phase diagram chart below), however the last bar of pressure may sublimate; we should investigate further. So we will need to scale by some experimentally determined factor. Assume a cylindrical recovery volume having a length to diameter ratio equal.

$$V_{cyl} := \pi r^2 \cdot 2r \qquad r_{rv} := \sqrt[3]{\frac{V_{Xe\_s}}{2\pi}} \qquad r_{rv} = 0.187\,m \qquad A_{rv} := \pi r_{rv}^{\;2}$$



$$\tau_{rv} := \frac{2 \cdot Q_T^{\;2}}{\rho_{Xe\_s} \cdot C_T \cdot K_T \cdot \Delta T_{gs}^{\;2} \cdot A_{rv}^{\;2}} \qquad \tau_{rv} = 15.9\,day$$

One could possibly add vertical fins in the vessel, however the density change from liquid to solid is pronounced, and shrinkage may damage them.