Gas velocity and pressure drop though SiPM plate D. Shuman 5/14/12

Gas flow is constricted primarily as it flows between the dice boards. The pressure drop associated with this flow is the primary load transmitted to the support structure, as the support structure has far more open area. Dice boards are assumed here 79mm square, with a 1mm gap between them (spacing = 80mm), and they are ~5mm thick

DB width	spacing	gap		thickness
$w_{db} := 79mm$	$s_{db} := 80mm$	$g_{db} := s_{db} - w_{db}$	$g_{db} = 1\,mm$	$t_{db} := 3mm$

Main drift volume dimensions:

$r_{Xe} = 0.53\,m$ $\quad A_{tp} := \pi r_{Xe}^2$ $\quad A_{tp} = 0.882\,m^2$

Area ratio, DB gaps to main volume is:

$$R_a := \left(\frac{g_{db}}{s_{db}}\right)^2 \qquad R_a = 1.563 \times 10^{-4}$$

Gas flow area through dice board gaps:

$A_{dbg} := A_{tp} \cdot R_a$ $\quad A_{dbg} = 1.379\,cm^2$

Gas velocity, through dice board gaps $\quad v_{drift} = 0.274\,\frac{mm}{s}$ (200SLPM@ 15 bar)

$$v_{dbg} := \frac{v_{drift}}{R_a} \qquad v_{dbg} = 1.753\,\frac{m}{s}$$

Hydraulic diameter of dice board gap = 2x gap

$D_{h\_dbg} := 2 \cdot g_{db}$ $\quad D_{h\_dbg} = 2\,mm$ (limit of Dh = (4*area/perimeter))

Reynolds number:

$$Re_{dbg} := \frac{v_{dbg} \cdot D_{h\_dbg}}{\nu_{Xe}} \qquad Re_{dbg} = 1.346 \times 10^4$$

turbulent flow, but is likely still laminar, as duct is short, , flow not yet developed, so use Hagen-Poisseulle flow formula

$$\Delta P := 8 \cdot \frac{\mu \cdot L \cdot Q}{\pi r^4}$$

mass flow rate: $\quad M_{a\_Xe} = 0.136\,\frac{kg}{mol}$

$q_{Xe} := 350SLPM$ $\quad q_{Xe} \cdot M_{a\_Xe} = 0.035\,\frac{kg}{s}$

Volumetric flow rate:

$$dVdt_{Xe} := q_{Xe} \cdot \frac{R \cdot 293K}{15bar} \qquad dVdt_{Xe} = 4.229 \times 10^{-4}\,\frac{m^3}{s}$$

dynamic viscosity

$$\mu_{Xe} := \nu_{Xe} \cdot \rho_{Xe} \qquad \mu_{Xe} = 2.369 \times 10^{-5}\,\frac{kg}{m\,s}$$

Pressure drop across SiPM plane

$$\Delta P_{dbg} := \frac{8\cdot\mu_{Xe}\cdot t_{db}\cdot dVdt_{Xe}}{\pi\left(0.5D_{h\_dbg}\right)^4} \qquad \Delta P_{dbg} = 76.537\,\mathrm{Pa}$$

If there are leaks around the edges, gas will not flow through the plate as needed, and stagnation conditions may be developed, or perhaps a radial dependent flow profile, which would not be desirable. We can use the periphery to install flapper valves which will open in case of fast vent to minimize stress and deflection, which might push the SiPMs into the mesh.

Plate Deflection Estimate:

using formulas from Roark's Formulas for Stress and Strain

$q := \Delta P_{dbg}$ $\qquad a := 53\mathrm{cm}$

moduli of elasticity

$E_{Cu} := 115\mathrm{GPa}$ $\qquad E_{PE} := 1\mathrm{GPa}$ $\qquad E_{pl} := E_{Cu}$ $\qquad t_{pl} := 1\mathrm{cm}$ $\qquad \nu_{pl} := .3$

$$D_{pl} := 0.5\frac{E_{pl}\cdot t_{pl}^3}{12\left(1-\nu_{pl}^2\right)} \qquad D_{pl} = 5.266\times10^3\,\mathrm{N\cdot m}$$

factor of 0.5 to account for missing material

$K_\delta := -.06307$ $\qquad K_M := .206$ $\qquad$ table 31, case 10a simple supports, circ. plate

$$\delta_{cpl} := K_\delta\cdot\frac{q\cdot a^4}{D_{pl}} \qquad M_{cpl} := K_M\cdot q\cdot a^2 \qquad M_{cpl} = 4.429\,\mathrm{N}$$

$$\delta_{cpl} = -0.072\,\mathrm{mm} \qquad \sigma_{pl} := \frac{6M_{cpl}}{t_{pl}^2} \qquad \sigma_{pl} = 0.266\,\mathrm{MPa}$$

Tracking Plane should not deflect more than 0.5mm under this condition, to maintain uniform distance between EL mesh and SiPMs. We may have various pressures and flow rates so we would like to avoid

For **emergency vent condition** (to vacuum chamber) , vent flow (choked flow condition) through vent valve is sized to be:

$$m_{emg} := 25\frac{\mathrm{kg}}{\mathrm{s}}$$

= 10x the max flow through a broken 41 pin feedthrough (this is greater than through a broken PMT window/conduit assembly)

Assume this flow is through a valve on the energy head, and we do not use a secondary vent valve on the tracking head (or it fails to open). Assume there are volumes:

total gas volume	drift volume	head volume	
$V_{total} := 1.6\mathrm{m}^3$	$V_{dv} := 1.1\mathrm{m}^3$	$V_{hd} := 0.5\left(V_{total}-V_{dv}\right)$	$V_{hd} = 0.25\,\mathrm{m}^3$

So volume ratio of gas behind tracking plane to total volume is:

$$R_v := \frac{V_{hd}}{V_{total}} \qquad R_v = 0.156$$

and maximum flow through tracking plane is then:

$$m_{tp\_max} := R_v\cdot m_{emg} \qquad m_{tp\_max} = 3.906\frac{\mathrm{kg}}{\mathrm{s}}$$

$$q_{tp\_max} := \frac{m_{tp\_max}}{M_{a\_Xe}} \qquad q_{tp\_max} = 28.722\frac{\mathrm{mol}}{\mathrm{s}}$$

$$dVdt_{tp\_max} := q_{tp\_max}\cdot\frac{R\cdot 293\mathrm{K}}{15\mathrm{bar}} \qquad dVdt_{tp\_max} = 0.047\frac{\mathrm{m}^3}{\mathrm{s}}$$

$$v_{tp\_max} := \frac{dVdt_{tp\_max}}{A_{dbg}} \qquad v_{tp\_max} = 338.287\frac{\mathrm{m}}{\mathrm{s}}$$

$Re_{tp\_max} := \dfrac{v_{tp\_max} \cdot D_{h\_dbg}}{\nu_{Xe}}$ $\quad Re_{tp\_max} = 2.599 \times 10^{6}$ highly turbulent flow (for tubes)

$\Delta P := 2 \cdot \dfrac{C_f \cdot L \cdot \mu \cdot v^2}{D}$ Darcy formula:

$C_f := .0791 Re^{-.25}$ Blasius formula, smooth tubes

$C_f := .0018 + .152 Re^{-.35}$ Lee formula

$C_{f\_B} := .0791 Re_{tp\_max}^{-.25}$ $\quad C_{f\_B} = 1.97 \times 10^{-3}$

$C_{f\_L} := .0018 + .152 \cdot Re_{tp\_max}^{-.35}$ $\quad C_{f\_L} = 2.664 \times 10^{-3}$

these are in reasonably good agreement so we take the average

$C_f := 0.5 \cdot (C_{f\_B} + C_{f\_L})$

$\Delta P_{tp\_max} := \dfrac{2 \cdot C_f \cdot t_{db} \cdot \rho_{Xe} \cdot v_{tp\_max}^{2}}{g_{db}}$ $\quad \Delta P_{tp\_max} = 1.448\,\text{bar}$

we will need an internal PR valve (both directions) to limit this. This can can be spring loaded flap doors alternately opening in opposing directions mounted in the periphery of the SiPM.

Tracking plane should not deflect more than 5mm under this condition (nor be damaged), to avoid contact with EL mesh. If we add a second vent valve having a maximum (choked) flow rate:

$m_{tp\_max} = 3.906 \dfrac{\text{kg}}{\text{s}}$

then theoretically, there will be no flow across the tracking plane under emergency vent condition. In reality, I would expect the best we would achieve would be ~10x reduction in flow across tracking plane, even if we are careful. This would still give a pressure drop:

$\Delta P_{tp\_2v} := 0.1 \Delta P_{tp\_max}$ $\quad \Delta P_{tp\_2v} = 0.145\,\text{bar}$

which is still quite high for a thin plate; the flapper plates will still be needed. I suggest the opening pressure for these valves should be ~10x the nominal operating pressure

$\Delta P_{dbg} = 76.537\,\text{Pa}$

$\Delta P_{fv} := 10 \cdot \Delta P_{dbg}$ $\quad \Delta P_{fv} = 765.37\,\text{Pa}$ $\quad \Delta P_{fv} = 0.008\,\text{bar}$