Lecture 1: General Background

- What is photonics?
- Optical fields and Maxwell's equations
- The wave equation
- Plane harmonic waves. Phase velocity
- Polarization of light
- Maxwell's equations in matters
- Optical power and energy
- Reflection and transmission at a dielectric interface
- Photon nature of light

References: Photonic Devices, Jia-Ming Liu, Chapter 1
Introduction to Modern Optics, G. R. Fowles, Chapters 1-2
A Student's Guide to Maxwell's Equations, Daniel Fleisch
Applied Electromagnetism, 3rd Ed., Shen and Kong, Chapters 2-4

What is photonics?

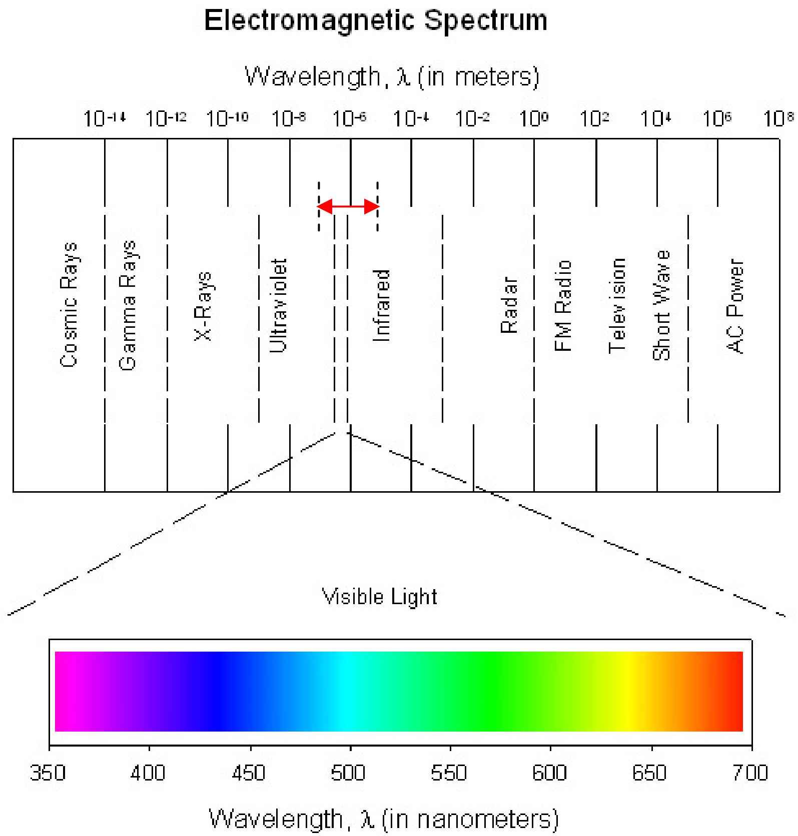
What is photonics?

- **Photonics** is the technology of generating / controlling / detecting light and other forms of radiant energy whose *quantum* unit is the ***photon***.

- ***The uniqueness of photonic devices*** is that both **wave** and **quantum** characteristics of light have to be considered for the function and applications of these devices.

- The photon nature (***quantum mechanics***) of light is important in the operation of photonic devices for *generation*, *amplification*, *frequency conversion*, or *detection of light*, while the wave nature (***Maxwell's equations***) is important in the operation of *all* photonic devices but is particularly so for devices used in *transmission*, *modulation*, or *switching* of light.

What is photonics?



- The spectral range of concern in photonics is usually in a wavelength range between ~10 μm (mid-IR) and ~100 nm (deep-UV).

What is photonics?

In free space (i.e. vacuum or air) $\lambda\upsilon = c = 3 \times 10^8$ m/s

e.g. λ = 0.5 μm = 500 nm = 0.5×10^{-6} m, gives $\upsilon = 6 \times 10^{14}$ Hz = 600×10^{12} Hz = 600 THz

Optical carrier frequency ~ 100 THz, which is **5 orders of magnitude larger than microwave carrier frequency of GHz**. Potentially ~THz information can be modulated on a single optical carrier!

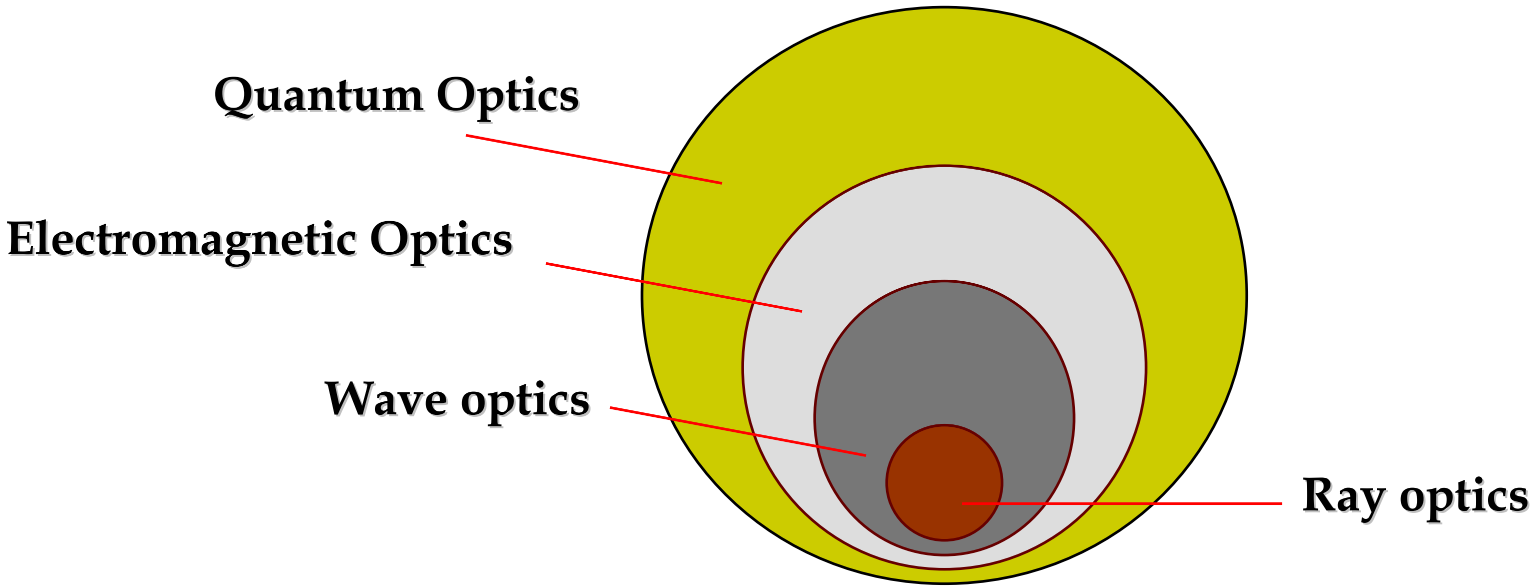
Photonic technologies at a glance

- **Communications** --- fiber optic communications, optical interconnect
- **Computing** --- chip-to-chip optical interconnect, on-chip optical interconnect communications
- **Energy** ("*Green*") --- solid-state lighting, solar
- **Human-Machine interface** --- CCD/CMOS camera, displays, pico-projectors
- **Medicine** --- laser surgery, optical coherence tomography (OCT)
- **Bio** --- optical tweezers, laser-based diagnostics of cells/tissues
- **Nano** --- integrated photonics, sub-diffraction-limited optical microscopy, optical nanolithography
- **Defense ---** laser weapons, bio-aerosols monitoring
- **Sensing ---** fiber sensors, bio-sensing, LIDAR
- **Data Storage** --- CD/DVD/Blu-ray, holography
- **Manufacturing ---** laser-based drilling and cutting
- **Fundamental Science ---** femto-/atto-second science
- **Space Science ---** adaptive optics
- **Entertainment** --- light shows
- **And many more!!**

A Brief Historical Note

- **Beyond the middle ages**:
  –Newton (1642-1726) and Huygens (1629-1695) fight over nature of light
- **18th–19th centuries**
  –Fresnel, Young experimentally observe diffraction, defeat Newton's particle theory
  –Maxwell formulates electro-magnetic equations, Hertz verifies antenna emission principle (1899)
- **20th-21st century**
  –Quantum theory explains wave-particle duality
  –Invention of holography (1948)
  –Invention of laser principle (1954)
  –1st demonstration of laser (1960)
  –Proposal of fiber optic communications (1966)
  –1st demonstration of low-loss optical fibers (1970)
  –Optical applications proliferate **into the 21st century**:
  nonlinear optics, fiber optics, laser-based spectroscopy, computing, communications, fundamental science, medicine, biology, manufacturing, entertainment, … (Let all flowers blossom!)

The nature of light: Models



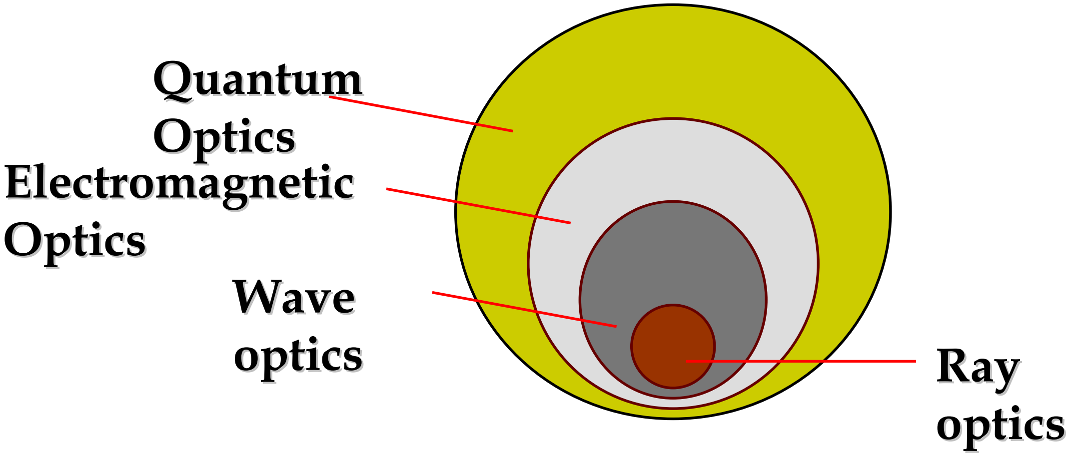
Ray optics ⇒ Limit of wave optics when wavelength is very short compared with simple optical components and systems.

Wave optics ⇒ Scalar approximation of EM optics.

EM Optics ⇒ Most complete treatment of light within the confines of classical optics

Quantum Optics ⇒ Explanation of virtually all optical phenomena

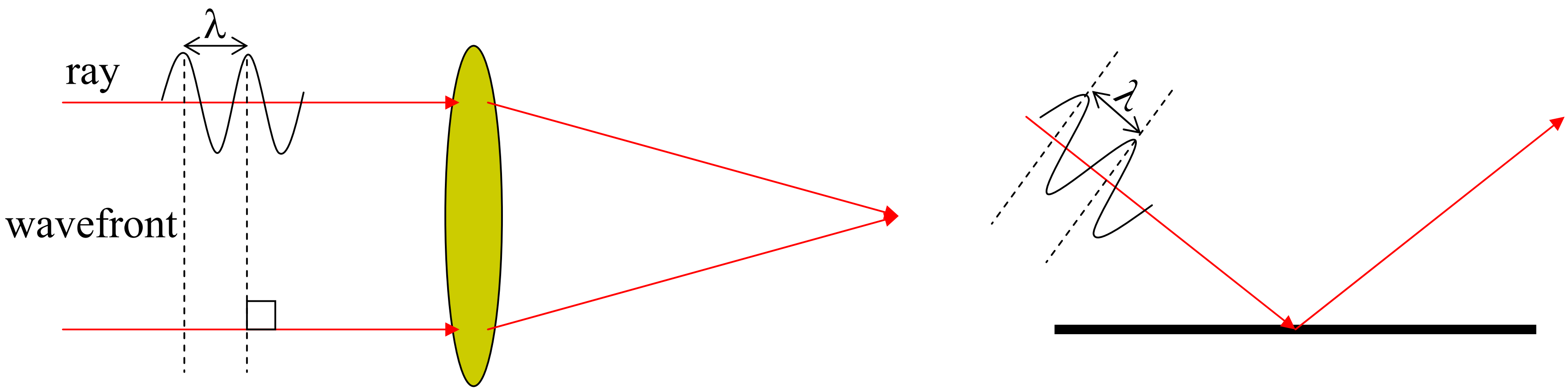
The nature of light



- ***Ray optics:*** propagation of **light rays** through simple optical components and systems.
- ***Wave optics:*** propagations of **light waves** through optical components and systems.
- ***Electromagnetic optics***: description of light waves in terms of **electric and magnetic fields**.
- ***Quantum optics***: emission/absorption of **photons**, which are characteristically quantum mechanical in nature and cannot be explained by classical optics (e.g. lasers, light-emitting diodes, photodiode detectors, solar cells)

Remark on Ray Optics or Geometrical Optics

Wavelength λ << size of the optical component



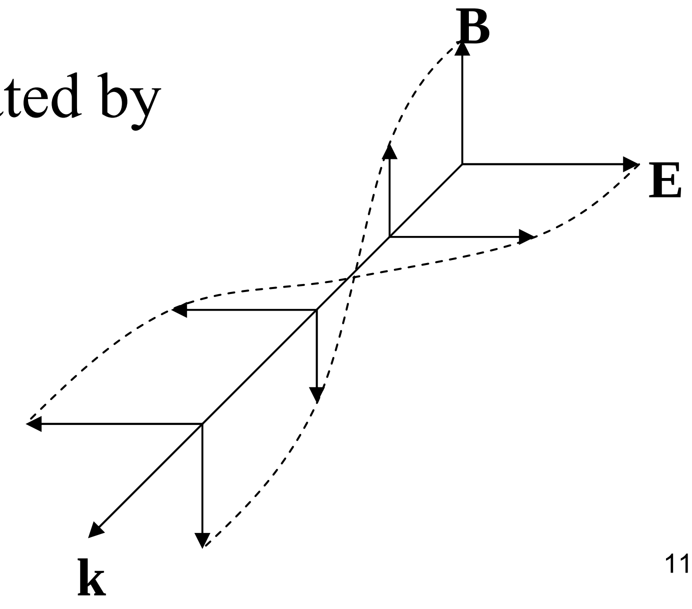
- In many applications of interest the *wavelength λ* of light is *short* compared with the relevant length scales of the optical components or system (e.g. mirrors, prisms, lenses).
- This branch of optics is referred to as **Ray optics** or **Geometrical Optics**, where energy of light is propagated along rays.
- The rays are perpendicular to the *wavefronts*.

Light as electromagnetic waves

- The electromagnetic wave equation derived from ***Maxwell's equations*** show that light and all other electromagnetic waves travel with the same velocity in free space ($c \approx 3 \times 10^8$ m/s).
- Two variables in an electromagnetic wave – the electric and magnetic fields **E** and **B**, both are *vector* quantities, both *transverse* to the direction of propagation, and *mutually perpendicular*, and *mutually coupled*
- In *free space* their magnitudes are related by

$$\boxed{\mathbf{E} = c\mathbf{B}}$$

c is the velocity of light in free space



Optical fields and Maxwell's equations

Electromagnetic field

- The electromagnetic field is generally characterized by the following four field quantities:

Electric field	$\boldsymbol{E}(\mathbf{r}, t)$	V m^{-1}
Magnetic induction	$\boldsymbol{B}(\mathbf{r}, t)$	T or Wb m^{-2}
Electric displacement	$\boldsymbol{D}(\mathbf{r}, t)$	C m^{-2}
Magnetic field	$\boldsymbol{H}(\mathbf{r}, t)$	A m^{-1}

(The units are in SI units)
(coulomb C = A•s)
(weber Wb = V•s)

- ***E*** and ***B*** are <u>fundamental microscopic fields</u>, while ***D*** and ***H*** are *macroscopic* fields that include the *response of the medium*.

They are functions of both position and time. e.g. $\boldsymbol{E}(\mathbf{r}, t) = \mathbf{E}(\mathbf{r})\, e^{-i\omega t}$, where $\mathbf{r} = x\mathbf{e}_\mathbf{x} + y\mathbf{e}_\mathbf{y} + z\mathbf{e}_\mathbf{z}$

Maxwell's Equations in free space

$$\nabla \bullet E = \frac{\rho}{\varepsilon_0}$$ Gauss' law for electric fields

$$\nabla \bullet B = 0$$ Gauss' law for magnetic fields

$$\nabla \times E = -\frac{\partial B}{\partial t}$$ Faraday's law

$$\nabla \times B = \mu_0 \left(J + \varepsilon_0 \frac{\partial E}{\partial t} \right)$$ Ampere-Maxwell law

ρ (Cm^{-3}): total charge density, J (Am^{-2}): total current density

The permittivity and permeability of free space

- The constant proportionality in Gauss' law for electric fields is the *permittivity of free space* (or *vacuum permittivity*).

$$\varepsilon_0 \approx 8.85 \times 10^{-12} \approx 1/36\pi \times 10^{-9} \text{ C/Vm (F/m)}$$

- Gauss' law as written in this form is *general*, and applies to electric fields within *dielectrics* and those in *free space*, provided that you account for *all* of the enclosed charge including charges that are *bound* to the atoms of the material.

- The constant proportionality in the Ampere-Maxwell law is that of the permeability of free space (or *vacuum permeability*).

$$\mu_0 = 4\pi \times 10^{-7} \text{ Vs/Am (H/m)}$$

- The presence of this quantity does *not* mean that the Ampere-Maxwell law applies only to sources and fields in a vacuum. This form of the Ampere-Maxwell law is general, so long as you consider *all* currents (*bound* and free).

Conservation of charge

- Consider the Ampere-Maxwell law again:

$$\nabla \times B = \mu_0 \left(J + \varepsilon_0 \frac{\partial E}{\partial t} \right)$$

- Apply the vector identity:

$$\nabla \bullet (\nabla \times a) \equiv 0$$

- $\nabla \bullet$ both sides of the Ampere-Maxwell law, interchange the time-space derivatives

$$\nabla \bullet J + \varepsilon_0 \frac{\partial}{\partial t} \nabla \bullet E = 0$$

$$\Rightarrow \quad \nabla \bullet J + \frac{\partial \rho}{\partial t} = 0$$

Law of conservation of electric charge

Conservation of charge

- ***Law of conservation of electric charge*** is a direct consequence of Maxwell's equations.
- What is the physical meaning of the conservation law?

$$\nabla \bullet J = -\frac{\partial \rho}{\partial t}$$

Flow of electric current out of a differential volume

Rate of decrease of electric charge in the volume

- This implies that electric charge is conserved – it can neither be created nor be destroyed. Therefore, it is also known as the ***continuity equation***.

Electromagnetic fields in a source-free region

- In a *source-free* region, Maxwell's equations are

$$\nabla \times \boldsymbol{E} = -\partial \boldsymbol{B}/\partial t$$

$$\nabla \times \boldsymbol{B} = \mu_0 \varepsilon_0 \partial \boldsymbol{E}/\partial t$$

$$\nabla \bullet \boldsymbol{E} = 0$$

$$\nabla \bullet \boldsymbol{B} = 0$$

These are equations normally used for optical fields as optical fields are usually *not* generated directly by free currents or free charges.

The wave equation

The wave equation in free space

- Now we are ready to get the ***wave equation from Maxwell's equations***. First, take the *curl* of both sides of the differential form of Faraday's law:

$$\nabla \times (\nabla \times E) = \nabla \times \left(-\frac{\partial B}{\partial t} \right) = -\frac{\partial (\nabla \times B)}{\partial t}$$

- Next we need a vector operator identity which says that the *curl* of the *curl* of any vector field equals the gradient of the divergence of the field minus the *Laplacian* of the field:

$$\nabla \times (\nabla \times A) = \nabla (\nabla \bullet A) - \nabla^2 A$$

where

$$\nabla^2 A = \frac{\partial^2 A_x}{\partial x^2} + \frac{\partial^2 A_y}{\partial y^2} + \frac{\partial^2 A_z}{\partial z^2}$$

This is the Laplacian operator.

The wave equation

- Thus, $$\nabla \times (\nabla \times E) = \nabla(\nabla \bullet E) - \nabla^2 E = -\frac{\partial(\nabla \times B)}{\partial t}$$

- You know the curl of the magnetic field from the differential form of the Ampere-Maxwell law:

$$\nabla \times B = \mu_0 \left(J + \varepsilon_0 \frac{\partial E}{\partial t} \right)$$

- So

$$\nabla \times (\nabla \times E) = \nabla(\nabla \bullet E) - \nabla^2 E = -\frac{\partial \left(\mu_0 (J + \varepsilon_0 \frac{\partial E}{\partial t}) \right)}{\partial t}$$

The wave equation

- Using Gauss' law for electric fields $$\nabla \bullet E = \frac{\rho}{\varepsilon_0}$$
- Gives $$\nabla \times (\nabla \times E) = \nabla(\frac{\rho}{\varepsilon_0}) - \nabla^2 E = -\frac{\partial\left(\mu_0 (J + \varepsilon_0 \frac{\partial E}{\partial t})\right)}{\partial t}$$
- Putting terms containing the electric field on the left side of the equation gives

$$\nabla^2 E - \mu_0 \varepsilon_0 \frac{\partial^2 E}{\partial t^2} = \nabla\left(\frac{\rho}{\varepsilon_0}\right) + \mu_0 \frac{\partial J}{\partial t}$$

- In a *charge*- and *current-free* region, ρ = 0 and J = 0,

$$\nabla^2 E = \mu_0 \varepsilon_0 \frac{\partial^2 E}{\partial t^2}$$

Characteristics of the wave equation

- A similar analysis beginning with the *curl* of both sides of the Ampere-Maxwell law leads to

$$\nabla^2 B = \mu_0 \varepsilon_0 \frac{\partial^2 B}{\partial t^2}$$

- The wave equation is a *linear*, *second-order*, *homogeneous partial differential equation* that describes a field that travels from one location to another --- a *propagating wave*.

  - ***Linear***: The time and space derivatives of the wave function (***E*** or ***B***) appear to the first power and without cross terms
  - ***Second-order***: the highest derivative present is the second derivative
  - ***Homogeneous***: all terms involve the wave function or its derivatives, no forcing or source terms are present
  - ***Partial***: the wave function is a function of multiple variables (<u>space</u> and <u>time</u> in this case)

Phase velocity

- This form of the wave equation does not just tell you that you have a wave --- it provides the *velocity* of propagation as well !
- The general form of the wave equation is (*same for mechanical waves, sound waves*, etc.)

$$\nabla^2 A = \frac{1}{v^2}\frac{\partial^2 A}{\partial t^2}$$

Speed of propagation of the wave
(known as phase velocity)

- For the electric and magnetic fields

$$\frac{1}{v^2} = \mu_0 \varepsilon_0 \rightarrow v = \sqrt{\frac{1}{\mu_0 \varepsilon_0}}$$

Phase velocity in free space

- Recall $\varepsilon_0 \approx 8.854 \times 10^{-12}$ C/Vm $\approx (1/36\pi) \times 10^{-9}$ C/Vm

- And $\mu_o = 4\pi \times 10^{-7}$ Vs/Am

$$(\mu_0\, \varepsilon_0)^{-1/2} = (4\pi \times 10^{-7} \times (1/36\pi) \times 10^{-9})^{-1/2}\, (s^2 m^{-2})^{-1/2} = 3 \times 10^8\ ms^{-1}$$

*It was the agreement of the calculated velocity of propagation with the measured speed of light that caused Maxwell to write,

"*light is an electromagnetic disturbance propagated through the field according to electromagnetic laws.*"

Plane harmonic waves, phase velocity

Waves in one dimension

$$\partial^2\psi/\partial z^2 = (1/v^2)\,\partial^2\psi/\partial t^2 \quad \text{(1-D wave equation)}$$

$$\text{assume } \psi = A\cos[2\pi\,(z/\lambda - t/\tau)]$$

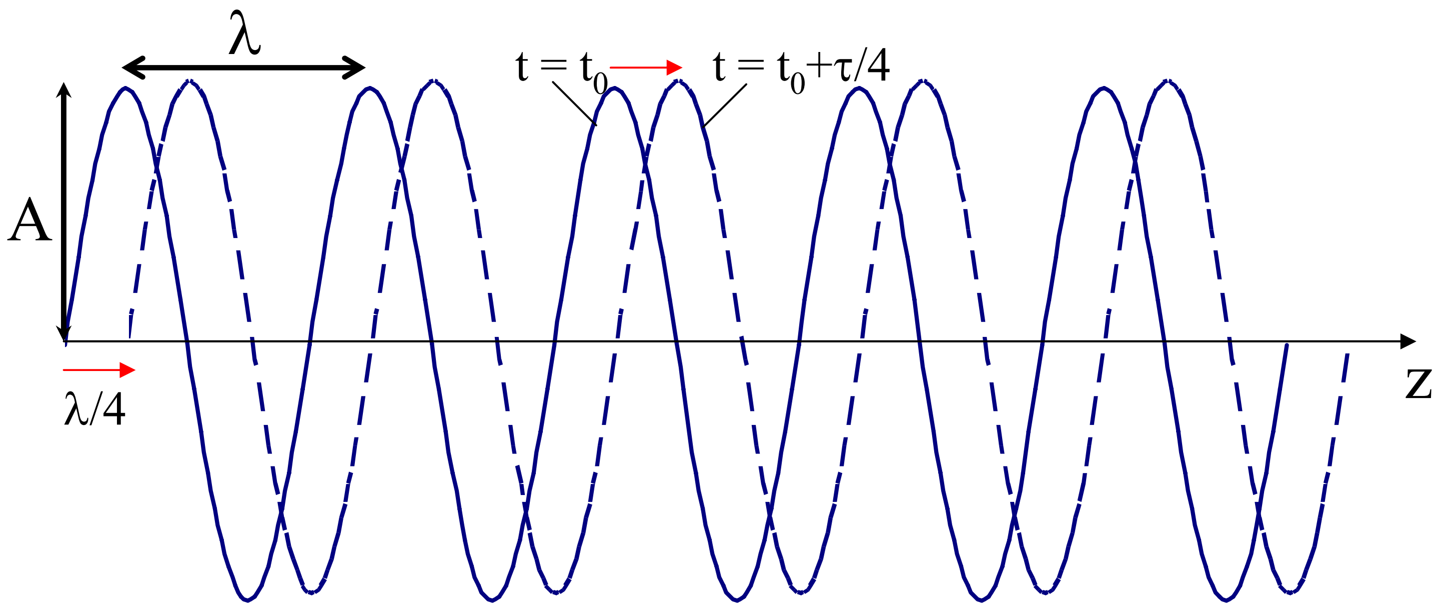
$$\partial^2\psi/\partial t^2 = -(2\pi/\tau)^2 A\cos[2\pi\,(z/\lambda - t/\tau)]$$

$$\partial^2\psi/\partial z^2 = -(2\pi/\lambda)^2 A\cos[2\pi\,(z/\lambda - t/\tau)]$$

$$\Rightarrow \quad v^2(2\pi/\lambda)^2 = (2\pi/\tau)^2$$

$$\Rightarrow \quad v = \lambda/\tau = \lambda\upsilon$$

Plane harmonic waves



$$\psi = A \cos\left[2\pi\left(z/\lambda - t/\tau\right)\right]$$

- *At any <u>point</u> a harmonic wave varies sinusoidally with <u>time</u> t.*
- *At any <u>time</u> a harmonic wave varies sinusoidally with <u>distance</u> z.*

Key parameters of harmonic waves

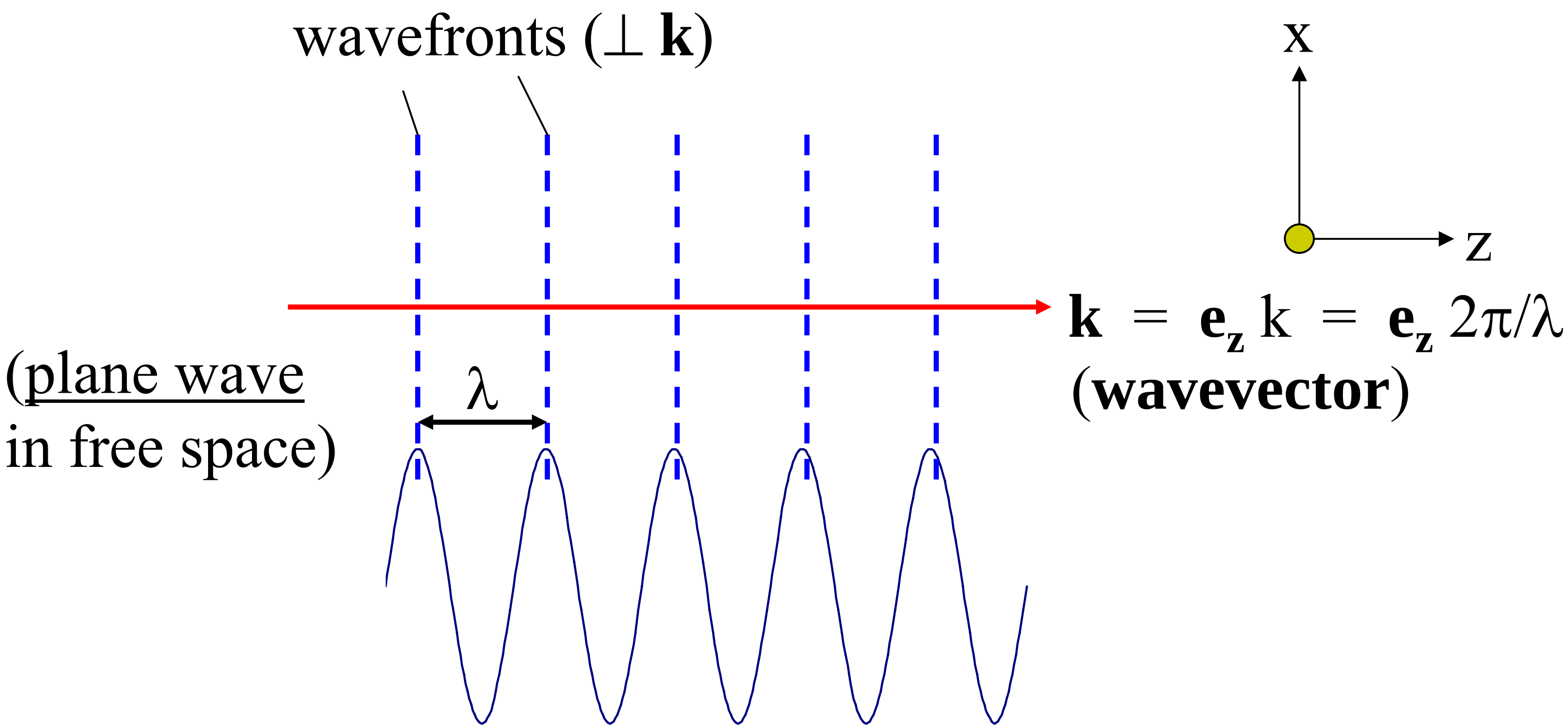
- The *frequency* of oscillation is $\upsilon = 1/\tau$.
- It is often convenient to use an ***angular frequency*** $\omega = 2\pi\upsilon$.
- A ***propagation constant*** or ***wave number***

$$k = 2\pi/\lambda$$

- In terms of k and ω: $\psi = A \cos (kz - \omega t)$
- The vector quantity $\mathbf{k} = (2\pi/\lambda)\mathbf{n}$, where **n** is the unit vector in the direction of **k**, is also termed the ***wavevector***.
- Here $\psi = A \cos (kz - \omega t)$ describes the plane wave is moving in the direction +z, so **k** is pointing in the +**z** direction.

Plane harmonic waves

- $\psi = A \cos(kz - \omega t)$



Wavefronts: surfaces of constant phase

Phase velocity

- For a plane optical wave traveling in the z direction, the electric field has a phase varies with z and t

$$\phi = kz - \omega t$$

- For a point of constant phase on the space- and time-varying field, ϕ = constant and thus $kdz - \omega dt = 0$. If we track this point of constant phase, we find that it is moving with a velocity of

$$v_p = dz/dt = \omega/k \qquad \text{phase velocity}$$

- In *free space*, the *phase velocity* $v_p = c = \omega/k = \upsilon\lambda$

the *propagation constant* $k = \omega/c$

Complex exponentials

- Another powerful way of writing harmonic plane wave solutions of the wave equation is in terms of complex exponentials

$$\psi = A \exp i(kz - \omega t)$$

- The complex exponentials form can vastly simplify the math of *combining waves of different amplitudes and phases* (phasor analysis).

- The Euler identity:

$$\exp i(kz - \omega t) = \cos (kz - \omega t) + i \sin (kz - \omega t)$$

Plane wave as the basic solution

Consider a *plane wave* propagating in *free space* in the **z** direction,

$$\boldsymbol{E} = \boldsymbol{E}_o \exp i(kz - \omega t)$$

1-D wave equation $\partial^2\boldsymbol{E}/\partial z^2 = \mu_0\varepsilon_0\, \partial^2\boldsymbol{E}/\partial t^2$

$$k^2\,\boldsymbol{E} = \mu_0\varepsilon_0\,\omega^2\,\boldsymbol{E}$$

$$k^2/\omega^2 = \mu_0\varepsilon_0$$

$$k^2/\omega^2 = (2\pi/\lambda)^2 / (2\pi\upsilon)^2 = 1/(\lambda\upsilon)^2 = 1/c^2 = \mu_0\varepsilon_0$$

- Consider the complex exponential expression for a plane harmonic wave in three dimensions

$$\exp i\left(k \bullet r - \omega t\right)$$

- Taking the time derivative

$$\frac{\partial}{\partial t}\exp i\left(k \bullet r - \omega t\right) = -i\omega \exp i\left(k \bullet r - \omega t\right)$$

- Taking the partial derivative with respect to one of the space variables, say x

$$\frac{\partial}{\partial x}\exp i\left(k \bullet r - \omega t\right) = \frac{\partial}{\partial x}\exp i\left(k_x x + k_y y + k_z z - \omega t\right)$$

$$= ik_x \exp i\left(k \bullet r - \omega t\right)$$

- Hence on application of the del operator

$$\nabla = \hat{x}\frac{\partial}{\partial x} + \hat{y}\frac{\partial}{\partial y} + \hat{z}\frac{\partial}{\partial z}$$

- It follows that

$$\nabla \exp i(k \bullet r - \omega t) = ik \exp i(k \bullet r - \omega t)$$

- Thus we have the following operator relations

$$\frac{\partial}{\partial t} \rightarrow -i\omega \qquad\qquad \nabla \rightarrow ik$$

which are valid for plane harmonic waves

Maxwell's equations for plane harmonic waves

- Using the relations $\nabla \to ik, \frac{\partial}{\partial t} \to -i\omega$

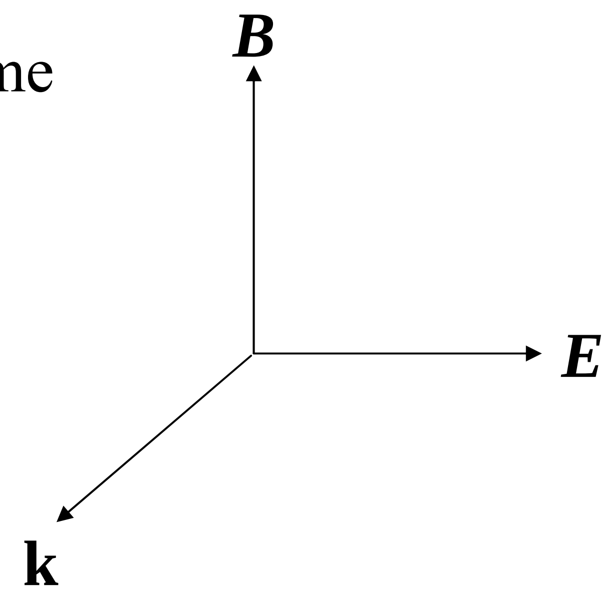
  Maxwell's equations in free space become

$$k \times \mathrm{E} = \omega B$$

$$k \times B = -\mu_0 \varepsilon_0 \omega E$$

$$k \bullet \mathrm{E} = 0$$

$$k \bullet B = 0$$



- Based on Maxwell's equations, we can show that **E** and **B** are both perpendicular to the direction of propagation **k**. Such a wave is called a ***transverse wave***. Furthermore, **E** and **B** are mutually perpendicular – **E**, **B**, and **k** form a mutually orthogonal triad.

Maxwell's equations for plane harmonic waves

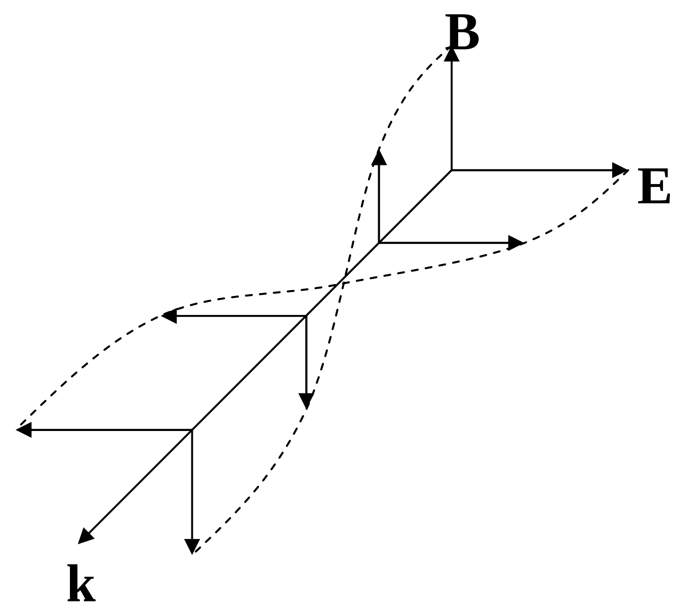
- We can also write

$$k \times \mathrm{E} = \omega B \longrightarrow B = \frac{1}{c} \hat{k} \times \mathrm{E}$$

$$k \times B = -\omega \mu_0 \varepsilon_0 \mathrm{E} \longrightarrow E = cB \times \hat{k}$$

where $\hat{k} = k / |k|$

$$\boxed{\mathbf{E} = c\mathbf{B}}$$



Polarization of light

Polarization

- At a fixed point in space, the ***E*** vector of a time-harmonic electromagnetic wave varies sinusoidally with time.
- The ***polarization*** of the wave is described by the *locus of the tip of the* **E** *vector as time progresses*.
- If the locus is a straight line the wave is said to be ***linearly polarized***.
- It is ***circularly polarized*** if the locus is a circle and ***elliptically polarized*** if the locus is an ellipse.
- An electromagnetic wave, e.g. sunlight or lamplight, may also be randomly polarized. In such cases, the wave is ***unpolarized***. An unpolarized wave can be regarded as a wave containing many linearly polarized waves with their polarization randomly oriented in space.
- A wave can also be ***partially polarized***, such as skylight or light reflected from the surface of an object – i.e. *glare*. A partially polarized wave can be thought of as a mixture of polarized waves and unpolarized waves.

Polarization

- The plane harmonic wave discussed so far is *linearly polarized*.

$$\boldsymbol{E}(z, t) = \boldsymbol{x}\, E_0 \cos(kz - \omega t)$$

- Tracing the tip of the vector ***E*** at any point z shows that the tip always stays on the **x** axis with maximum displacement E_0. => the plane wave is linearly polarized.
- Now consider a plane wave with the following electric-field vector:

$$\boldsymbol{E} = \boldsymbol{x}\, a \cos(\omega t - kz + \phi_a) + \boldsymbol{y}\, b \cos(\omega t - kz + \phi_b)$$

- The ***E*** vector has **x** and **y** components. a and b are real constants.

Linear polarization

- Condition for linear polarization

$$\phi = \phi_b - \phi_a = 0 \text{ or } \pi$$

- When this relation holds between the phases E_x and E_y,

$$E_y = \pm(b/a)E_x$$

- This result is a straight line with slope $\pm(b/a)$. The +ve sign applies to the case $\phi = 0$, and the –ve sign to $\phi = \pi$.

Circular polarization

- Conditions for circular polarization

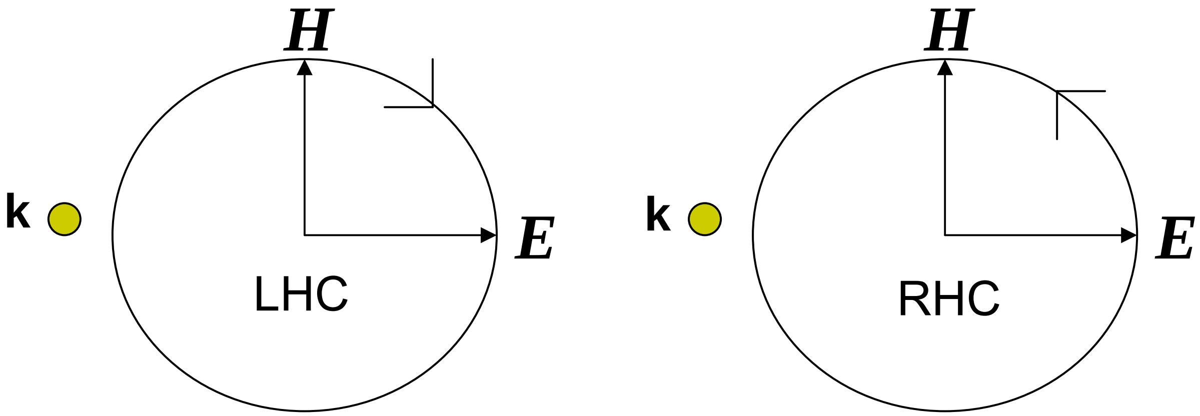
$$\phi = \phi_b - \phi_a = \pm\pi/2 \qquad \text{and} \qquad A = b/a = 1$$

- Consider the case $\phi = \pi/2$ and $A = 1$.

$$E_x = a \cos(\omega t - kz + \phi_a)$$
$$E_y = -a \sin(\omega t - kz + \phi_a)$$

- Elimination of t yields: $E_x{}^2 + E_y{}^2 = a^2$
- This result is a circle in the E_x-E_y plane, and the circle radius is equal to a. The tip of ***E*** moves *clockwise* along the circle as time progresses. If we use left-hand fingers to follow the tip's motion, the thumb will point in the direction of wave propagation. We call this wave ***left-hand circularly polarized***. The wave is ***right-hand circularly polarized*** when $\phi = -\pi/2$ and $A = 1$.

Left or right circularly polarized



- Consider an observer located at some arbitrary point toward which the wave is approaching.

- For convenience, we choose this point at $z = \pi/k$ at $t = 0$.

$\mathbf{E_x}\ (z, t) = \mathbf{-e_x}\ E_0,\ \mathbf{E_y}\ (z, t) = 0$ **E** lies along the –x axis.

Left or right circularly polarized

- At a later time, say $t = \pi/2\omega$, the electric field vector has rotated through 90^o and now lies along the +y axis.

- Thus, as the wave moves toward the observer with increasing time, **E** rotates *clockwise* at an angular frequency ω. It makes one complete rotation as the wave advances through one wavelength. Such a light wave is *left circularly polarized*.

- If we choose the negative sign for ϕ, then the electric field vector is given by

$$\mathbf{E} = E_0 \left[\mathbf{e_x} \cos(\omega t - kz) + \mathbf{e_y} \sin(\omega t - kz)\right]$$

- Now **E** rotates *counterclockwise* and the wave is *right circularly polarized*.

Elliptical polarization

- The wave is ***elliptically polarized*** if it is neither linearly nor circularly polarized.

- E.g. $\phi = -\pi/2$ and $A = b/a = 2$.

$$E_x = a \cos(\omega t - kz + \phi_a)$$
$$E_y = 2a \sin(\omega t - kz + \phi_a)$$

- Eliminating t yields

$$(E_x/a)^2 + (E_y/2a)^2 = 1$$

This result is an ellipse.

- For other ϕ and A values, the wave is generally elliptically polarized.

Elliptical polarization

- For general values of ϕ the wave is elliptically polarized.

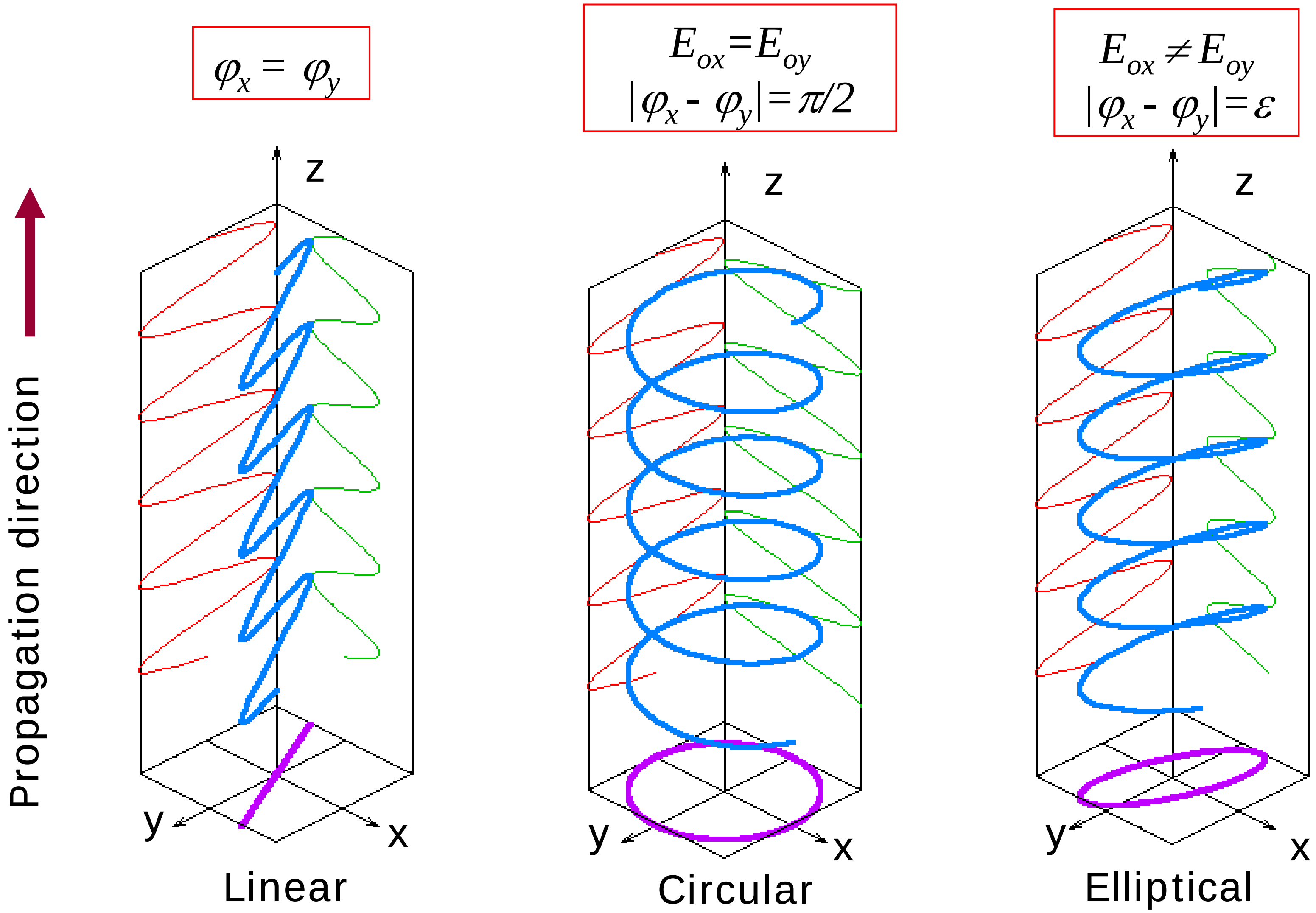
- The resultant field vector **E** will both rotate and change its magnitude as a function of the angular frequency ω. We can show that for a general value of ϕ

$$(E_x/E_{0x})^2 + (E_y/E_{0y})^2 - 2(E_x/E_{0x})\,(E_y/E_{0y})\cos\phi = \sin^2\phi$$

  which is the general equation of an ellipse.

- This ellipse represents the trajectory of the **E** vector = state of polarization (SOP)

Polarization of light wave



Maxwell’s equations in matters

Maxwell's Equations in matters

- Maxwell's equations apply to electric and magnetic fields in *matters* and in free space.
- When you are dealing with fields inside matters, remember the following:
  - *ALL* charge – *bound* and free should be considered
  - *ALL* currents – *bound* and *polarization* and *free* should be considered
- The *bound charge* is accounted for in terms of **electric polarization *P*** in the **displacement field *D***.
- The *bound current* is accounted for in terms of **magnetic polarization *M*** in the **magnetic field strength *H***.

Response of a medium

- The response of a medium to an electromagnetic field generates the *polarization* and the *magnetization*:

  Polarization (electric polarization) $\boldsymbol{P}(\mathbf{r}, t)$ $\mathrm{Cm^{-2}}$

  Magnetization (magnetic polarization) $\boldsymbol{M}(\mathbf{r}, t)$ $\mathrm{Am^{-1}}$

- They are connected to the field quantities through the following *constitutive relations*:

$$\boldsymbol{D}(\mathbf{r}, t) = \varepsilon_0 \boldsymbol{E}(\mathbf{r}, t) + \boldsymbol{P}(\mathbf{r}, t)$$

$$\boldsymbol{B}(\mathbf{r}, t) = \mu_0 \boldsymbol{H}(\mathbf{r}, t) + \mu_0 \boldsymbol{M}(\mathbf{r}, t)$$

where $\varepsilon_0 \approx 1/36\pi \times 10^{-9}$ $\mathrm{Fm^{-1}}$ or $\mathrm{AsV^{-1}m^{-1}}$ is the *electric permittivity of free space* and $\mu_0 = 4\pi \times 10^{-7}$ $\mathrm{Hm^{-1}}$ or $\mathrm{VsA^{-1}m^{-1}}$ is the *magnetic permeability of free space*.

Response of medium

- Polarization and magnetization in a medium are generated by the response of the medium to the electric and magnetic fields.

Therefore, $\boldsymbol{P}(\mathbf{r}, t)$ depends on $\boldsymbol{E}(\mathbf{r}, t)$,

$\boldsymbol{M}(\mathbf{r}, t)$ depends on $\boldsymbol{B}(\mathbf{r}, t)$

- At *optical frequencies* (10^{14} Hz), the magnetization vanishes, $\boldsymbol{M} = 0$.
- *Consequently, for optical fields, the following relation is always true*:

$$\boldsymbol{B}(\mathbf{r}, t) = \mu_0 \boldsymbol{H}(\mathbf{r}, t)$$

Response of medium

- This is not true at low frequencies.

- It is possible to change the properties of a medium through a magnetization induced by a DC or low-frequency magnetic field, leading to the functioning of *magneto-optic* devices.

- Even for magneto-optic devices, magnetization is induced by a DC or low-frequency magnetic field that is separate from the optical fields.

- *No magnetization is induced by the magnetic components of the optical fields.*

Response of medium

- Except for magneto-optic devices, most photonic devices are made of *dielectric* materials that have *zero magnetization at all frequencies*.
- The optical properties of such materials are completely determined by the relation between ***P***(**r**, t) and ***E***(**r**, t).
- This relation is generally characterized by an *electric susceptibility tensor*, χ,

$$P(r,t) = \varepsilon_0 \int_{-\infty}^{\infty} dr' \int_{-\infty}^{t} dt' \chi(r-r',t-t') \cdot E(r',t')$$

$$D(r,t) = \varepsilon_0 E(r,t) + \varepsilon_0 \int_{-\infty}^{\infty} dr' \int_{-\infty}^{t} dt' \chi(r-r',t-t') \cdot E(r',t')$$

$$= \int_{-\infty}^{\infty} dr' \int_{-\infty}^{t} dt' \varepsilon(r-r',t-t') \cdot E(r',t')$$

where ε is the *electric permittivity tensor* of the medium.

Response of medium

χ and ε represent the response of a medium to the optical field and thus completely characterize the macroscopic electromagnetic properties of the medium.

1. Both χ and ε are generally tensors because the vectors ***P*** and ***D*** are, in general, not parallel to vector ***E*** due to material anisotropy. In the case of an isotropic medium, both χ and ε can be reduced to scalars.

2. **The convolution in time** accounts for the fact that the response of a medium to excitation of an electric field is generally *not* instantaneous or local in time and will not vanish for some time after the excitation is over.

   Because time is unidirectional, **causality** exists in physical processes. An earlier excitation can have an effect on the property of a medium at a later time, but not a later excitation on the property of the medium at an earlier time. Therefore, the upper limit in the time integral is t, not infinity.

Response of medium

- **The convolution in space** accounts for the spatial nonlocality of the material response. Excitation of a medium at a location **r**' can result in a change in the property of the medium at another location **r**.

  E.g. The property of a semiconductor at one location can be changed by electric or optical excitation at another location through carrier diffusion.

- Because space is *not* unidirectional, there is no spatial causality, in general, and spatial convolution is integrated over the entire space.

- The temporal nonlocality of the optical response of a medium results in ***frequency dispersion*** of its optical property, while the spatial nonlocality results in ***momentum dispersion.***

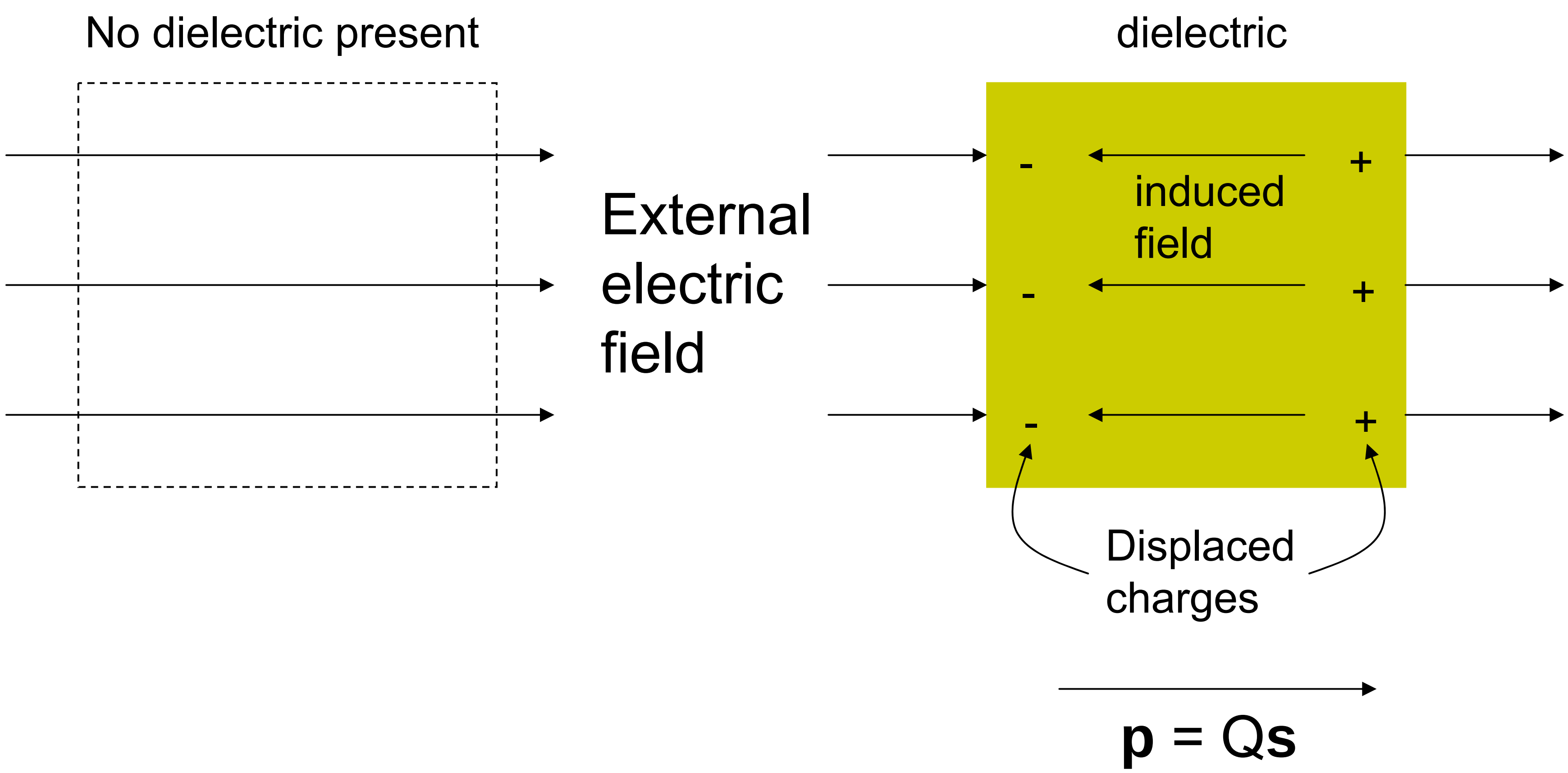
Dipole moment

- Within a dielectric material, positive and negative charges may become slightly displaced when an electric field is applied.
- When a positive charge Q is separated by distance s from an equal negative charge –Q, the electric “dipole moment” is given by

$$\mathbf{p} = Q\mathbf{s}$$

where **s** is a vector directed from the *negative* to the positive charge with magnitude equal to the distance between the charges.

Electric field and dipole moment induced in a dielectric



Electric polarization

- For a dielectric material with N molecules per unit volume, the *dipole moment per unit volume* is

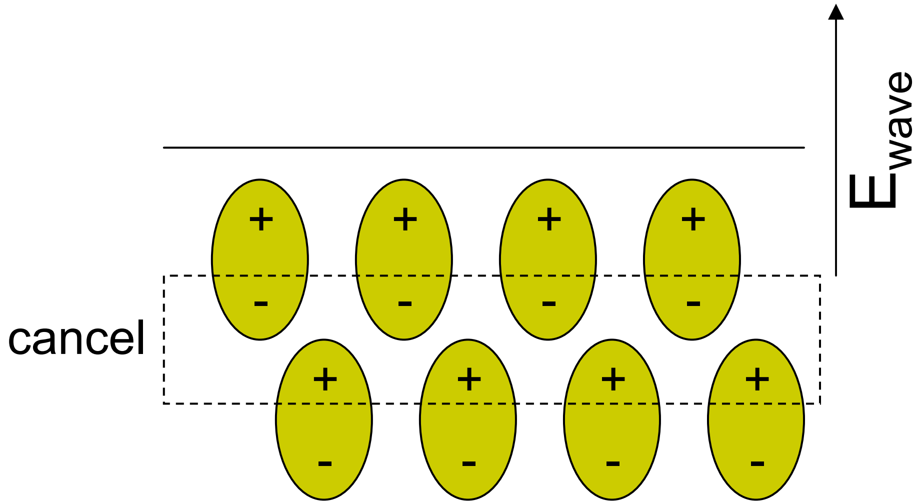
$$\mathbf{P} = N\mathbf{p}$$

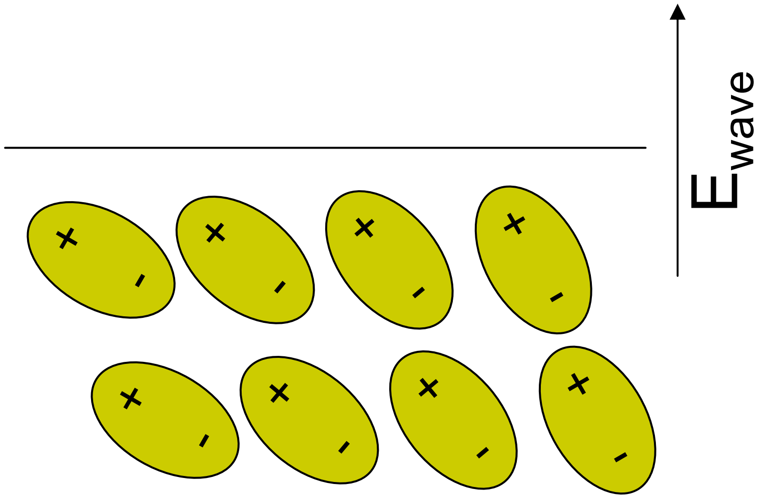
- A quantity which is also called the "electric polarization" of the material.
- *If the polarization is uniform*, bound *charge* appears only on the surface of the material.
- *If the polarization varies from point to point* within the dielectric, there are accumulations of charge within the material, with *volume charge density* given by

$$\rho_b = -\nabla \bullet P$$

where ρ_b represents the volume density of bound charge (*charge that is displaced by the electric field but does not move freely through the material*).

Bound charge





- *The polarization is uniform*, bound *charge* appears only on the surface of the material.
- *The polarization is non-uniform, bound charge appears* within the material.

Gauss' law for electric fields

- In the differential form of Gauss' law, the divergence of the electric field is

$$\nabla \bullet E = \frac{\rho}{\varepsilon_0}$$

where ρ is the ***total*** charge density.

- *Within matter*, the *total* charge density consists of both ***free*** and ***bound*** charge densities:

$$\rho = \underbrace{\rho_f}_{\text{free charge density}} + \underbrace{\rho_b}_{\text{bound charge density}}$$

Gauss' law for electric fields

- Thus, Gauss' law may be written as

$$\nabla \bullet E = \frac{\rho}{\varepsilon_0} = \frac{\rho_f + \rho_b}{\varepsilon_0}$$

- Substituting the negative divergence of the polarization for the bound charge and multiplying through by the permittivity of free space gives

$$\nabla \bullet \varepsilon_0 E = \rho_f + \rho_b = \rho_f - \nabla \bullet P$$

or

$$\nabla \bullet \varepsilon_0 E + \nabla \bullet P = \rho_f$$

The displacement field

- Collecting terms within the divergence operator gives

$$\nabla \bullet (\varepsilon_0 E + P) = \rho_f$$

- In this form of Gauss' law, the term in parentheses is often written as a vector called the **"displacement,"** which is defined as

$$D = \varepsilon_0 E + P$$

=> $$\nabla \bullet D = \rho_f$$

- This is a version of the differential form of Gauss' law that depends only on the density of *free* charge.

Electric susceptibility and relative permittivity

- The relation between $\boldsymbol{E}$ and $\boldsymbol{P}$ is through the *electric susceptibility function* χ.

$$\boldsymbol{P}(\mathbf{r}, t) = \varepsilon_0 \chi\, \boldsymbol{E}(\mathbf{r}, t)$$

$$\boldsymbol{D} = \varepsilon_0\, (1+ \chi)\, \boldsymbol{E}(\mathbf{r}, t) = \varepsilon_0\, \varepsilon_r\, \boldsymbol{E}(\boldsymbol{r}, t) = \varepsilon\, \boldsymbol{E}(\mathbf{r}, t)$$

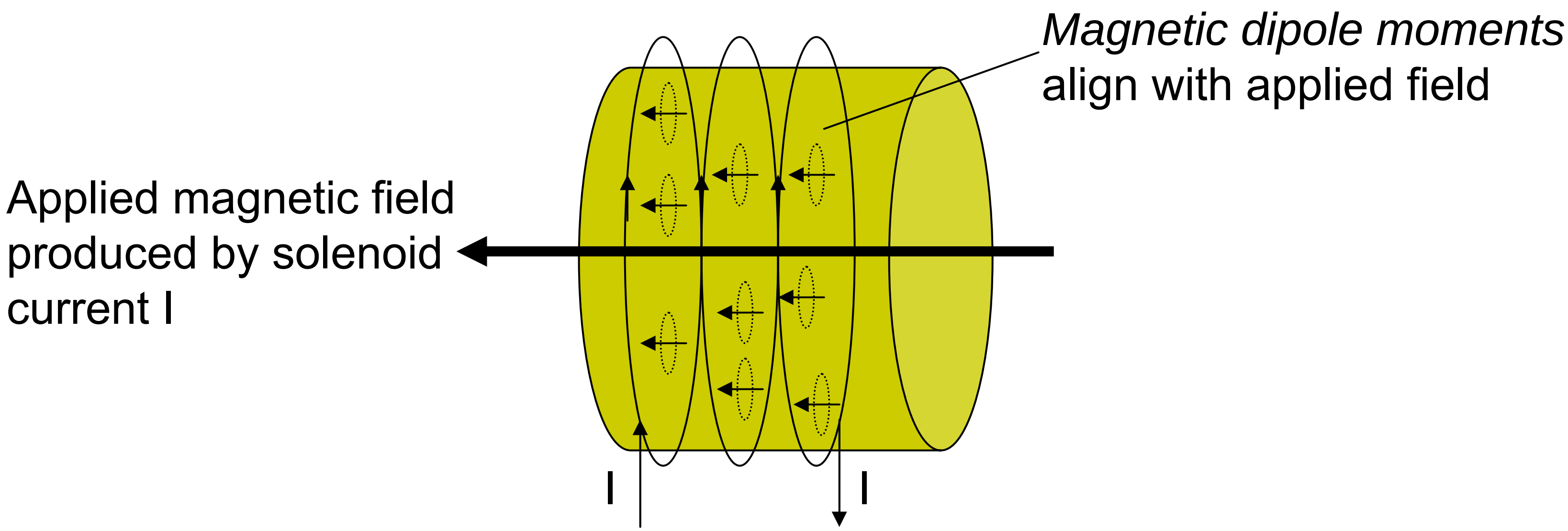
  where the *relative permittivity* (*dielectric constant*) ε_r is defined as $1+\chi$, and the *permittivity of the medium* $\varepsilon = \varepsilon_r\, \varepsilon_0$.

- For *isotropic* medium, χ and ε_r are *scalars* so that ***E*** // ***P*** *and* ***D*** // ***E***. ($\nabla \cdot E = (1/\varepsilon)\, \nabla \cdot D = 0$ in *source-free* media)

- In general, χ and ε_r are <u>second-rank tensors</u> (expressed in 3×3 matrices), in which case the medium they describe is *anisotropic*. (***E*** not // ***P***, ***D*** *not //* ***E***, in general $\nabla \cdot E \neq 0$)

Effect of magnetic materials on magnetic fields

- One interesting difference between the effect of dielectrics on electric fields and the effect of magnetic materials on magnetic fields is that the magnetic field is actually *stronger* than the applied field within many magnetic materials.
- These materials become *magnetized* when exposed to an external magnetic field, and the induced magnetic field is in the same direction as the applied field.



Bound current

- Just as applied electric fields induce polarization (*electric dipole moment per unit volume*) within dielectrics, applied magnetic fields induce “magnetization” (*magnetic dipole moment per unit volume*) within magnetic materials.
- Just as bound electric charges act as the source of additional electric fields within the material, *bound currents* may act as the source of additional magnetic fields.
- The *bound current density* is given by the curl of the magnetization:

$$J_b = \nabla \times M$$

where $\mathbf{J_b}$ is the bound current density and $\boldsymbol{M}$ represents the magnetization of the material.

Polarization current

- Another contribution to the current density within matters comes from the *time rate of change of the polarization*, as *any movement of charge constitutes an electric current.*
- The *polarization current density* is given by

$$J_P = \frac{\partial P}{\partial t}$$

- Thus, the *total* current density includes not only the *free* current density, but the *bound* and *polarization* current densities:

$$J = \underbrace{J_f}_{\text{free}} + \underbrace{J_b}_{\text{bound}} + \underbrace{J_P}_{\text{polarization}}$$

The Ampere-Maxwell law

- Thus, the Ampere-Maxwell law in differential form

$$\nabla \times B = \mu_0 (J_f + J_b + J_P + \varepsilon_0 \frac{\partial E}{\partial t})$$

- Inserting the expressions for the *bound* and *polarization* current and dividing by the permeability of free space

$$\frac{1}{\mu_0} \nabla \times B = J_f + \nabla \times M + \frac{\partial P}{\partial t} + \varepsilon_0 \frac{\partial E}{\partial t}$$

- Gathering curl terms and time-derivative terms gives

$$\nabla \times \frac{B}{\mu_0} - \nabla \times M = J_f + \frac{\partial P}{\partial t} + \frac{\partial (\varepsilon_0 E)}{\partial t}$$

The Ampere-Maxwell law

- Moving the terms inside the curl and derivative operators gives

$$\nabla \times \left(\frac{B}{\mu_0} - M \right) = J_f + \frac{\partial(\varepsilon_0 E + P)}{\partial t}$$

- In this form of the Ampere-Maxwell law, the term

$$H = \frac{B}{\mu_0} - M$$

is often called the **"magnetic field intensity"** or **"magnetic field strength"**

- Thus, the differential form of the Ampere-Maxwell law in terms of ***H***, ***D*** and the free current density is

$$\nabla \times H = J_f + \frac{\partial D}{\partial t}$$

Maxwell's Equations in a medium

$$\nabla \bullet D = \rho_{free}$$ Gauss' law for electric fields

$$\nabla \bullet B = 0$$ Gauss' law for magnetic fields

$$\nabla \times E = -\frac{\partial B}{\partial t}$$ Faraday's law

$$\nabla \times H = J_{free} + \frac{\partial D}{\partial t}$$ Ampere-Maxwell law

ρ_{free} (Cm^{-3}): free charge density, J_{free} (Am^{-2}): free current density

Maxwell's Equations in a medium free of sources

$$\nabla \bullet D = 0$$ Gauss' law for electric fields

$$\nabla \bullet B = 0$$ Gauss' law for magnetic fields

$$\nabla \times E = -\frac{\partial B}{\partial t}$$ Faraday's law

$$\nabla \times H = \frac{\partial D}{\partial t}$$ Ampere-Maxwell law

- These are the equations normally used for optical fields because optical fields are usually *not* generated directly by free currents or free charges.

Wave equation

- Now we are ready to get the wave equation. First, take the curl of Faraday's law and using $\boldsymbol{B} = \mu_0 \boldsymbol{H}$ and $\nabla \times \boldsymbol{H} = \partial \boldsymbol{D}/\partial t$:

$$\nabla \times (\nabla \times E) + \mu_0 \frac{\partial^2 D}{\partial t^2} = 0$$

- Using $\boldsymbol{D} = \varepsilon_0 \boldsymbol{E} + \boldsymbol{P}$,

$$\nabla \times (\nabla \times E) + \mu_0 \varepsilon_0 \frac{\partial^2 E}{\partial t^2} = -\mu_0 \frac{\partial^2 P}{\partial t^2}$$

$$\nabla \times (\nabla \times E) + \frac{1}{c^2} \frac{\partial^2 E}{\partial t^2} = -\mu_0 \frac{\partial^2 P}{\partial t^2}$$

Speed of light in free space

Polarization in a medium drives the evolution of an optical field

$$(\mu_0 \, \varepsilon_0)^{-1/2} = (4\pi \times 10^{-7} \times (1/36\pi) \times 10^{-9})^{-1/2} \, (s^2 m^{-2})^{-1/2} = 3 \times 10^8 \ \mathrm{ms}^{-1}$$

Propagation in an isotropic medium free of sources

- For an isotropic medium, ε(ω) is reduced to a scalar and

$$\nabla \bullet \mathrm{E} = \frac{1}{\varepsilon(\varpi)} \nabla \bullet D = 0$$

- By using the vector identity $\nabla \times \nabla \times = \nabla\nabla \bullet - \nabla^2$

- The wave equation

$$\nabla^2 \mathrm{E} - \mu_0 \varepsilon(\varpi) \frac{\partial^2 \mathrm{E}}{\partial t^2} = 0$$

- Note that for an *anisotropic* medium, the above wave equation is generally not valid because ε(ω) is a tensor and $\nabla \bullet \mathrm{E} \neq 0$

Phase velocity in dielectric media

$$v_p = 1/\sqrt{(\mu_0\varepsilon)} = 1/\sqrt{(\mu_0\varepsilon_0\varepsilon_r)}$$

- The velocity of light in a dielectric medium is therefore

$$v_p = c/\sqrt{\varepsilon_r}$$

where we used the relation $\mu_0\varepsilon_0 = 1/c^2$ and c is the speed of light.

$$v_p = c/n$$

$$n = \sqrt{\varepsilon_r}$$

**The refractive index n is rooted in the material relative permittivity.*

Remark on dispersion

- The index of refraction is in general ***frequency or wavelength dependent***. This is true for *all* transparent optical media.

- The variation of the index of refraction with frequency is called ***dispersion***. The dispersion of glass is responsible for the familiar splitting of light into its component colors by a prism.

- In order to explain the dispersion it is necessary to take into account the actual motion of the electrons in the optical medium through which the light is traveling. We will discuss the theory of dispersion in detail in Lecture 2.

Optical power and energy

Optical power and energy

- By multiplying ***E*** by Ampere-Maxwell law and multiplying ***H*** by Faraday's law

$$E \bullet (\nabla \times H) = E \bullet J + E \bullet \frac{\partial D}{\partial t}$$

$$H \bullet (\nabla \times E) = -H \bullet \frac{\partial B}{\partial t}$$

- Using the vector identity $B \bullet (\nabla \times A) - A \bullet (\nabla \times B) = \nabla \bullet (A \times B)$
- We can combine the above relations

$$-\nabla \bullet (E \times H) = E \bullet J + E \bullet \frac{\partial D}{\partial t} + H \bullet \frac{\partial B}{\partial t}$$

$$E \bullet J = -\nabla \bullet (E \times H) - \frac{\partial}{\partial t}\left(\frac{\varepsilon_0}{2}|E|^2 + \frac{\mu_0}{2}|H|^2\right) - \left(E \bullet \frac{\partial P}{\partial t} + \mu_0 H \bullet \frac{\partial M}{\partial t}\right)$$

Optical power and energy

- Recall that power in an electric circuit is given by voltage times current and has the unit of W = V A (watts = volts × amperes).
- In an electromagnetic field, we find similarly that E•J is the power density that has the unit of V A m^{-3} or W m^{-3}.
- Therefore, the *total power* dissipated by an electromagnetic field in a volume V is

$$\int_V E \bullet J dV$$

$$\int_V E \bullet J dV = -\oint_A E \times H \bullet \hat{n} dA - \frac{\partial}{\partial t}\int_V \left(\frac{\varepsilon_0}{2}|E|^2 + \frac{\mu_0}{2}|H|^2 \right) dV - \int_V \left(E \bullet \frac{\partial P}{\partial t} + \mu_0 H \bullet \frac{\partial M}{\partial t} \right) dV$$

Surface integral over the closed surface A of volume V, ***n*** is the outward-pointing unit normal vector of the surface

(Each term has the unit of power.)

Optical power and energy

- The vector quantity

$$\boldsymbol{S} = \boldsymbol{E} \times \boldsymbol{H}$$

is called the ***Poynting vector*** of the electromagnetic field. It represents the instantaneous magnitude and direction of the *power flow* of the field.

- The scalar quantity $u_0 = \dfrac{\varepsilon_0}{2}\left|E\right|^2 + \dfrac{\mu_0}{2}\left|H\right|^2$

has the unit of *energy per unit volume* and is the *energy density* stored in the *propagating field*. It consists of two components, thus accounting for energies stored in both electric and magnetic fields at any instant of time.

Optical power and energy

- The quantity $$W_p = E \bullet \frac{\partial P}{\partial t}$$

  is the *power density expended by the electromagnetic field on the polarization*. It is the rate of energy transfer from the electromagnetic field to the medium by inducing electric polarization in the medium.

- The quantity $$W_m = \mu_0 H \bullet \frac{\partial M}{\partial t}$$

  is the power density expended by the electromagnetic field on the magnetization.

Optical power and energy

- Hence the relation

$$\int_V E \bullet J dV = -\oint_A E \times H \bullet \hat{n} dA - \frac{\partial}{\partial t}\int_V \left(\frac{\varepsilon_0}{2}|E|^2 + \frac{\mu_0}{2}|H|^2 \right) dV - \int_V \left(E \bullet \frac{\partial P}{\partial t} + \mu_0 H \bullet \frac{\partial M}{\partial t} \right) dV$$

simply states the *law of conservation of energy* in any arbitrary volume element V in the medium: the total energy in the medium equals that in the propagating field plus that in the electric and magnetic polarizations.

- For an optical field, **J** = 0 and **M** = 0,

$$-\oint_A S \bullet \hat{n} dA = \frac{\partial}{\partial t}\int_V u_0 dV + \int_V W_p dV$$

which states that the total power flowing into volume V through its boundary surface A is equal to the rate of increase with time of the energy stored in the propagating fields in V plus the power transferred to the polarization of the medium in this volume.

Energy flow and the Poynting vector

- The *time rate of flow of electromagnetic energy per unit area* is given by the vector ***S***, called the *Poynting vector*,

$$\mathbf{S} = \mathbf{E} \times \mathbf{H}$$

This vector specifies both the *direction* and the *magnitude* of the energy flux. (watts per square meter)

- Consider the case of plane harmonic waves in which the fields are given by the real expressions (*note that **E** and **H** are in phase*)

$$E(z,t) = \hat{x}E_0 \cos(kz - \omega t)$$

$$H(z,t) = \hat{y}H_0 \cos(kz - \omega t)$$

Average Poynting vector

- For the *instantaneous* value (~100 THz) of the Poynting vector:

$$S = E \times H = \hat{z} E_0 H_0 \cos^2(kz - \omega t)$$

- As the *average* value of the cosine squared is ½, then for the average value of the Poynting vector (*detector does not detect so fast*!)

$$\langle S \rangle_{time} = \hat{z} \frac{1}{2} E_0 H_0$$

- As the wavevector **k** is perpendicular to both ***E*** and ***H***, **k** has the same direction as the Poynting vector ***S***.

Irradiance

- An alternative expression for the average Poynting flux is

$$\langle \boldsymbol{S} \rangle = I\,\mathbf{k}/k$$

magnitude of the average Poynting flux

unit vector in the direction of propagation

- I is called the *irradiance* (often termed *intensity*), given by

$$I = \tfrac{1}{2}\,\boldsymbol{E_o H_o} = (n/2Z_o)\,|E_o|^2 \propto |E_o|^2$$

$$[\mathrm{W/cm^2}] = [\mathrm{V^2/(\Omega \cdot cm^2)}] = [1/\Omega]\,[\mathrm{V/cm}]^2$$

- Thus, the rate of flow of energy is proportional to the *square of the amplitude* of the electric field. Z_0 is the *intrinsic impedance* of free space in units of Ω.

Impedance

- We can write in a medium of index n

$$k \times \mathrm{E} = \omega\mu_0 H \longrightarrow H = \frac{n}{Z_0}\hat{k} \times \mathrm{E}$$

$$k \times H = -\omega\varepsilon\mathrm{E} \longrightarrow E = \frac{Z_0}{n} H \times \hat{k}$$

where $\hat{k} = k / |k|$

- $Z_0 = (\mu_0/\varepsilon_0)^{1/2} \approx 120\pi\ \Omega \approx 377\ \Omega$ is the *free-space impedance*.

- The concept of this impedance is analogous to the concept of the impedance of a transmission line.

Propagation in a lossless isotropic medium

- In this case, $\varepsilon(\omega)$ is reduced to a positive real scalar.
- All of the results obtained for free space remain valid, except that ε_0 is replaced by $\varepsilon(\omega)$.
- This change of the electric permittivity from a vacuum to a material is measured by the *relative electric permittivity*, $\varepsilon/\varepsilon_0$, which is a dimensionless quantity also known as the *dielectric constant* of the material.
- Therefore, the *propagation constant* in the medium

$$k = \omega\sqrt{\mu_0 \varepsilon} = \frac{n\omega}{c} = \frac{2\pi n \nu}{c} = \frac{2\pi n}{\lambda}$$

where $n = (\varepsilon/\varepsilon_0)^{1/2}$ is the *index of refraction* or refractive index of the medium.

Lossless medium

- In a medium that has an index of refraction n, the optical frequency is still υ, but the optical wavelength is λ/n, and the speed of light is υ = c/n.
- Because n(ω) in a medium is generally frequency dependent, the speed of light in a medium is also frequency dependent.
- This results in various *dispersive* phenomena such as the separation of different colors by a prism and the broadening or shortening of an optical pulse traveling through the medium.
- We also note that the impedance $Z = Z_0/n$ in a medium.
- The light intensity or irradiance

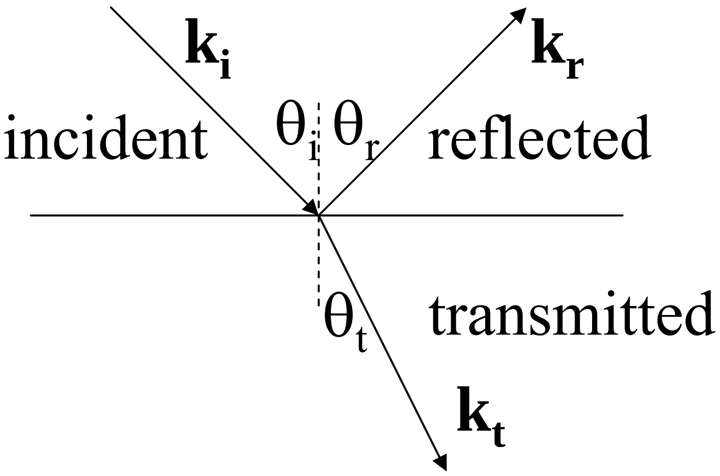
$$I = 2\frac{|\mathrm{E}|^2}{Z} = 2Z\,|\mathrm{H}|^2$$

Reflection and transmission at a dielectric interface

The laws of reflection and refraction

- We now review the phenomena of reflection and refraction of light from the standpoint of electromagnetic theory.

- Consider a plane harmonic wave incident upon a plane boundary separating two different optical media.



*The space-time dependence of these three waves, *aside from constant amplitude factors*, is given by

$\exp i(\mathbf{k}_i \bullet \mathbf{r} - \omega t)$	incident
$\exp i(\mathbf{k}_r \bullet \mathbf{r} - \omega t)$	reflected
$\exp i(\mathbf{k}_t \bullet \mathbf{r} - \omega t)$	transmitted

The law of reflection

- Assume that the interface is at z = 0.
- As **r** varies along the interface, the exponentials change.
- In order that *any constant relation* can exist for *all* points of the boundary and for *all* values of t, it is necessary that the three exponential functions be equal at the boundary.

$$e^{ik_t \bullet r} = e^{ik_i \bullet r} = e^{ik_r \bullet r}$$

- The equality of exponentials can *only* hold so long as

$$ik_t \bullet r = ik_i \bullet r = ik_r \bullet r$$

(For z = 0, **r** is confined to x-y plane)

The law of reflection and Snell's law

- The dot product gives the projection of **k** onto the x-y plane.

$$k_t \bullet r = k_i \bullet r = k_r \bullet r \rightarrow k_t r \sin\theta_t = k_i r \sin\theta_i = k_r r \sin\theta_r$$

$$\Rightarrow \quad k_t \sin\theta_t = k_i \sin\theta_i = k_r \sin\theta_r$$

- Note that $k_i = k_0 n_1 = k_r$

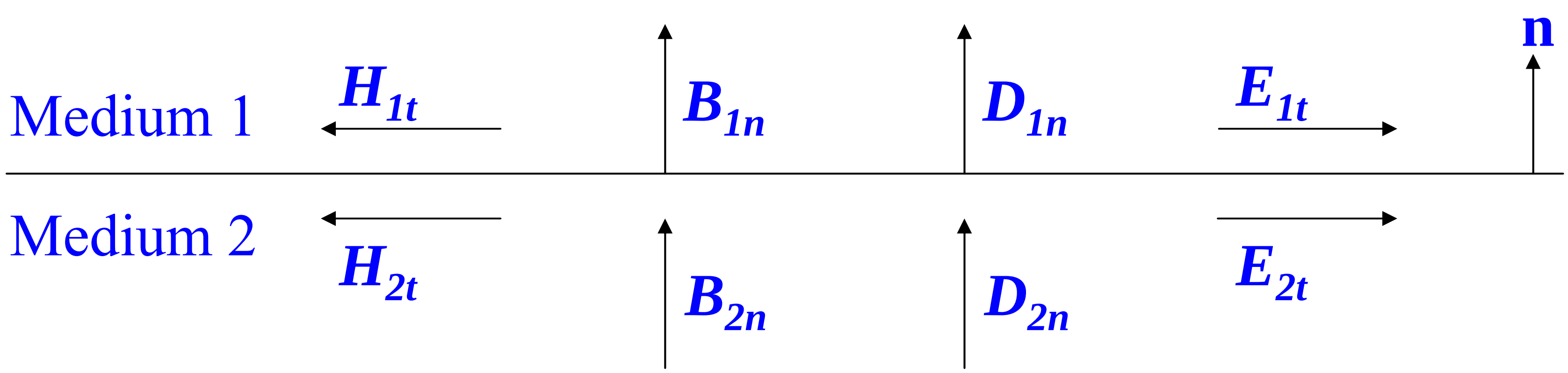
$$k_i \sin\theta_i = k_r \sin\theta_r \rightarrow \theta_i = \theta_r$$ (Law of Reflection)

- Note that $k_t = k_0 n_2$

$$k_t \sin\theta_t = k_i \sin\theta_i \rightarrow n_2 \sin\theta_t = n_1 \sin\theta_i$$ (Snell's Law)

Boundary conditions for the electric and magnetic fields

- We need ***boundary conditions*** when we solve Maxwell's equations for waveguides and reflection coefficients.
- Boundary conditions describe how the electric and magnetic fields behave as they move across interfaces between different materials.
- Here we consider *dielectric* media with *no* free charges or free currents:



Boundary conditions for dielectric media

- *Without* free surface charge or surface currents in the *absence* of magnetic media

$$\varepsilon_2 E_{2n} - \varepsilon_1 E_{1n} = 0$$

$$E_{2t} - E_{1t} = 0$$

$$H_{2n} - H_{1n} = 0$$

$$H_{2t} - H_{1t} = 0$$

Subscript n represents *normal* component to the boundary
Subscript t represents *tangential* component to the boundary

Boundary conditions for dielectric media

- The *tangential* components of ***E*** and ***H*** must be continuous across an interface, while the *normal* components of ***D*** and ***B*** are continuous.

- Because $\boldsymbol{B} = \mu_0 \boldsymbol{H}$ for optical fields, the *tangential* component of ***B*** and the *normal* component of ***H*** are also continuous.

- Consequently, *all of the magnetic field components in an optical field are continuous across a boundary*.

- *Possible discontinuities in an optical field exist only in the normal component of* **E** *or the tangential component of* **D**.

Boundary conditions for dielectric media

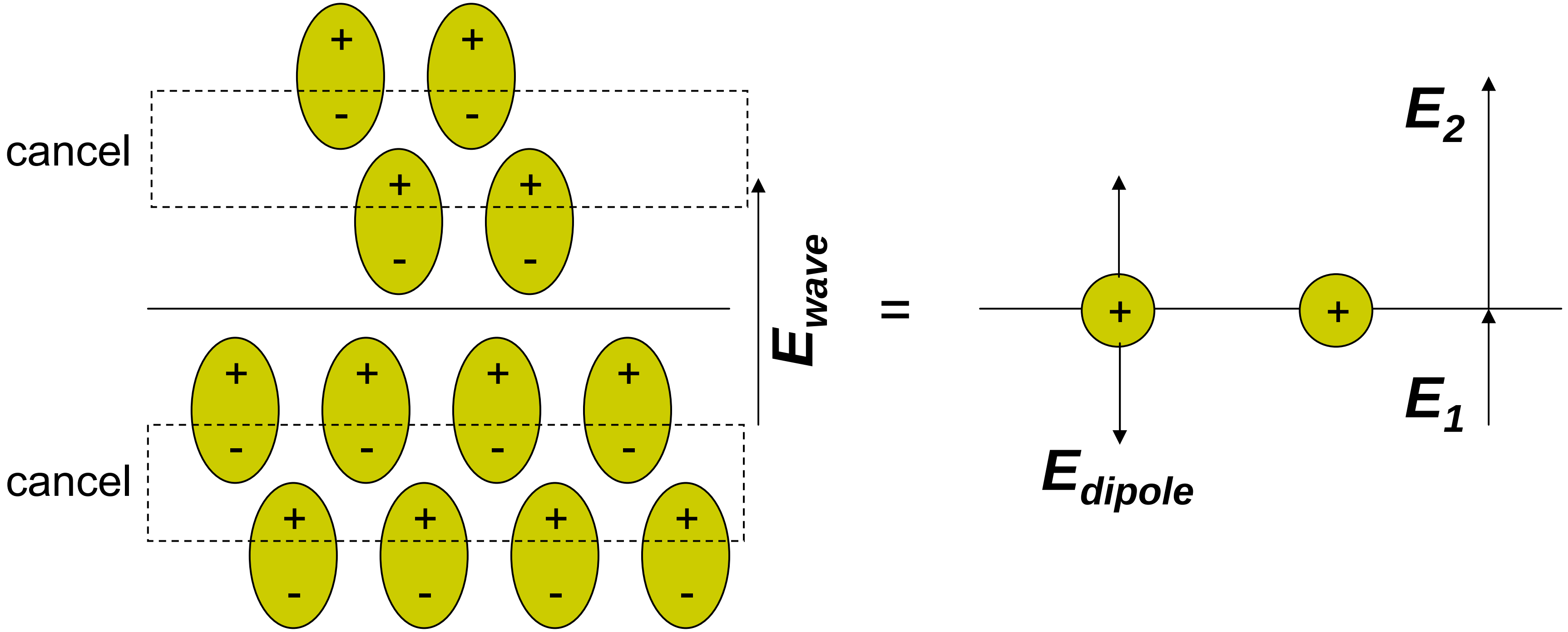
For the electric field

- The *normal* component of the electric field is *discontinuous* across a dielectric interface (*even when there is no free surface charge*).
- The *tangential* component of the electric field must always be continuous across a dielectric interface

For the magnetic field

- The *normal* component of the magnetic field is continuous across a dielectric interface (*for nonmagnetic materials*).
- The *tangential* component of the magnetic field must be continuous across a dielectric interface (*without surface currents*).

Dipole fields produce a discontinuity



- Dipole fields produce a discontinuity in the electric fields on either side of the interface. $(\sqrt{\varepsilon_1} = n_1, \sqrt{\varepsilon_2} = n_2)$

$$E_2 = (\varepsilon_1/\varepsilon_2)E_1 = (n_1/n_2)^2\, E_1$$

Boundary conditions in terms of electric fields

- Restate the last two boundary conditions in terms of the electric field for convenience.
- Recall the relation between the magnitude of the magnetic and electric fields in a dielectric

$$\mathbf{k} \times \boldsymbol{E} = \omega\mu_0 \boldsymbol{H}$$

- The four boundary conditions can be stated as

$$\varepsilon_2 E_{2n} - \varepsilon_1 E_{1n} = 0$$

$$E_{2t} - E_{1t} = 0$$

$$(k_2 \times E_2)_n - (k_1 \times E_1)_n = 0$$

$$(k_2 \times E_2)_t - (k_1 \times E_1)_t = 0$$

Fresnel reflectivity and transmissivity

- Here we derive the ***Fresnel reflectivity and transmissivity*** from the boundary conditions.

- The Fresnel reflectivity and transmissivity apply to electric fields rather than power.

- We must keep in mind that the fields in the boundary conditions represent the ***total*** field on either side of the boundary.

- We have *three* variables but only need to solve for two in terms of the third (the incident field). We therefore require *two* equations in the three variables.
  - We describe the reflected field in terms of the incident field; and the transmitted field in terms of the incident field.

Using the boundary conditions

- Assume region 2 refers to the transmission side of the interface while region 1 refers to the incidence side
- The *total* fields:

$$\boldsymbol{E_1} = \boldsymbol{E_i} + \boldsymbol{E_r}$$
$$\boldsymbol{E_2} = \boldsymbol{E_t}$$

$$\boldsymbol{H_1} = \boldsymbol{H_i} + \boldsymbol{H_r}$$
$$\boldsymbol{H_2} = \boldsymbol{H_t}$$

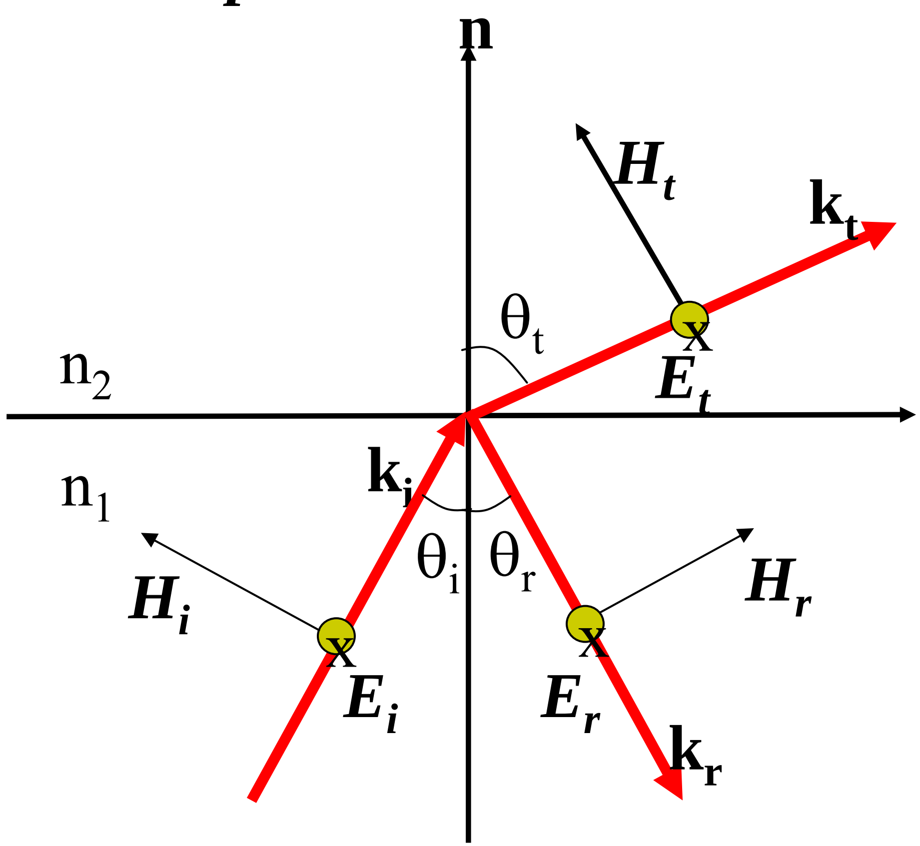
- Using the boundary conditions for the *tangential* components:

$$E_{2t} - E_{1t} = 0$$

$$(k_2 \times E_2)_t - (k_1 \times E_1)_t = 0$$

TE polarization (s-wave)

- The electric field is linearly polarized in a direction *perpendicular* to the plane of incidence, while the magnetic field is polarized to the plane of incidence. This is called ***transverse electric (TE)*** *polarization*. This wave is also called ***s-polarized***.



Fresnel reflectivity and transmissivity for TE fields

- Substitute the total fields to the boundary conditions
- Note that the E-fields are *transverse* to the plane of incidence

$$E_t - (E_i + E_r) = 0$$ (drop the tangential t subscript)

$$(k_t \times E_t)_t - (k_i \times E_i)_t - (k_r \times E_r)_t = 0$$

=> $$E_t k_0 n_2 \cos\theta_t - E_i k_0 n_1 \cos\theta_i + E_r k_0 n_1 \cos\theta_i = 0$$

The reflected k-vector makes an angle of π-θ with respect to the vertically pointing unit vector.

• The *reflection coefficient*, r_{TE}, and the *transmission coefficient*, t_{TE}, of the TE electric field are given by the following ***Fresnel equations***:

$$r_{TE} \equiv E_r/E_i = \frac{n_1 \cos\theta_i - n_2 \cos\theta_t}{n_1 \cos\theta_i + n_2 \cos\theta_t} = \frac{n_1 \cos\theta_i - (n_2{}^2 - n_1{}^2 \sin^2\theta_i)^{1/2}}{n_1 \cos\theta_i + (n_2{}^2 - n_1{}^2 \sin^2\theta_i)^{1/2}}$$

$$t_{TE} \equiv E_t/E_i = \frac{2n_1 \cos\theta_i}{n_1 \cos\theta_i + n_2 \cos\theta_t} = \frac{2n_1 \cos\theta_i}{n_1 \cos\theta_i + (n_2{}^2 - n_1{}^2 \sin^2\theta_i)^{1/2}}$$

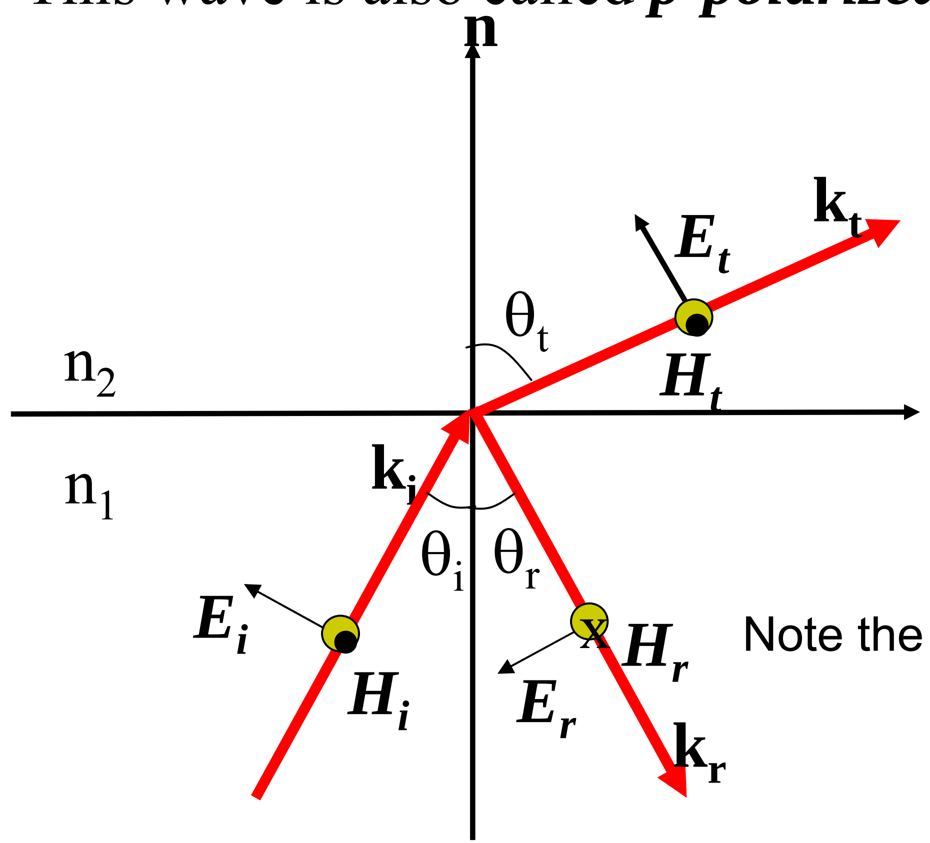
• The intensity *reflectance* and *transmittance*, *R* and *T*, which are also known as *reflectivity* and *transmissivity*, are given by

$$R_{TE} \equiv I_r/I_i = \left| \frac{n_1 \cos\theta_i - n_2 \cos\theta_t}{n_1 \cos\theta_i + n_2 \cos\theta_t} \right|^2$$

$$T_{TE} \equiv I_t/I_i = 1 - R_{TE}$$

TM polarization (p-wave)

- The electric field is linearly polarized in a direction *parallel* to the plane of incidence while the magnetic field is polarized perpendicular to the plane of incidence. This is called ***transverse magnetic (TM)*** *polarization*. This wave is also called ***p-polarized***.



Note the reverse direction of $\mathbf{H}_r$

Fresnel reflectivity and transmissivity for TM fields

- Again, using *tangential components of **E** and **H** are continuous*

$$(k_t \times E_t)_t - (k_i \times E_i)_t - (k_r \times E_r)_t = 0$$

$$(E_t - (E_i + E_r))_t = 0$$

- Note that the ***H*** fields are all *perpendicular* to the unit vector **n** s.t. (*note the field vector directions*)

$$k_0 n_2 E_t - k_0 n_1 E_i + k_0 n_1 E_r = 0$$

$$E_t \cos\theta_t - E_i \cos\theta_i - E_r \cos\theta_i = 0$$

- The *reflection coefficient*, r_{TM}, and the *transmission coefficient*, t_{TM}, of the TM electric field are given by the following ***Fresnel equations***:

$$r_{TM} \equiv E_r/E_i = \frac{-n_2 \cos\theta_i + n_1 \cos\theta_t}{n_2 \cos\theta_i + n_1 \cos\theta_t} = \frac{-n_2^2 \cos\theta_i + n_1(n_2^2 - n_1^2 \sin^2\theta_i)^{1/2}}{n_2^2 \cos\theta_i + n_1(n_2^2 - n_1^2 \sin^2\theta_i)^{1/2}}$$

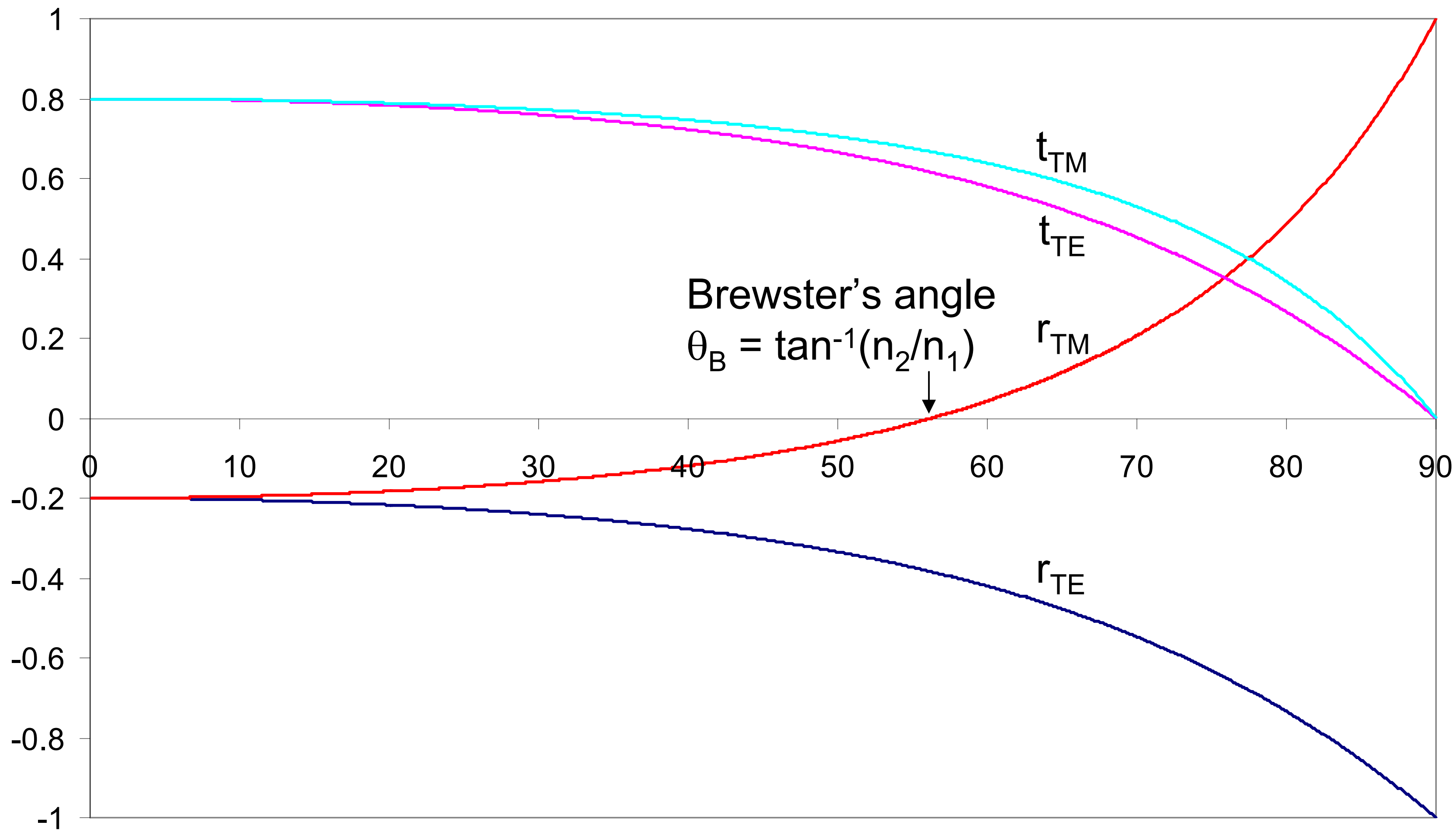
$$t_{TM} \equiv E_t/E_i = \frac{2n_1 \cos\theta_i}{n_2 \cos\theta_i + n_1 \cos\theta_t} = \frac{2n_1 n_2 \cos\theta_i}{n_2^2 \cos\theta_i + n_1(n_2^2 - n_1^2 \sin^2\theta_i)^{1/2}}$$

- The intensity *reflectance* and *transmittance* for TM polarization are given by

$$R_{TM} \equiv I_r/I_i = \left| \frac{-n_2 \cos\theta_i + n_1 \cos\theta_t}{n_2 \cos\theta_i + n_1 \cos\theta_t} \right|^2$$

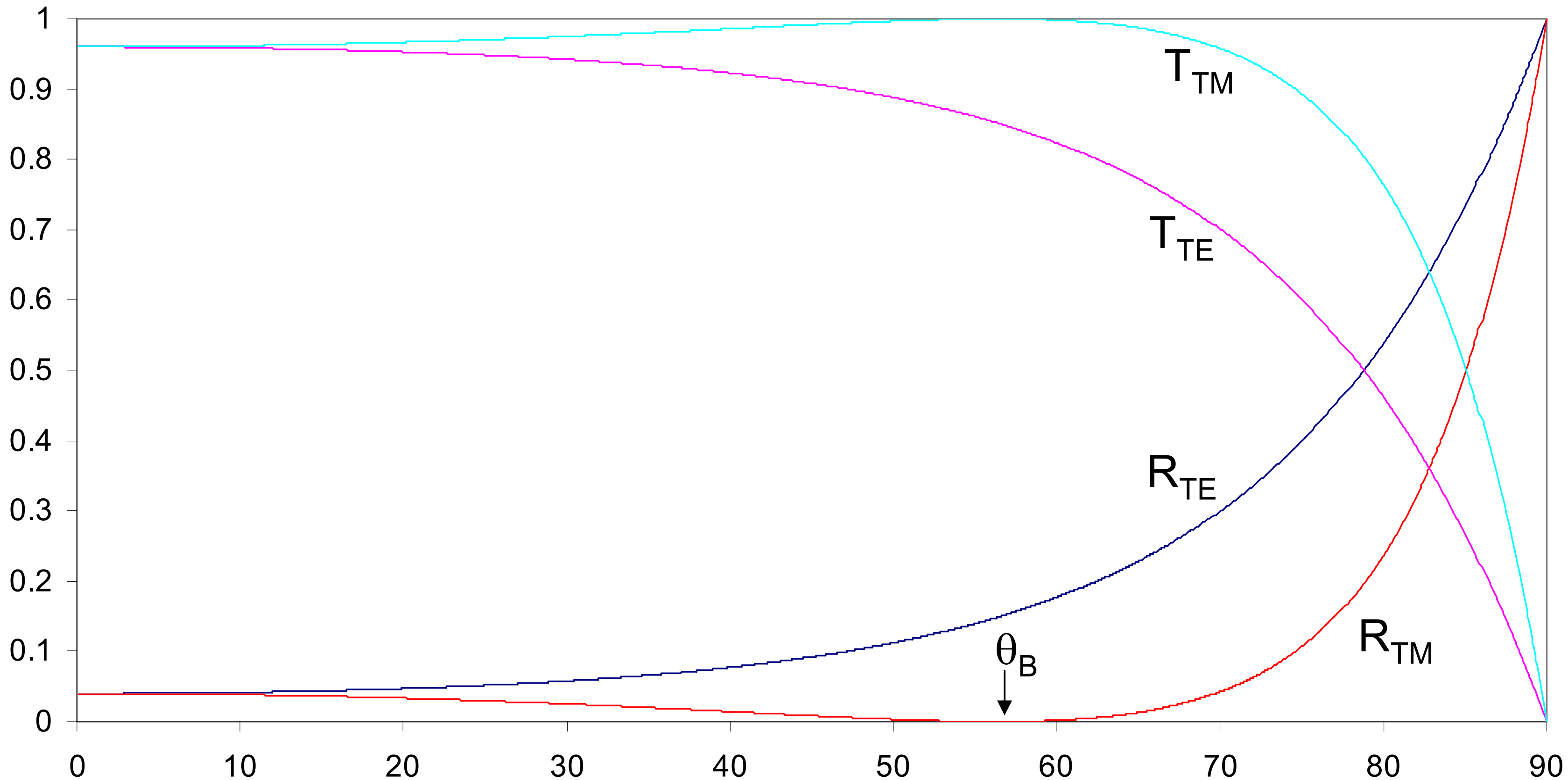
$$T_{TM} \equiv I_t/I_i = 1 - R_{TM}$$

Reflection and transmission coefficients



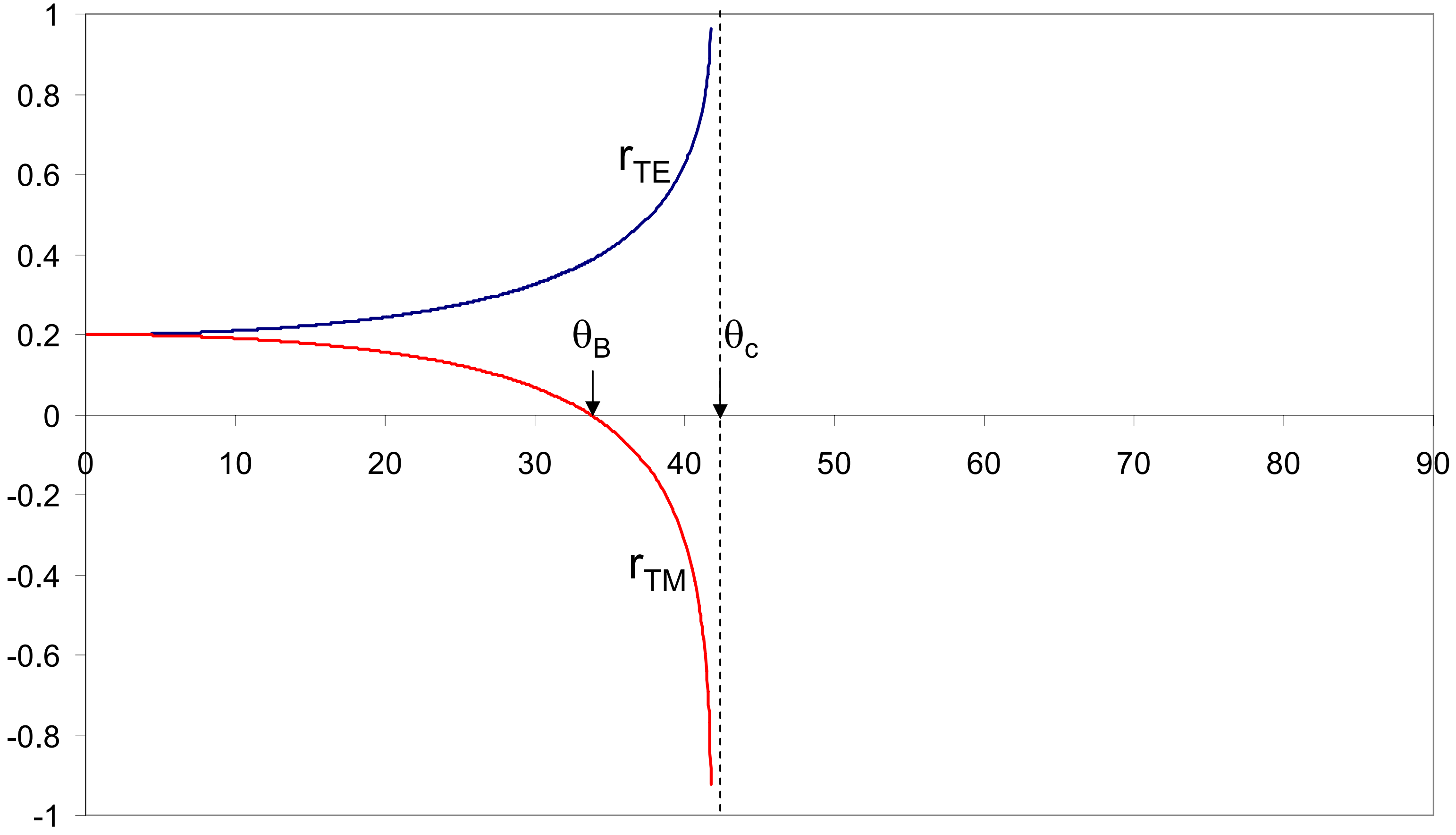
- The reflection and transmission coefficients versus the angle of incidence for $n_1 = 1$ and $n_2 = 1.5$

Reflectivity and transmissivity

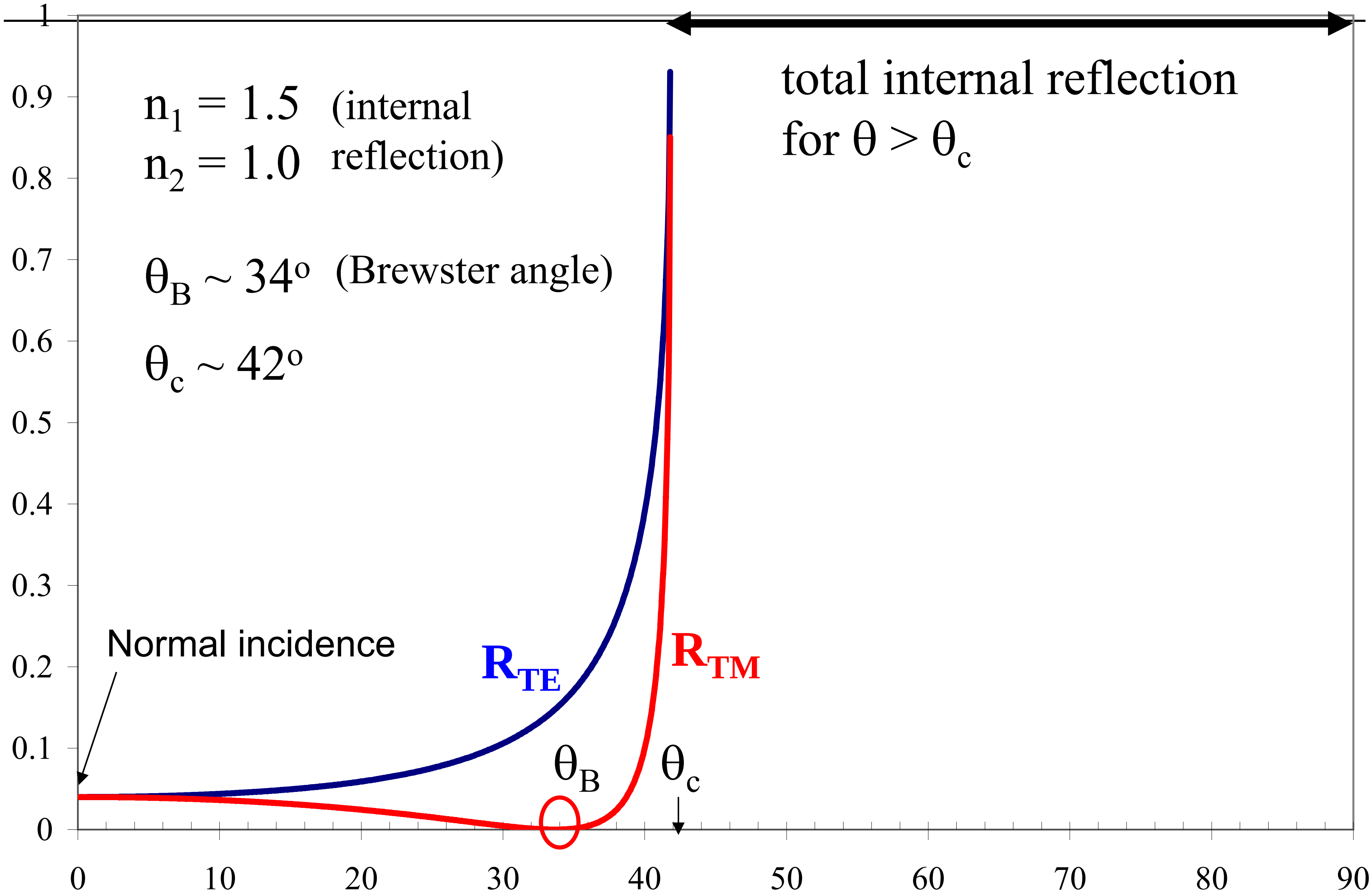


- The reflectivity (reflectance) and transmissivity (transmittance) versus the angle of incidence for $n_1 = 1$ and $n_2 = 1.5$

Internal reflection coefficients



- The reflection coefficients versus the angle of incidence for $n_1 = 1.5$ and $n_2 = 1$



1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
10
20
30
40
50
60
70
80
90
$n_1 = 1.5$ (internal
$n_2 = 1.0$ reflection)
$\theta_B \sim 34^o$ (Brewster angle)
$\theta_c \sim 42^o$
total internal reflection
for $\theta > \theta_c$
Normal incidence
R_{TE}
R_{TM}
θ_B
θ_c

Brewster angle

- For parallel polarization, we see that r = 0 gives

$$n_2 \cos\theta_b = n_1 \cos\theta_t$$

- And the phase matching condition,

$$n_1 \sin\theta_b = n_2 \sin\theta_t$$

- Solving both equations, we find $\theta_t + \theta_b = \pi/2$ and

$$\theta_b = \tan^{-1}(n_2/n_1) \qquad \textbf{\textit{Brewster angle}}$$

- If a wave is *arbitrarily* polarized and is incident on the boundary of the two dielectric media at the Brewster angle, the reflected wave contains only the perpendicular polarization because the parallel-polarized component of the wave is totally transmitted. For this reason, the *Brewster* angle is also called the ***polarization angle***.

Total Internal Reflection

For $\theta_i > \theta_c$ $\quad$ $\sin\theta_i > n_2/n_1$

$$|r_{TE}| = \left| \frac{n_1 \cos\theta_i - i\,(n_1{}^2 \sin^2\theta_i - n_2{}^2)^{1/2}}{n_1 \cos\theta_i + i\,(n_1{}^2 \sin^2\theta_i - n_2{}^2)^{1/2}} \right| = 1$$

$$|r_{TM}| = \left| \frac{-n_2{}^2 \cos\theta_i + i\, n_1(n_1{}^2 \sin^2\theta_i - n_2{}^2)^{1/2}}{n_2{}^2 \cos\theta_i + i\, n_1(n_1{}^2 \sin^2\theta_i - n_2{}^2)^{1/2}} \right| = 1$$

Phase changes in total internal reflection

- In the case of total internal reflection the *complex* values for the coefficients of reflection, given by the ***Fresnel*** coefficients r_{TE} and r_{TM}, imply that there is a *change of phase* which is a function of the angle of incidence.

- As the absolute values of r_{TE} and r_{TM} are both unity, we can write

$$r_{TE} = ae^{-i\alpha} / ae^{i\alpha} = \exp -i\varphi_{TE}$$

$$r_{TM} = -be^{-i\beta} / be^{i\beta} = -\exp -i\varphi_{TM}$$

where φ_{TE} and φ_{TM} are the phase changes for the TE and TM cases, and the complex numbers $ae^{-i\alpha}$ and $-be^{-i\beta}$ represent the numerators in r_{TE} and r_{TM}. Their complex conjugates appear in the denominators.

$$ae^{i\alpha} = n_1 \cos\theta_i + i\,(n_1{}^2 \sin^2\theta_i - n_2{}^2)^{1/2}$$

$$be^{+i\beta} = n_2{}^2 \cos\theta_i + i\, n_1(n_1{}^2 \sin^2\theta_i - n_2{}^2)^{1/2}$$

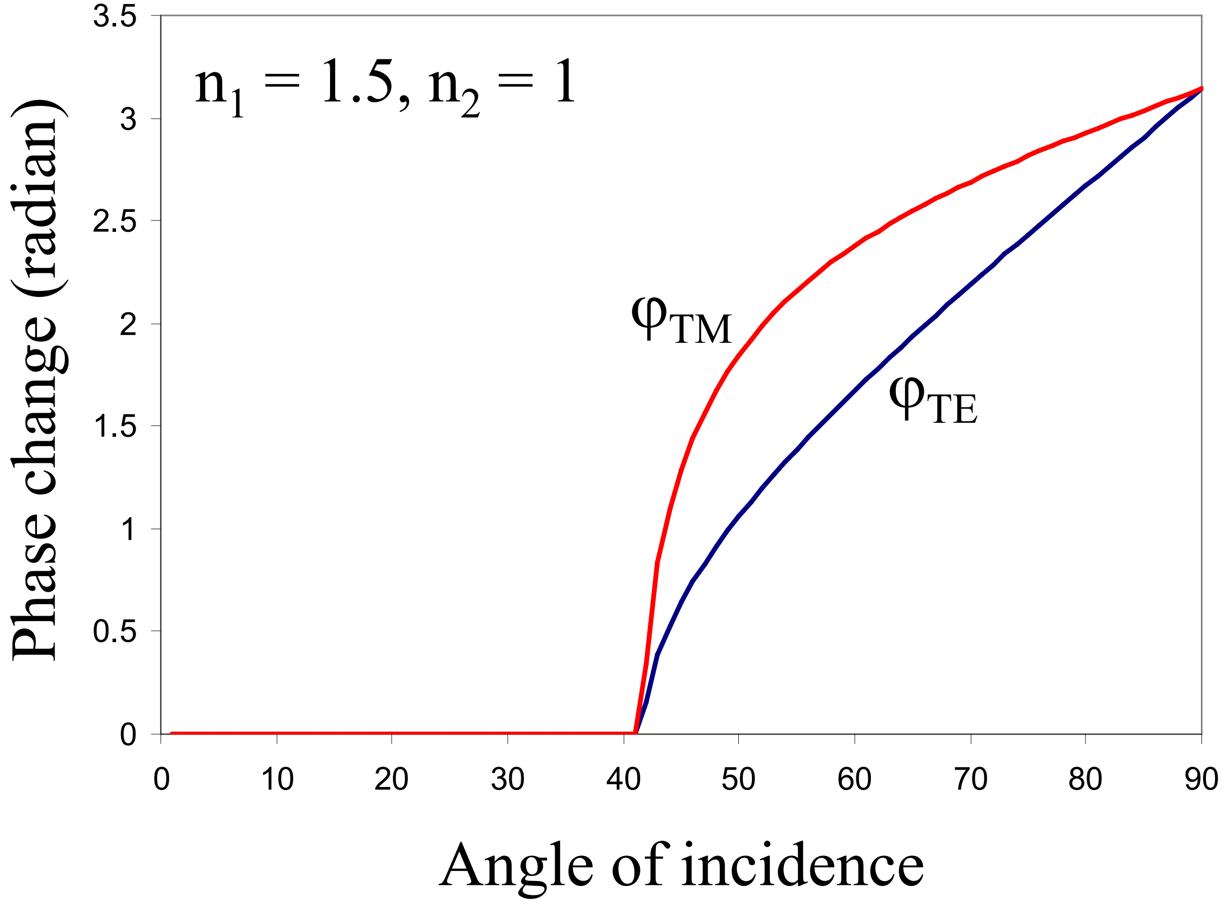
- We see that $\varphi_{TE} = 2\alpha$ and $\varphi_{TM} = 2\beta$. Accordingly, $\tan\alpha = \tan(\varphi_{TE}/2)$ and $\tan\beta = \tan(\varphi_{TM}/2)$.

- We therefore find the following expressions for the phase changes that occur in internal reflection:

$$\tan(\varphi_{TE}/2) = (n_1{}^2 \sin^2\theta_i - n_2{}^2)^{1/2} / (n_1 \cos\theta_i)$$

$$\tan(\varphi_{TM}/2) = n_1(n_1{}^2 \sin^2\theta_i - n_2{}^2)^{1/2} / (n_2{}^2 \cos\theta_i)$$

Total internal reflection phase shifts



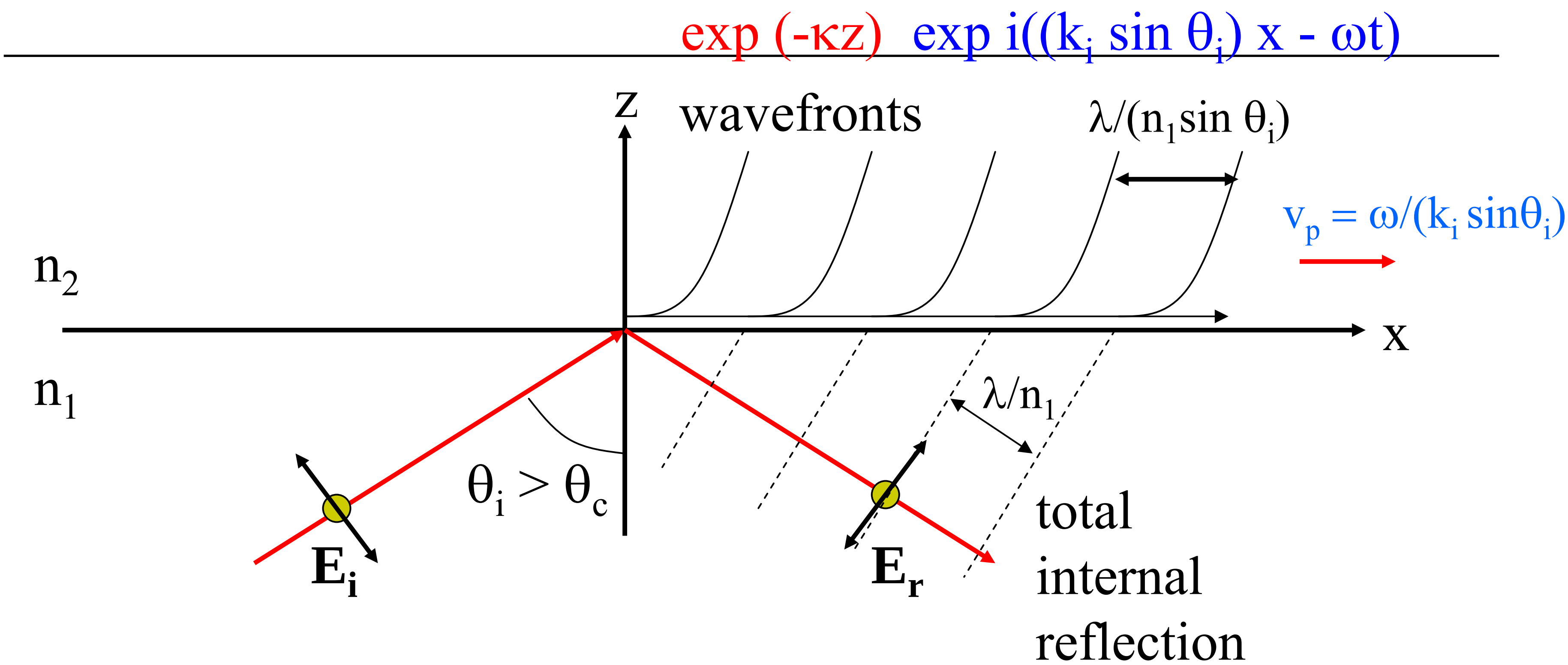
Evanescent wave

- In spite of the fact that the incident energy is totally reflected when the angle of incidence exceeds the critical angle, there is still an electromagnetic wave field in the region beyond the boundary. This field is known as the *evanescent wave*.

- Its existence can be understood by consideration of the wave function of the electric field of the transmitted wave:

$$\mathbf{E_t} = \boldsymbol{E_t} \exp i\,(\mathbf{k_t} \bullet \mathbf{r} - \omega t)$$

Choose the coordinate axis such that the plane of incidence is on the xz plane and the boundary is at z = 0.



$$\mathbf{k_t} \bullet \mathbf{r} = k_t\, x \sin \theta_t + k_t\, z \cos \theta_t$$

$$= k_t\, x\, (n_1/n_2) \sin \theta_i + k_t\, z\, (1 - (n_1/n_2)^2 \sin^2 \theta_i)^{1/2}$$

$$= k_i\, x \sin \theta_i + i\, k_t\, z\, ((n_1{}^2 \sin^2\theta_i/n_2{}^2) - 1)^{1/2}$$

The wave function for the electric field of the evanescent wave is

$$\mathbf{E}_{\mathbf{evan}} = \boldsymbol{E_t} \exp(-\kappa z) \exp i\,((k_i \sin\theta_i)\,x - \omega t)$$

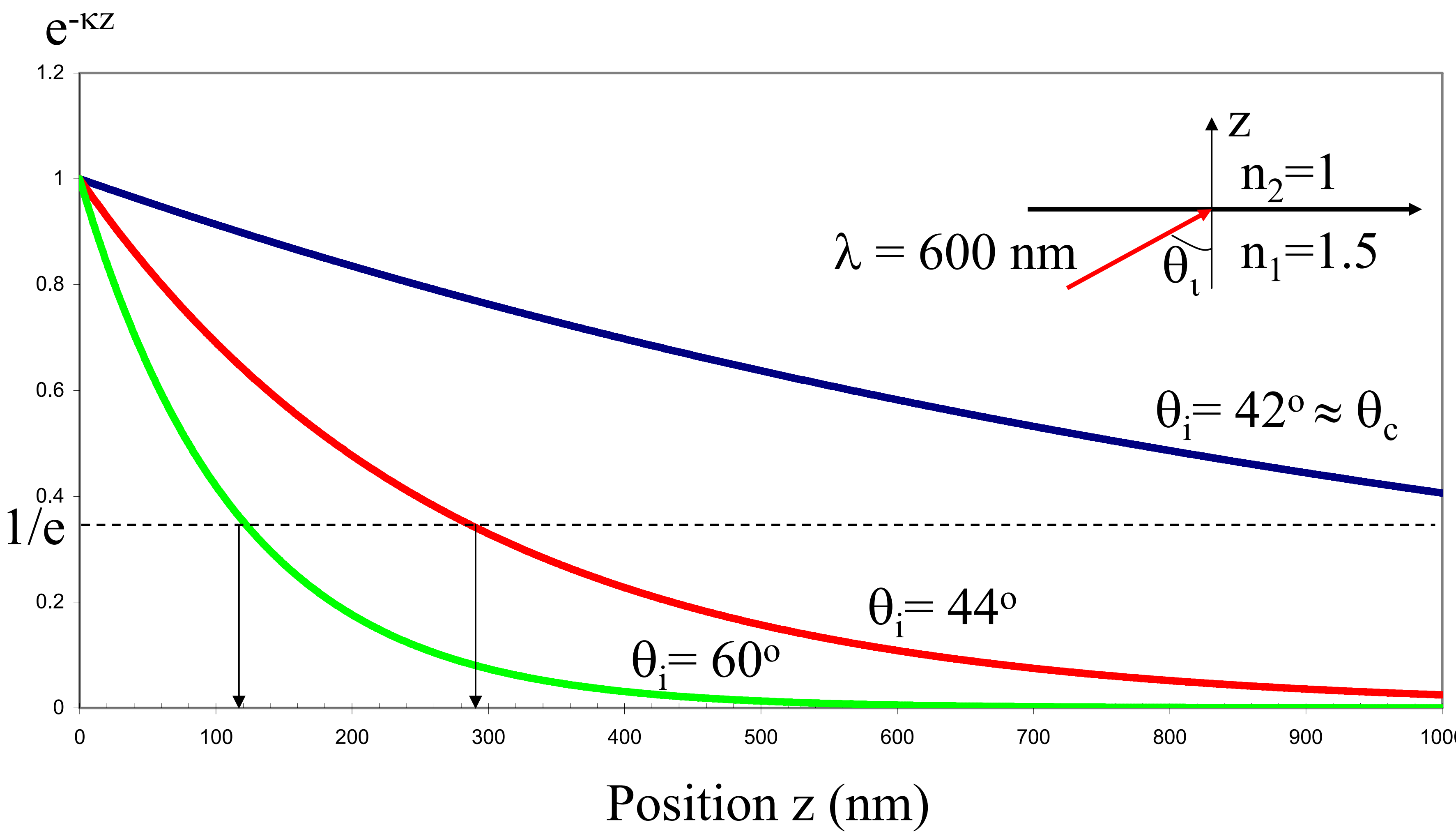
where $\kappa = k_t\,((n_1{}^2 \sin^2\theta_i / n_2{}^2) - 1)^{1/2}$

• The factor $\exp(-\kappa z)$ shows that the evanescent wave amplitude drops off very rapidly in the lower-index medium as a function of distance from the boundary.

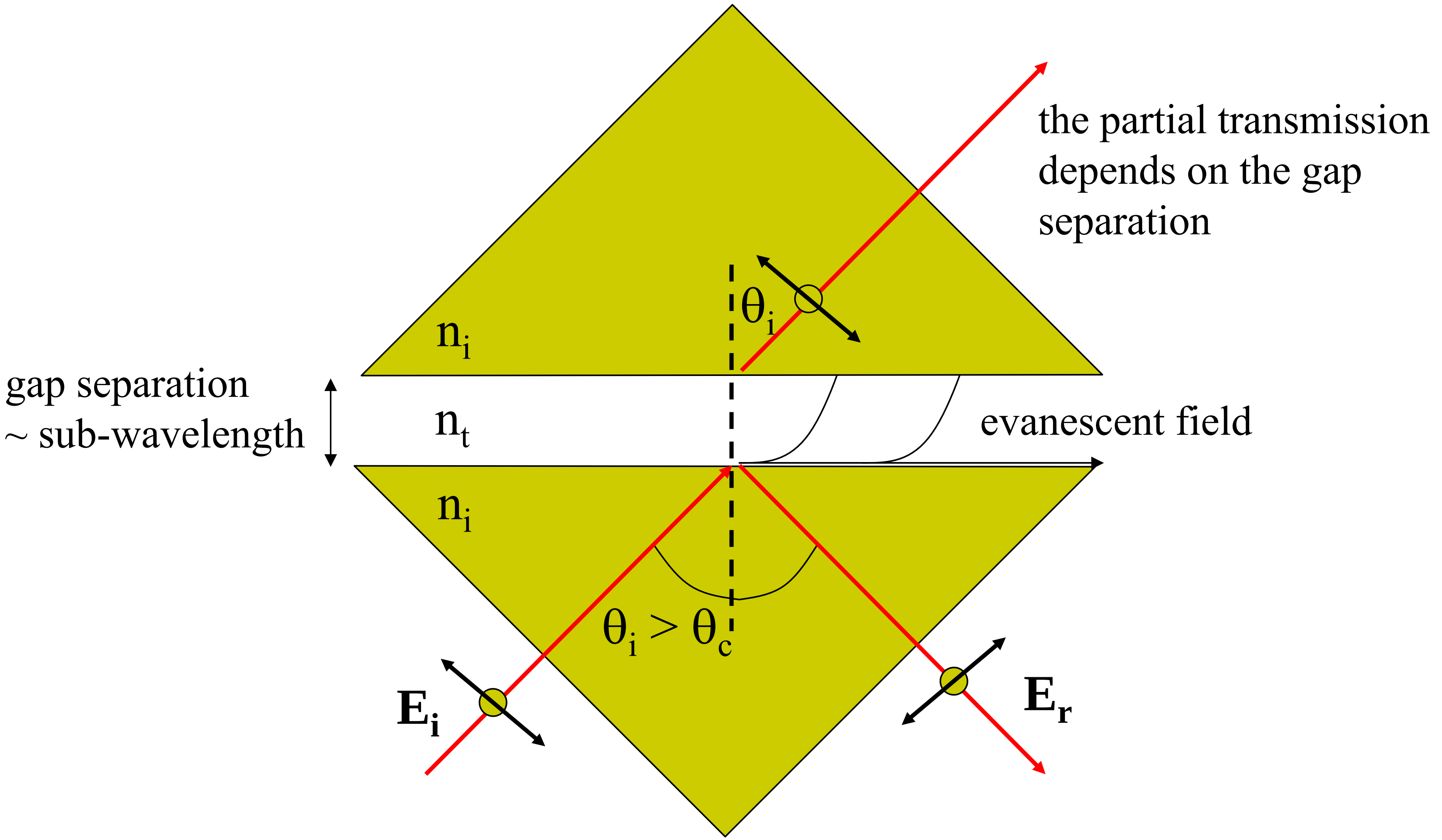
• The oscillatory term $\exp i\,((k_i \sin\theta_i)\,x - \omega t)$ indicates that the evanescent wave can be described in terms of *surfaces of constant phase* moving parallel to the boundary with phase velocity $\omega/(k_i \sin\theta_i)$.

• *The evanescent field stores energy and transports it in the direction of surface propagation, but does not transport energy in the transverse direction.* Therefore, evanescent wave is also known as *surface wave*.

Evanescent wave amplitude normal to the interface drops exponentially



Evanescent coupling between two components in close proximity



Photon nature of light

Photon nature of light

- When considering the function of a device that involves the emission or absorption of light, a purely electromagnetic wave description of light is not adequate.
- In this situation, the photon nature of light cannot be ignored.
- The material involved in this process also undergoes quantum mechanical transitions between its energy levels.
- The energy of a photon is determined by its frequency υ, or its angular frequency ω.
- Associated with the particle nature of a photon, there is a momentum determined by its wavelength λ, or its wavevector $\mathbf{k}$.

Quantum Mechanics: de Broglie's wavelength

- Wave and particle duality
- *All particles* have associated with them a wavelength (*confirmed experimentally in 1927 by Thomson and by Davisson and Germer*),

(de Broglie wavelength) $$\lambda = \frac{h}{p}$$

- For *any* particle with rest mass m_o, treated relativistically,

$$E^2 = p^2c^2 + m_o{}^2c^4$$

Photon de Broglie wavelength

- For photons, $m_o = 0$
- $E = pc$
- also $E = h\upsilon$

$$\lambda = \frac{h}{p} = \frac{h}{E/c} = \frac{h}{h\nu/c} = \frac{c}{\nu}$$

- But the relation $c = \lambda\upsilon$ is just what we expect for a harmonic wave (consistent with wave theory)

Photon in free space

- Speed $$c = \lambda \nu$$
- Energy $$h\nu = \hbar\omega = pc$$
- Momentum $$p = \frac{h\nu}{c} = \frac{h}{\lambda} = \hbar k$$
- The energy of a photon that has a wavelength λ in free space can be calculated as follows,

$$h\nu = \frac{hc}{\lambda} = \frac{1.2398}{\lambda}\mu m \bullet eV = \frac{1239.8}{\lambda} nm \bullet eV$$

e.g. at an optical wavelength of 1 μm, the photon energy is 1.2398 eV.

Photon energy

- Recall that $h = 6.626 \times 10^{-34}$ J•s
- For photon energy $h\upsilon = hc/\lambda$, we often use the energy unit of **Electron-volt** (eV) $= 1.602 \times 10^{-19}$ J
  (electronic charge (e) $= 1.602 \times 10^{-19}$ C, and $C \times V = J$)

- hc $\approx (6.626 \times 10^{-34} \times 1/1.602 \times 10^{19}$ eV•s$) \times (3 \times 10^{17}$ nm/s$)$
  ≈ 1240 nm•eV

The photon energy $\lambda = 400$ nm is ≈ 3.1 eV
$\lambda = 700$ nm is ≈ 1.77 eV
$\lambda = 1550$ nm is ≈ 0.8 eV

⇒ *These are in the range of the bandgaps of most semiconductors. Photon energy is an important factor that determines the behavior of an optical wave in a semiconductor photonic device.*