Lecture 5: Optical fibers

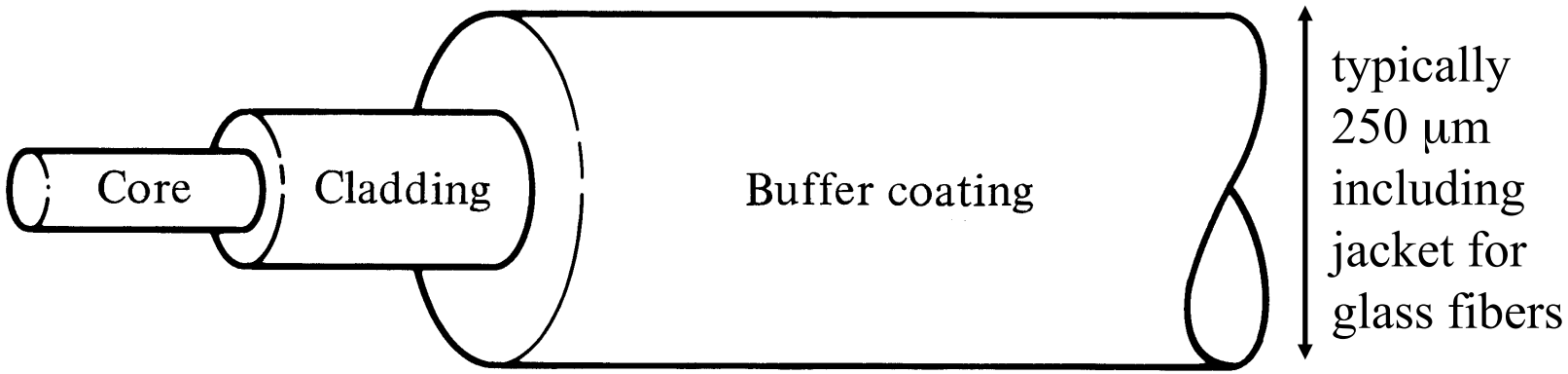
- Optical fiber basics
- Linearly polarized modes
- Field analysis/wave equation of weakly guiding fibers
- Attenuation in fibers
- Dispersion in fibers

References: Photonic Devices, Jia-Ming Liu, Chapter 3

*Most of the lecture materials here are adopted from ELEC342 notes.

Optical fiber structure

- A typical bare fiber consists of a core, a cladding, and a polymer jacket (buffer coating).



- The polymer coating is the first line of mechanical protection.

- The coating also reduces the internal reflection at the cladding, so light is only guided by the core.

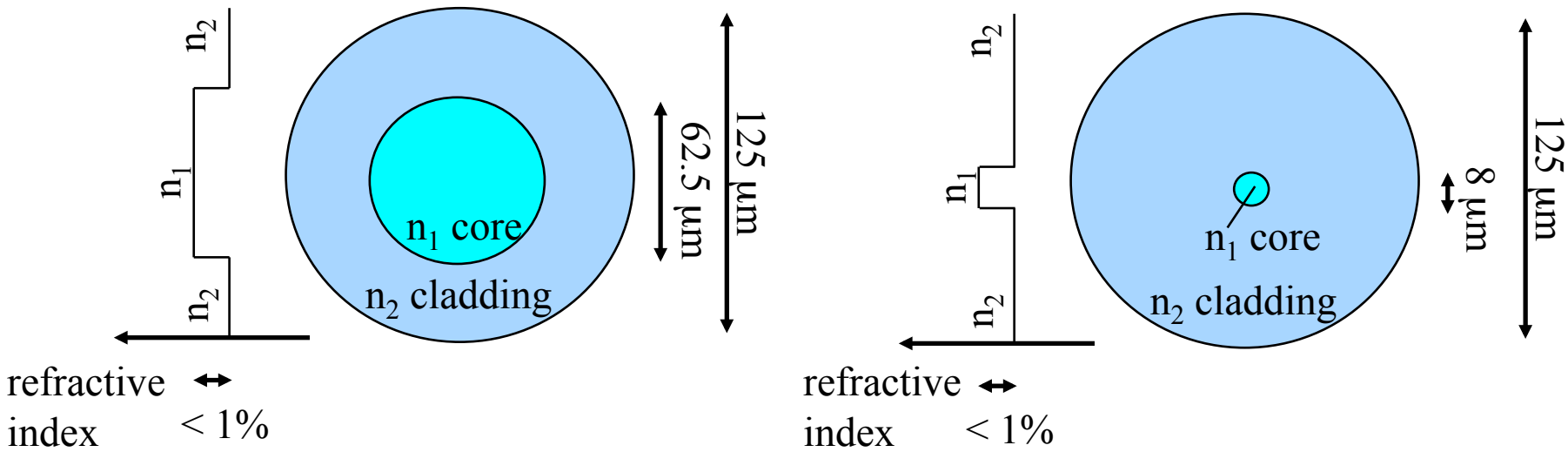
Silica optical fibers

- Both the core and the cladding are made from a type of glass known as silica (SiO_2) which is *almost transparent in the visible and near-IR*.

- In the case that the refractive index changes in a "step" between the core and the cladding. This fiber structure is known as step-index fiber.

- The higher core refractive index (~ 0.3% higher) is typically achieved by *doping* the silica core with germanium dioxide (GeO_2).

Step-index silica optical fiber cross-section



Multi-mode fiber

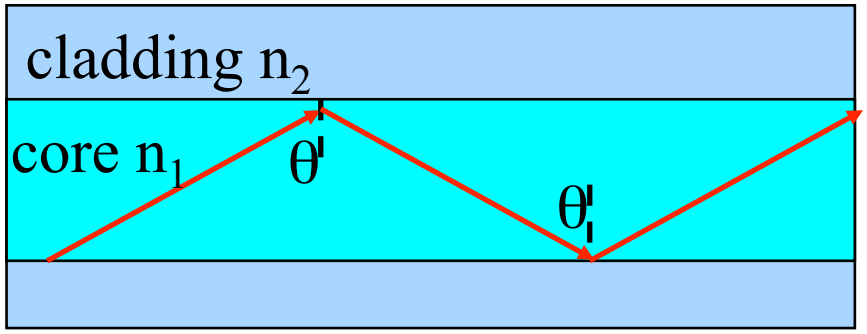
Single-mode fiber

- Multi-mode fiber: core dia. ~ 50 or 62.5 or 100 μm; cladding dia. ~ 125 μm
- Single-mode fiber: core dia. ~ 8 - 9 μm; cladding dia. ~ 125 μm

Both fiber types can have the same numerical aperture (NA) because NA is independent of the fiber core diameter!

Light ray guiding condition

- Light ray that satisfies *total internal reflection* at the interface of the higher refractive index core and the lower refractive index cladding can be guided along an optical fiber.



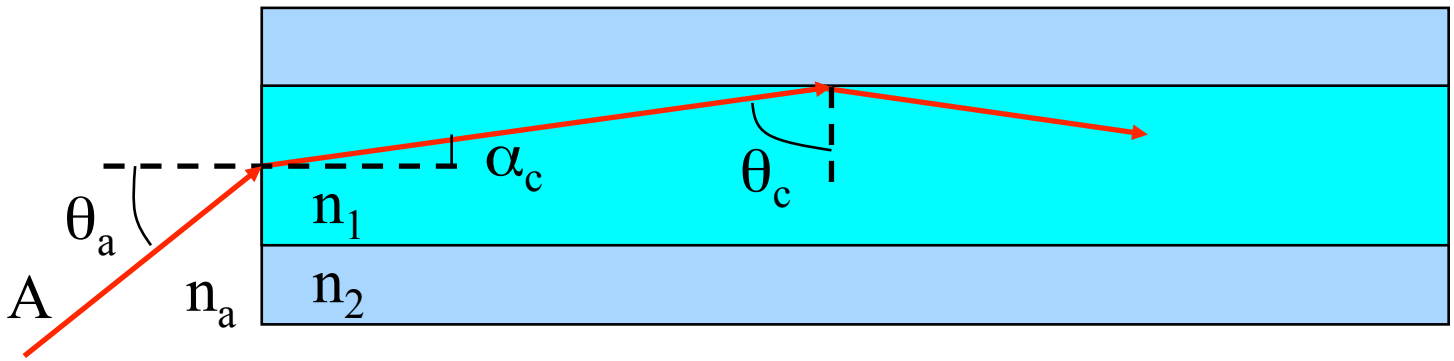
e.g. Under what condition will light be trapped inside the fiber core?
$n_1 = 1.46$; $n_2 = 1.44$

$$\theta > \theta_c$$

$$\theta_c = \sin^{-1}(n_2/n_1) = \sin^{-1}(1.44/1.46) = 80.5^o$$

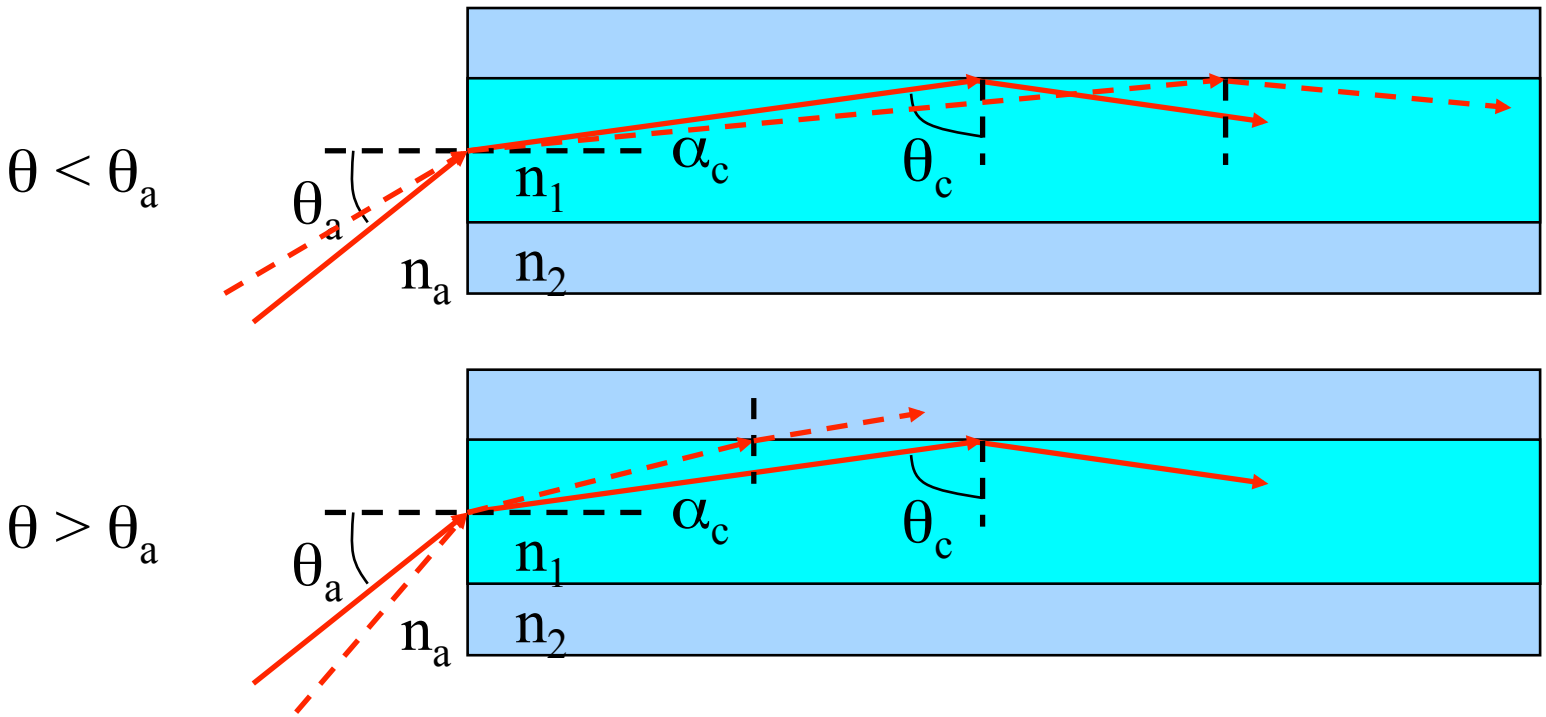
Acceptance angle

- Only rays with a sufficiently shallow grazing angle (i.e. with an angle to the normal greater than θ_c) at the core-cladding interface are transmitted by total internal reflection.



- Ray A incident at the critical angle θ_c at the core-cladding interface enters the fiber core at an angle θ_a to the fiber axis, and is refracted at the air-core interface.

Acceptance angle



- Any rays which are incident into the fiber core at an angle > θ_a have an incident angle less than θ_c at the core-cladding interface.

These rays will NOT be totally internal reflected, thus eventually loss to radiation (at the cladding-jacket interface).

Acceptance angle

- Light rays will be confined inside the fiber core if it is input-coupled at the fiber core end-face within the acceptance angle θ_a.

e.g. What is the fiber acceptance angle when $n_1 = 1.46$ and $n_2 = 1.44$?

$\theta_c = \sin^{-1}(n_2/n_1) = 80.5^o \Rightarrow \alpha_c = 90^o - \theta_c = 9.5^o$

using $\sin\theta_a = n_1 \sin\alpha_c$ (taking $n_a = 1$)

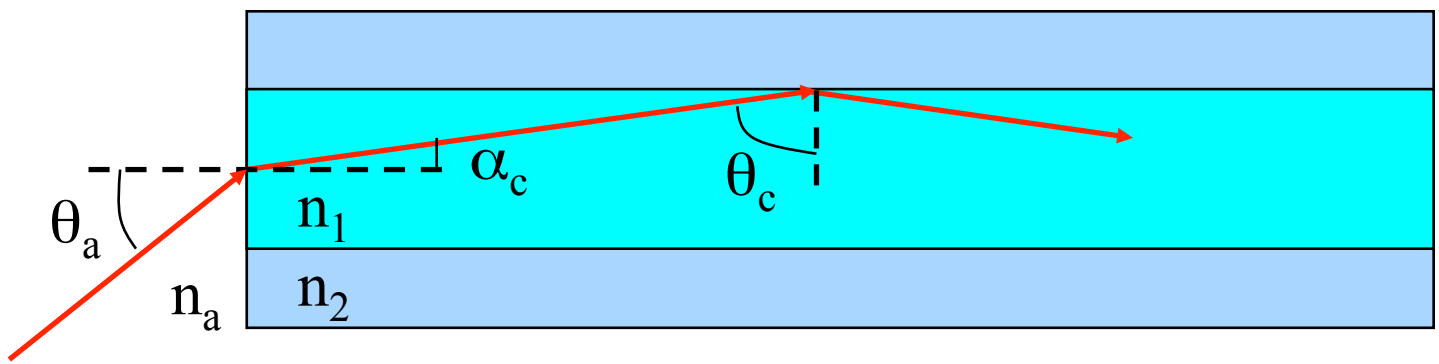
$\theta_a = \sin^{-1}(n_1 \sin\alpha_c) = \sin^{-1}(1.46 \sin 9.5^o) \sim 14^o$

$\Rightarrow$ the acceptance angle $\theta_a \sim 14^o$

Fiber numerical aperture

In fiber optics, we describe the fiber acceptance angle using **Numerical Aperture (NA):**

$$NA = n_a \sin\theta_a = \sin\theta_a = (n_1^2 - n_2^2)^{1/2}$$



- We can relate the <u>acceptance angle θ_a</u> and the <u>refractive indices of the core n_1, cladding n_2 and air n_a</u>.

Fiber numerical aperture

- Assuming the end face at the fiber core is *flat* and *normal* to the fiber axis (when the fiber has a "nice" cleave), we consider the refraction at the air-core interface using Snell's law:

$$\text{At } \theta_a: \quad n_a \sin\theta_a = n_1 \sin\alpha_c$$

launching the light from air: $(n_a \sim 1)$

$$\sin\theta_a = n_1 \sin\alpha_c$$

$$= n_1 \cos\theta_c$$

$$= n_1 (1 - \sin^2\theta_c)^{1/2}$$

$$= n_1 (1 - n_2^2/n_1^2)^{1/2}$$

$$= (n_1^2 - n_2^2)^{1/2}$$

Fiber numerical aperture

- Fiber NA therefore characterizes the fiber's ability to gather light from a source and guide the light.

e.g. What is the fiber numerical aperture when $n_1 = 1.46$ and $n_2 = 1.44$?

$$NA = \sin \theta_a = (1.46^2 - 1.44^2)^{1/2} = 0.24$$

- It is a common practice to define a relative refractive index Δ as:

$$\Delta = (n_1 - n_2) / n_1$$

$$(n_1 \sim n_2) \quad \Rightarrow \quad NA = n_1 (2\Delta)^{1/2}$$

i.e. Fiber NA only depends on n_1 and Δ.

Typical fiber NA

- Silica fibers for long-haul transmission are designed to have numerical apertures from about 0.1 to 0.3.

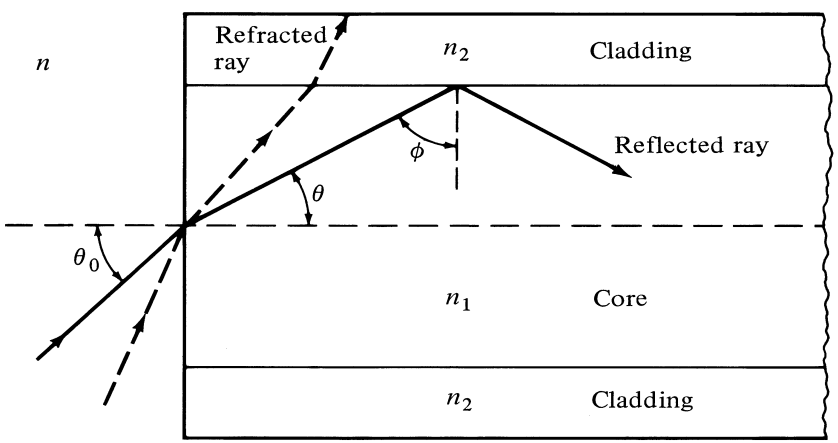
- The low NA makes coupling efficiency tend to be poor, but turns out to improve the fiber's bandwidth! (details later)

- Plastic, rather than glass, fibers are available for short-haul communications (e.g. within an automobile). These fibers are restricted to short lengths because of the relatively high attenuation in plastic materials.

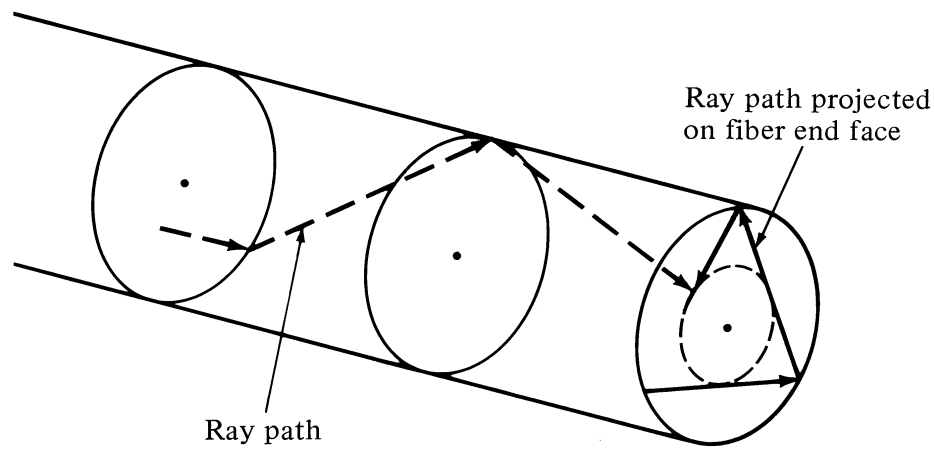
- Plastic optical fibers (POFs) are designed to have high numerical apertures (typically, 0.4 – 0.5) to improve coupling efficiency, and so partially offset the high propagation losses and also enable alignment tolerance.

Meridional and skew rays

- A *meridional* ray is one that has no ϕ component – it passes through the z axis, and is thus in direct analogy to a slab guide ray.
- Ray propagation in a fiber is complicated by the possibility of a path component in the ϕ direction, from which arises a *skew* ray.
- Such a ray exhibits a spiral-like path down the core, never crossing the z axis.



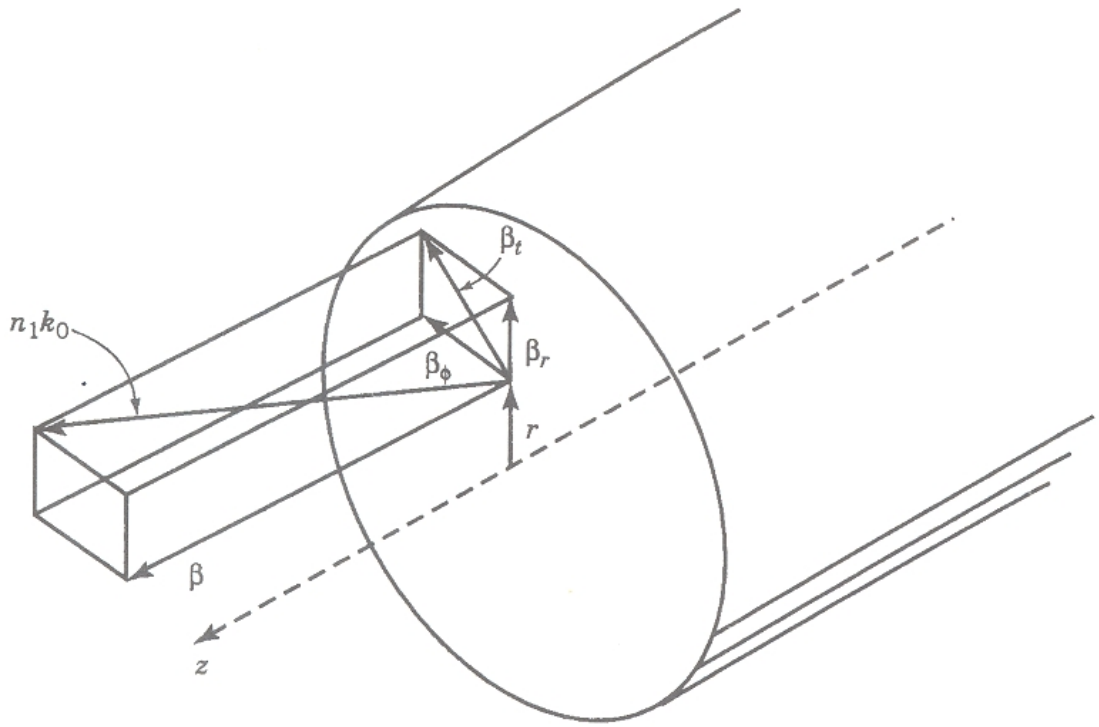
Meridional ray



Skew ray

Linearly polarized modes

Skew ray decomposition in the core of a step-index fiber



$$(n_1k_0)^2 = \beta_r^2 + \beta_\phi^2 + \beta^2 = \beta_t^2 + \beta^2$$

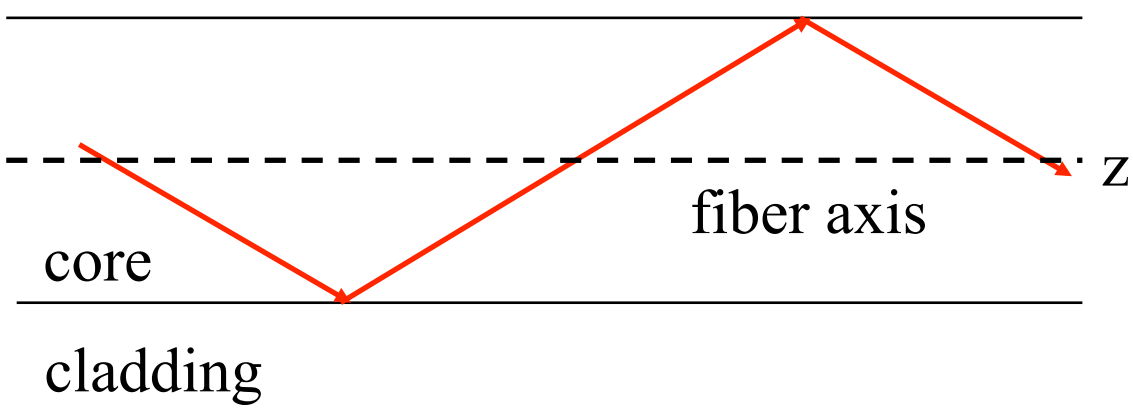
Vectorial characteristics of modes in optical fibers

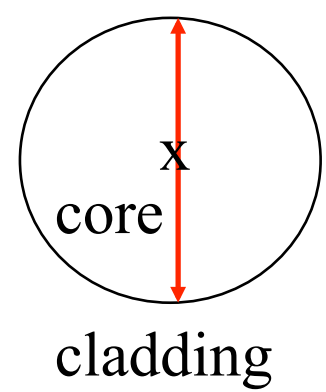
- **TE** (i.e. $E_z = 0$) and **TM** ($H_z = 0$) modes are also obtained within the circular optical fiber. These modes correspond to meridional rays (pass through the fiber axis).

- As the circular optical fiber is bounded in *two dimensions in the transverse plane*,

=> *two integers*, *l* and m, are necessary in order to specify the modes

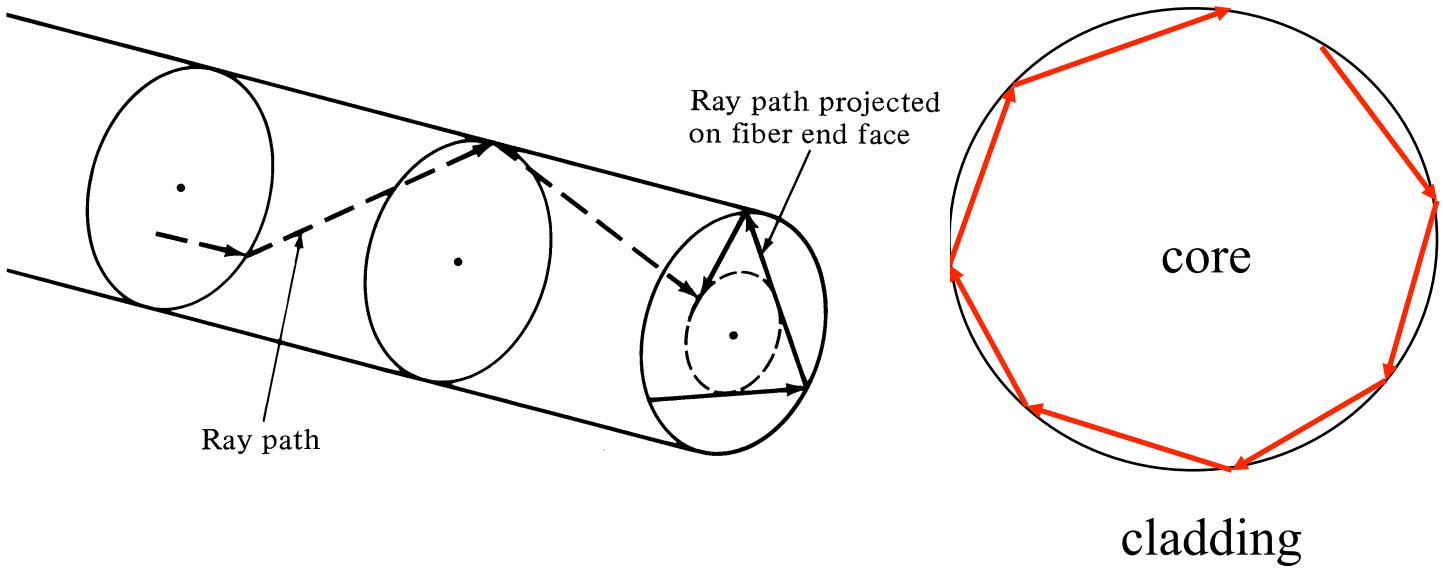
i.e. We refer to these modes as $\mathbf{TE}_{lm}$ and $\mathbf{TM}_{lm}$ modes.





Vectorial characteristics of modes in optical fibers

- ***Hybrid modes*** are modes in which *both E_z and H_z are nonzero*. These modes result from skew ray propagation (*helical path without passing through the fiber axis*). The modes are denoted as $\mathbf{HE}_{lm}$ and $\mathbf{EH}_{lm}$ depending on whether the components of **H** or **E** make the larger contribution to the transverse field.



- The full set of circular optical fiber modes therefore comprises: **TE**, **TM** (meridional rays), **HE** and **EH** (skew rays) modes.

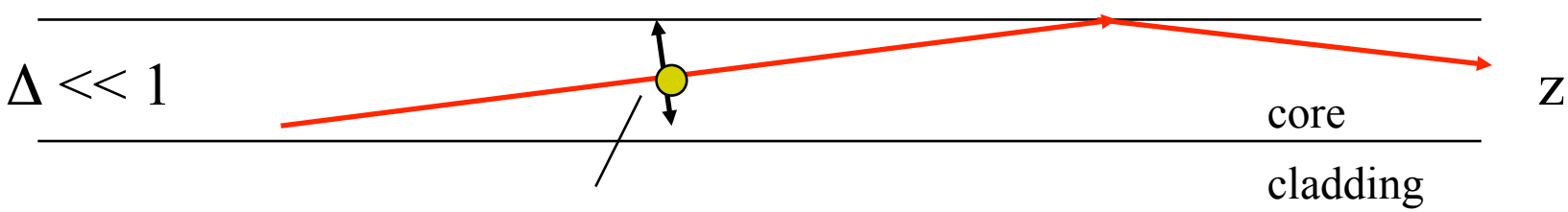
Weak-guidance approximation

- The analysis may be simplified when considering *telecommunications grade* optical fibers. These fibers have the *relative index difference* $\Delta << 1$ ($\Delta = (n_{core} - n_{clad})/n_{core}$ typically less than 1 %).

=> the propagation is *preferentially along the fiber axis* ($\theta \approx 90^o$).

i.e. the *field is therefore predominantly transverse*.

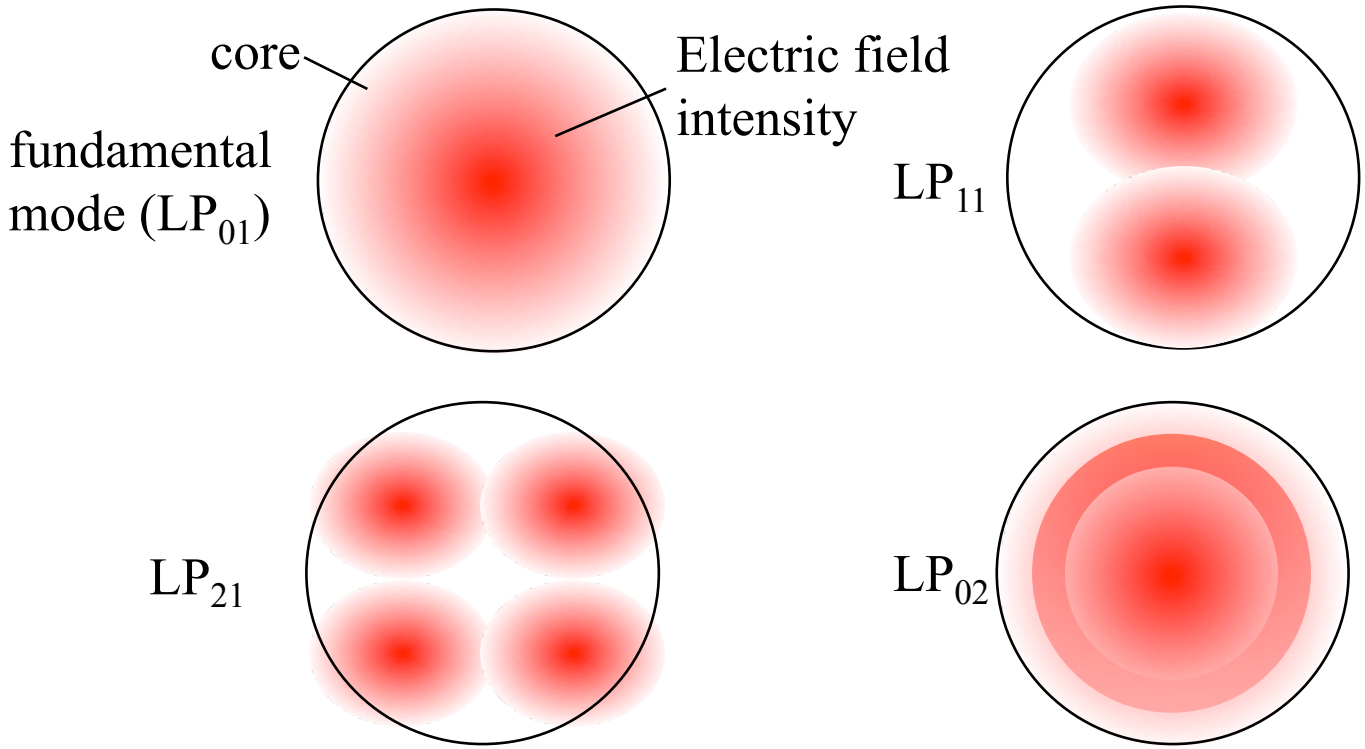
=> modes are *approximated* by *two linearly polarized components*.
(both E_z and H_z are *nearly* zero)



Two near linearly polarized modes

Linearly polarized modes

- These ***linearly polarized*** (**LP**) modes, designated as **LP**$_{lm}$, are *good approximations* formed by exact modes TE, TM, HE and EH.
- The mode subscripts l and m describe the electric field intensity profile. There are $2l$ field maxima around the the fiber core circumference and m field maxima along the fiber core radial direction.

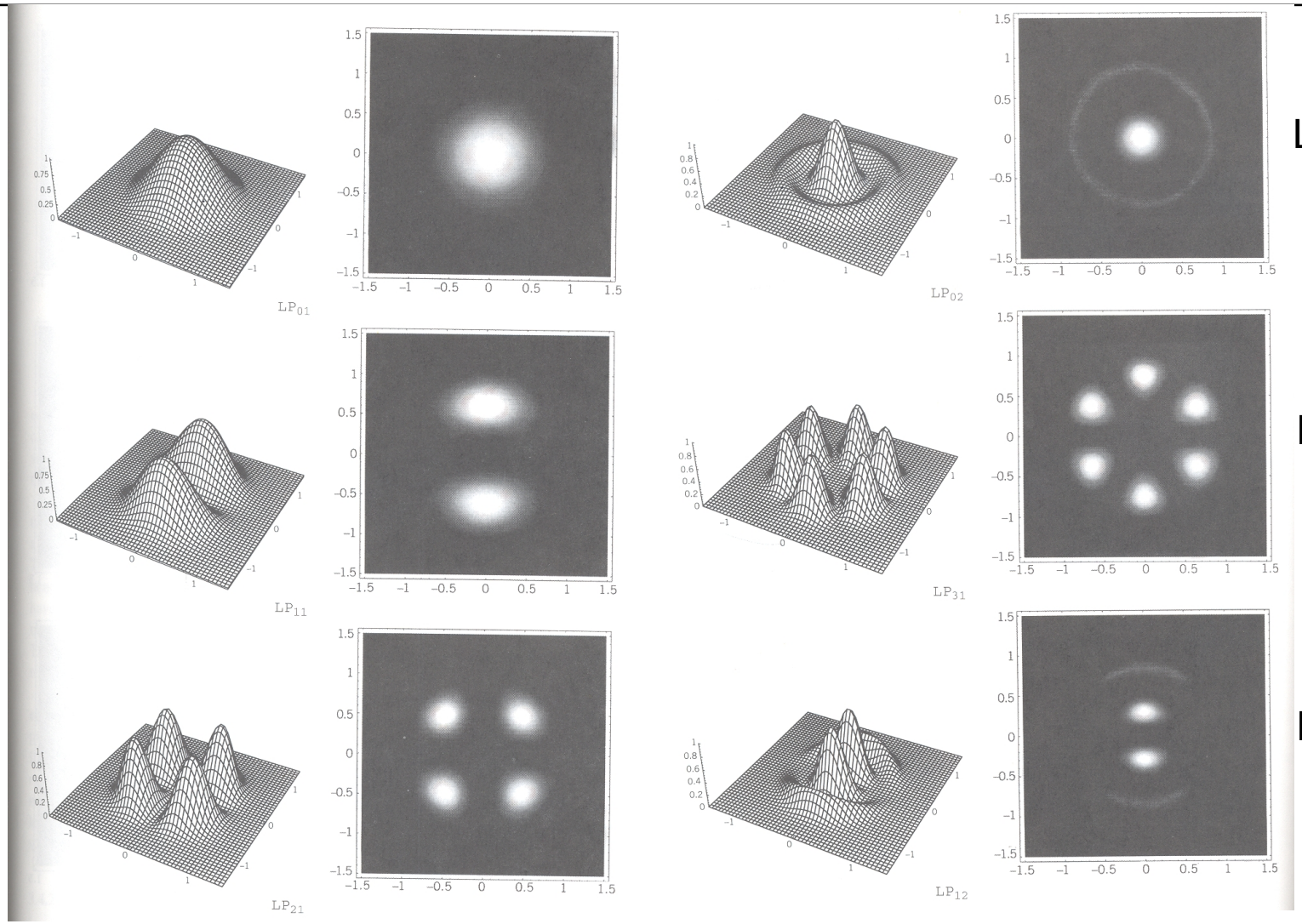


Intensity plots for the first six LP modes

LP_{01}

LP_{11}

LP_{21}

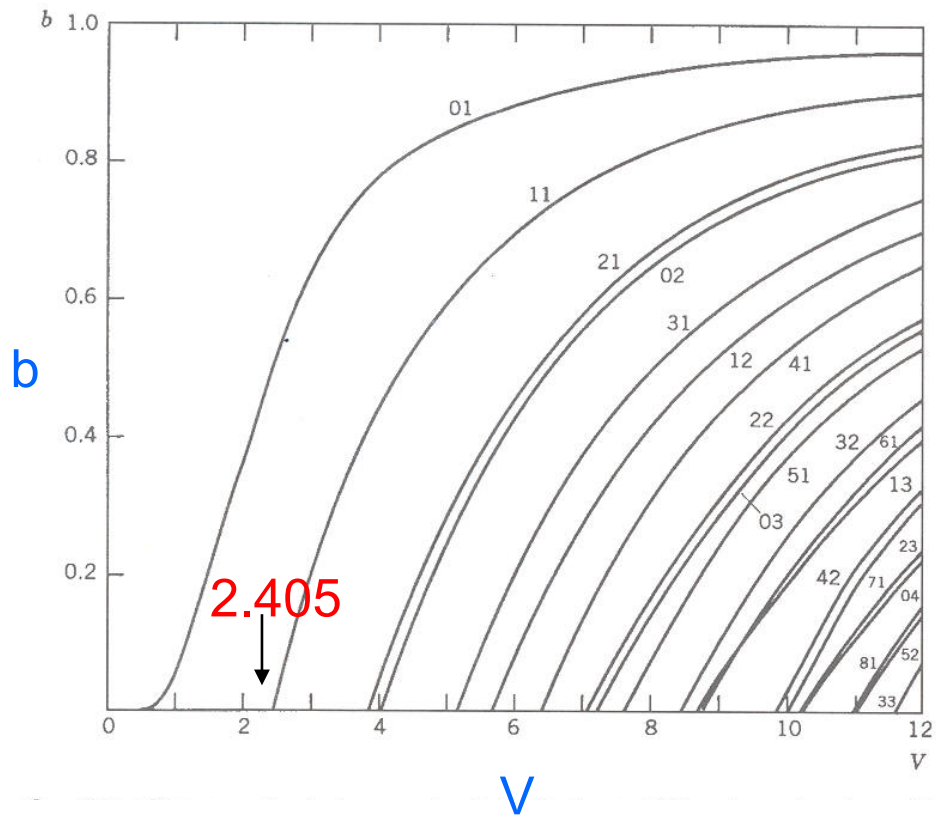


LP_{02}

LP_{31}

LP_{12}

Plot of the normalized propagation constant b as a function of V for various LP modes



$$V = (2\pi a/\lambda)\,(n_1^2 - n_2^2)^{1/2} = (u^2 + w^2)^{1/2}$$

(see p.41)

$$b = (\beta^2 - k_2^2)/(k_1^2 - k_2^2)$$

Number of guided modes

The total number of guided modes M for a *step-index* fiber is *approximately* related to the V number (for V > 20) as follows,

$$M \approx V^2 / 2$$

e.g. A multimode step-index fiber with a core diameter of 80 μm and a relative index difference of 1.5 % is operating at a wavelength of 0.85 μm. If the core refractive index is 1.48, estimate (a) the normalized frequency for the fiber; (b) the number of guided modes.

(a) $V = (2\pi/\lambda)\, a\, n_1\, (2\Delta)^{1/2} = 75.8$

(b) $M \approx V^2 / 2 = 2873$ (i.e. *nearly 3000 guided modes*!)

Cutoff wavelength

- The ***cutoff wavelength*** for any mode is defined as the *maximum* wavelength at which that mode propagates. It is the value of λ that corresponds to V_c for the mode concerns. For each LP mode, the two parameters are related

$$\lambda_c(lm) = (2\pi a/(V_c(lm))\ (n_1^2 - n_2^2)^{1/2}$$

  The range of wavelengths over which mode *l*m will propagate is thus $0 < \lambda < \lambda_c(lm)$.

- *For a fiber to operate single mode*, the operating wavelength must be *longer* than the cutoff wavelength for the LP_{11} mode. *This is an important specification* for a single-mode fiber, and is usually given the designation λ_c. We find λ_c by setting $V_c = 2.405$. The range of wavelengths for singlemode operation is $\lambda > \lambda_c$.

Singlemode condition

For single-mode operation, *only the fundamental LP_{01} mode exists*.

The ***cutoff normalized frequency*** (**V_c**) for the next higher order (LP_{11}) mode in step-index fibers occurs at **V_c = 2.405**.

=> single-mode propagation of the LP_{01} mode in step-index fibers:

$$V < 2.405$$

e.g. Determine the cutoff wavelength for a step-index fiber to exhibit single-mode operation when the core refractive index is 1.46 and the core radius is 4.5 μm, with the relative index difference of 0.25 %.

$$\lambda_c = (2\pi a n_1/2.405)\,(2\Delta)^{1/2} = 1214 \text{ nm.}$$

Hence, the fiber is *single-mode* for $\lambda > 1214$ nm.

Gaussian approximation for the LP_{01} mode field

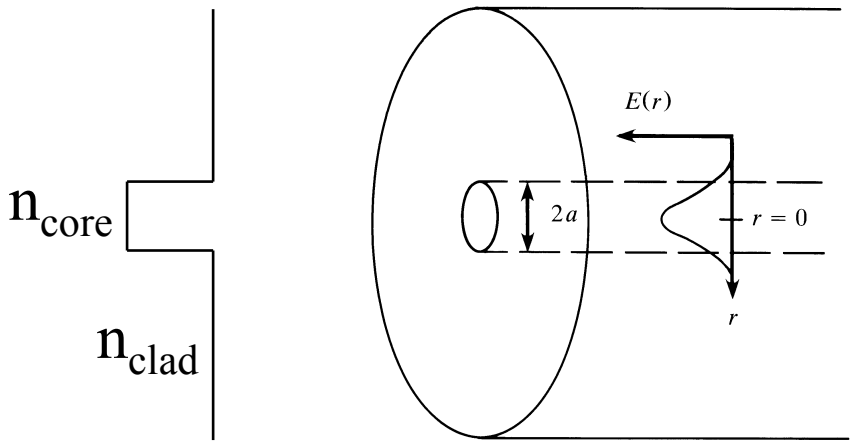
- The LP_{01} mode intensity varies with radius as $J_0^2(ur/a)$ inside the core and as $K_0^2(wr/a)$ in the cladding. The resultant intensity profile turns out to closely fits a *Gaussian* function having a width w_0, known as the *mode-field radius*.
- This is defined as the radial distance from the core center to the $1/e^2$ point of the Gaussian intensity profile.
- A similar Gaussian approximation can be applied to the fundamental symmetric slab waveguide mode.

$$E(r) = E(0) \exp(-r^2 / w_0^2)$$

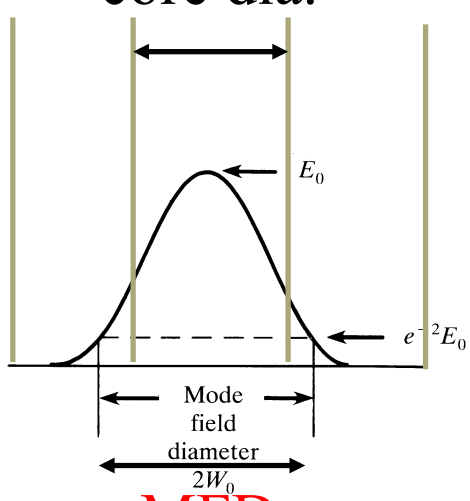
$$\Rightarrow \quad I(r) = I(0) \exp(-2r^2/w_0^2)$$

Mode-field diameter (MFD) = $2w_0$ (*rather than the core diameter*) characterizes the <u>functional properties of single-mode fibers</u>.
(w_0 is also called the *spot size*.)

Mode-field diameter



core dia.



MFD

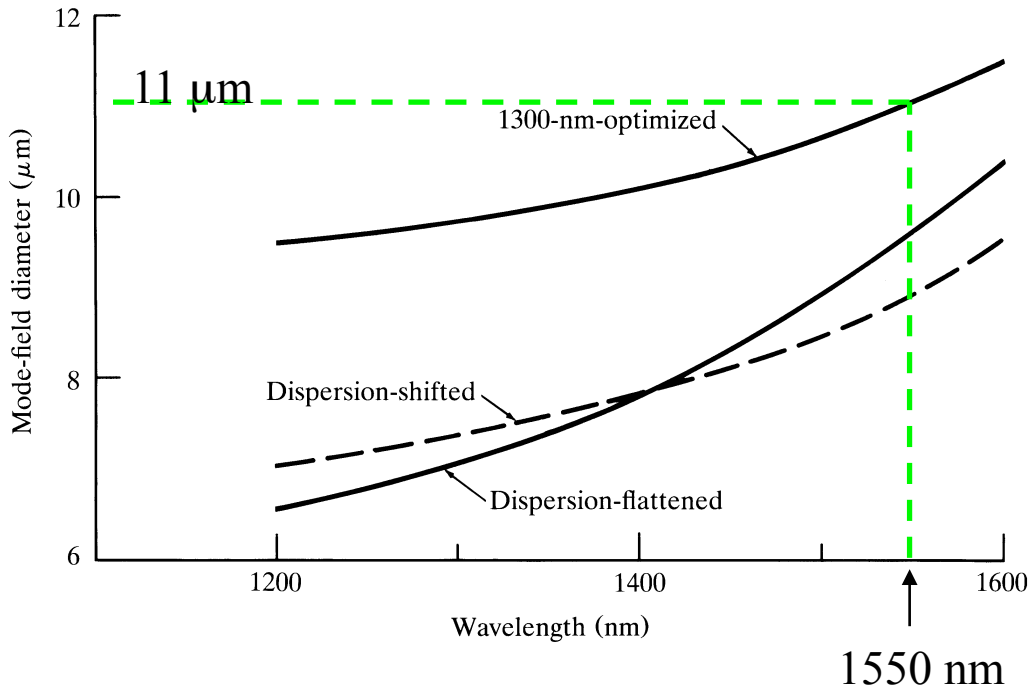
"Corning SMF-28" single-mode fiber has MFD:

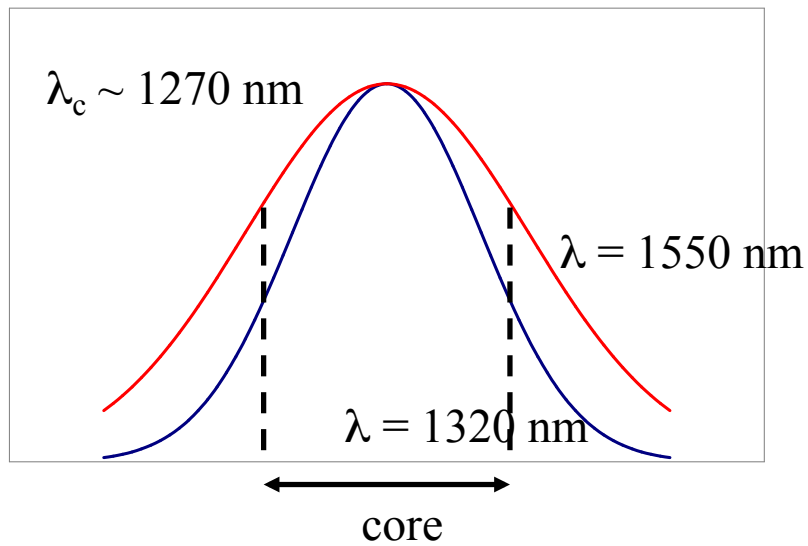
9.2 μm at 1310 nm
10.4 μm at 1550 nm

core diameter: 8.2 μm

MFD > core diameter

Mode-field diameter vs. wavelength





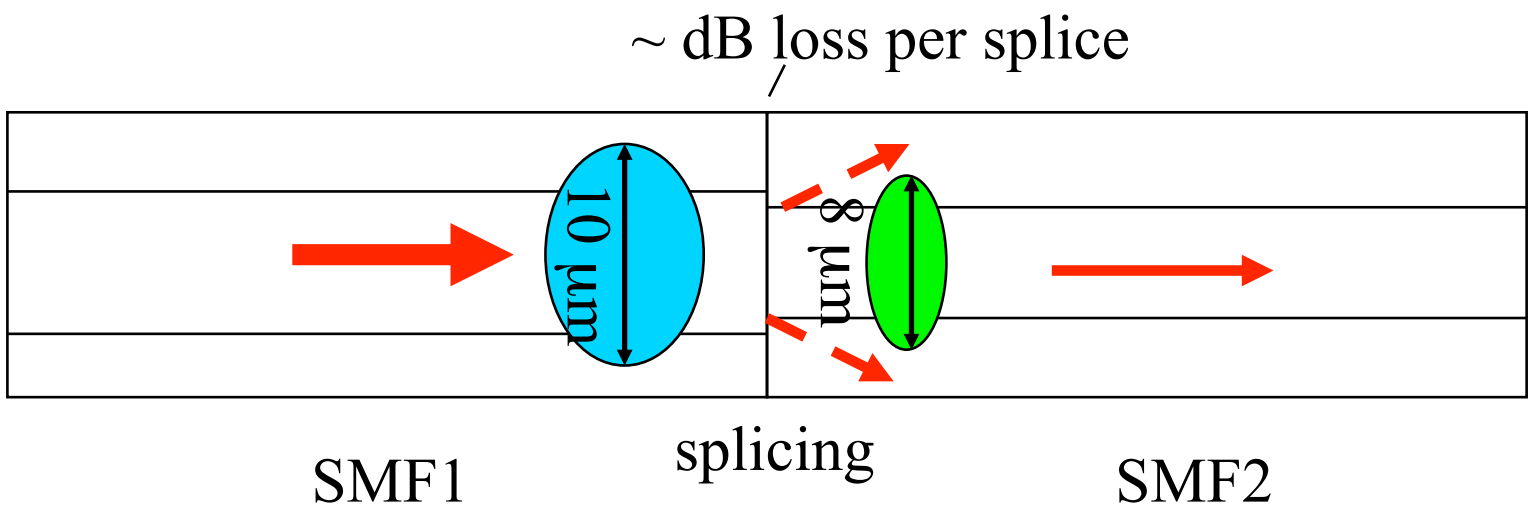
- Mode-field intensity distribution can be measured directly by *near-field imaging* the fiber output.

Why characterize the MFD for single-mode fibers?

Mode-field diameter mismatch

Ans.: Mismatches in mode-field diameter can increase fiber splice loss.

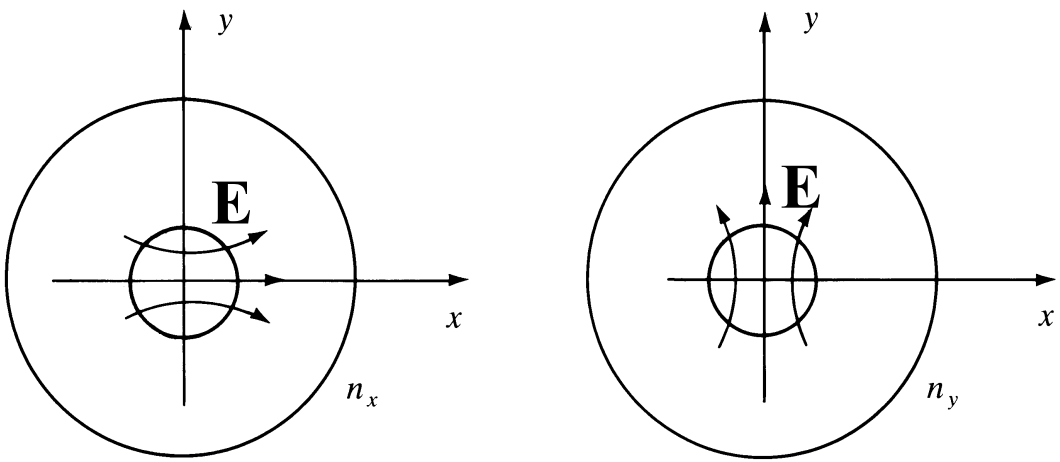
e.g. Splicing loss due to **MFD mismatch** between two different SMF's



(A related question: why do manufacturers standardize the cladding diameter?)

Remarks on single-mode fibers

- no cutoff for the fundamental mode
- there are in fact *two normal modes with orthogonal polarizations*



Horizontal mode Vertical mode

Fiber birefringence

- In *ideal* fibers with perfect rotational symmetry, the two modes are *degenerate* with equal propagation constants ($\beta_x = \beta_y$), and any polarization state injected into the fiber will propagate unchanged.
- In *actual* fibers there are *imperfections*, such as *asymmetrical lateral stresses*, *non-circular cores* and *variations in refractive-index profiles*. These imperfections break the circular symmetry of the ideal fiber and *lift the degeneracy* of the two modes.
- The modes propagate with different phase velocities, and the difference between their effective refractive indices is called the ***fiber birefringence***,

$$B = |n_y - n_x|$$

Real optical fiber geometry is by *no means perfect*

Corning SMF-28 single-mode fiber glass geometry

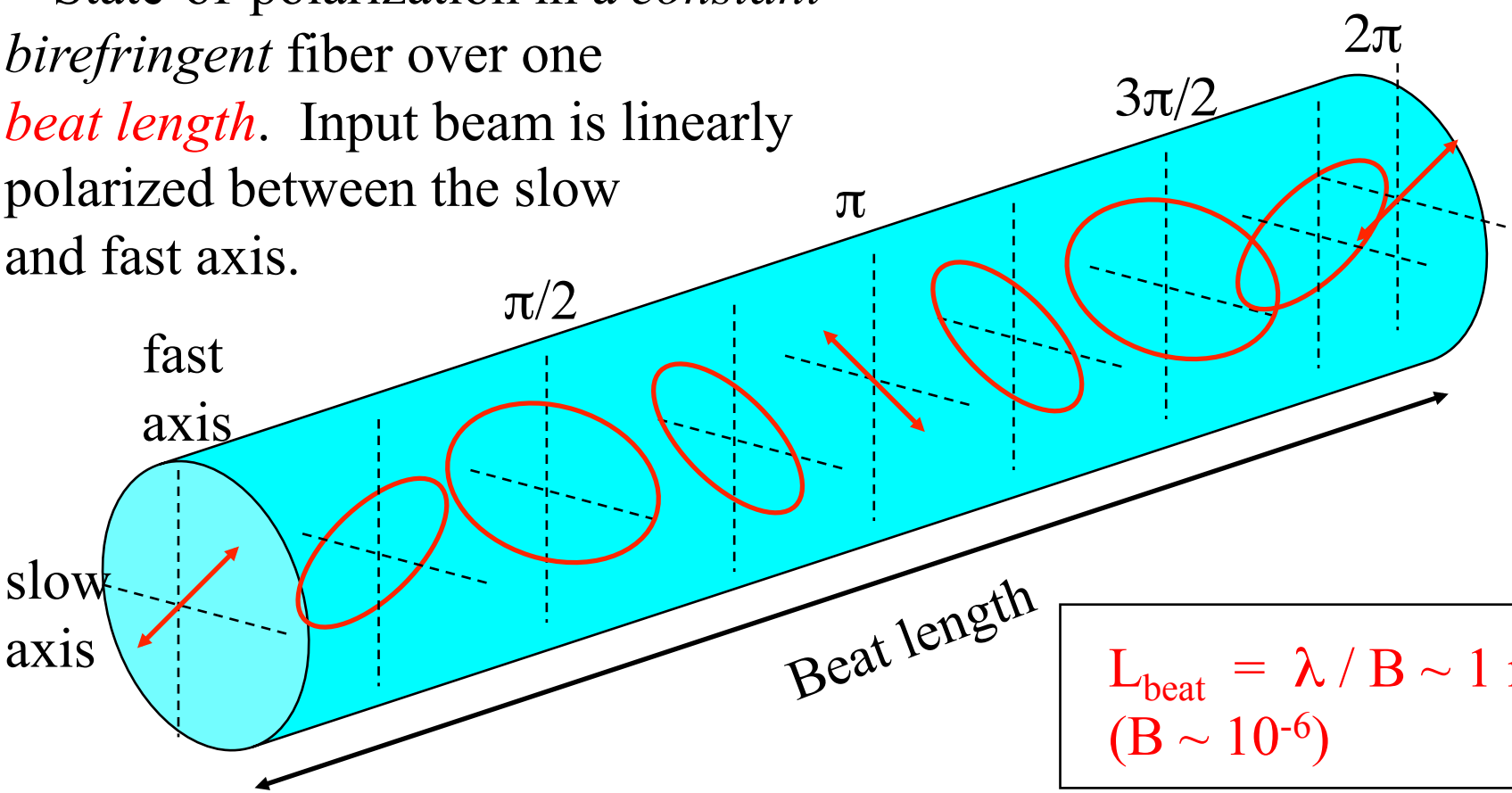
1. cladding diameter: 125.0 ± 0.7 μm

2. core-cladding *concentricity*: < 0.5 μm

3. cladding *non-circularity*: < 1%
[1- (min cladding dia./max clad dia.)]

Fiber birefringence

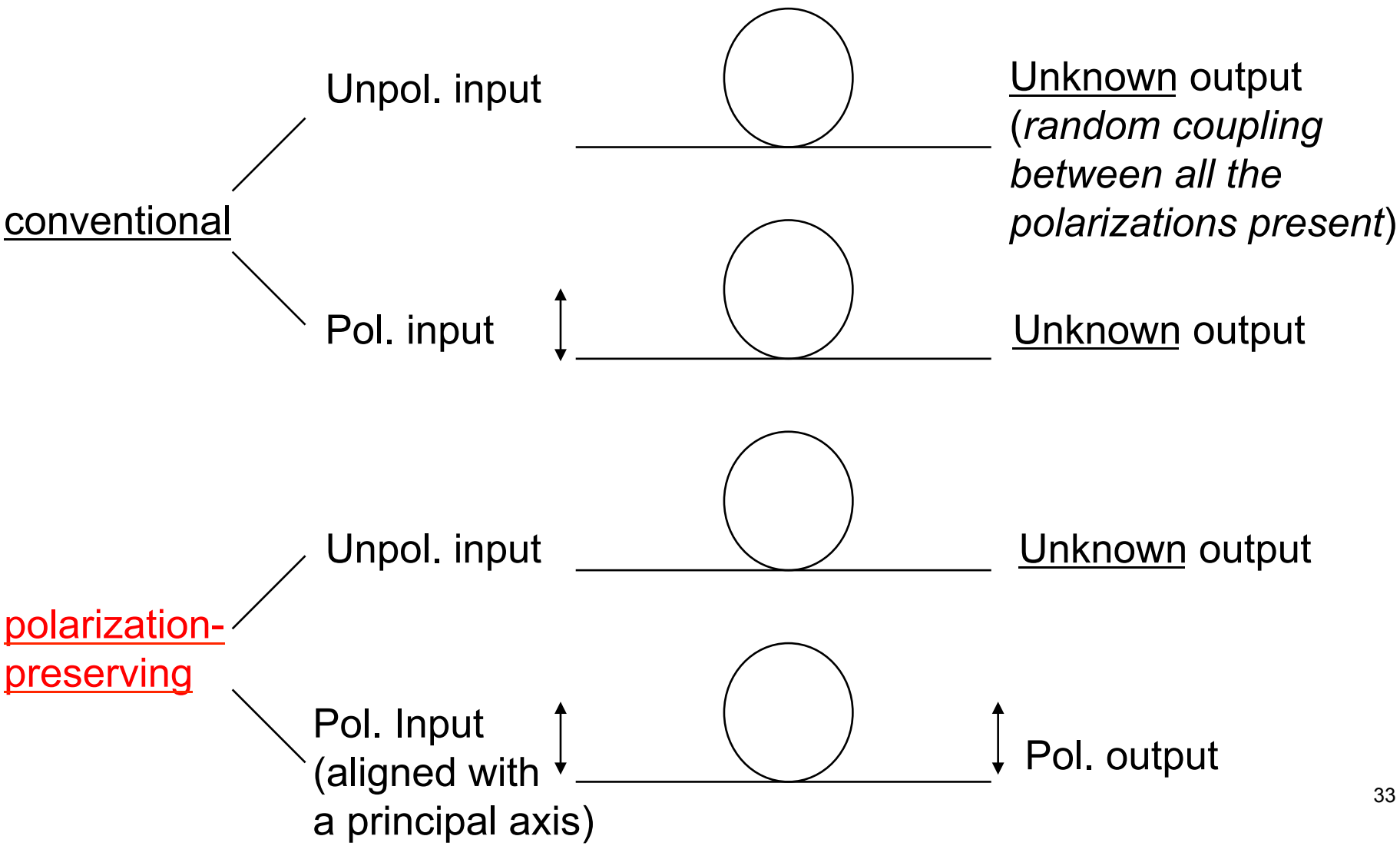
- State-of-polarization in a *constant birefringent* fiber over one *beat length*. Input beam is linearly polarized between the slow and fast axis.



$L_{beat} = \lambda / B \sim 1$ m
$(B \sim 10^{-6})$

*In optical pulses, the polarization state will also be different for different spectral components of the pulse.

Remark on polarizing effects of conventional / polarization-preserving fibers



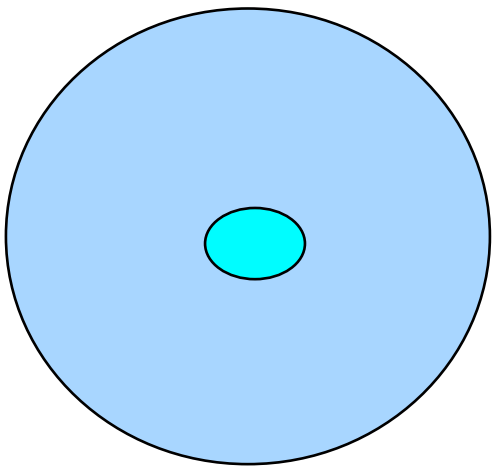
Polarization-preserving fibers

- The fiber birefringence is enhanced in single-mode *polarization-preserving* (*polarization-maintaining*) fibers, which are designed to maintain the polarization of the launched wave.

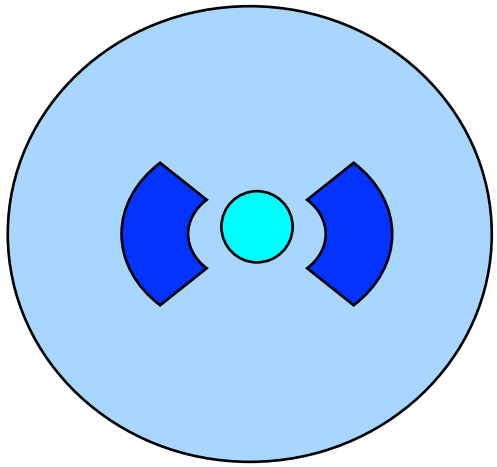
- *Polarization is preserved* because the two normal modes have significantly different propagation characteristics. This keeps them from exchanging energy as they propagate through the fiber.

- Polarization-preserving fibers are constructed by designing *asymmetries* into the fiber. Examples include fibers with <u>elliptical cores</u> (which cause waves polarized along the *major* and *minor* axes of the ellipse to have different effective refractive indices) and fibers that contain <u>nonsymmetrical stress-producing parts</u>.

Polarization-preserving fibers



Elliptical-core fiber



bow-tie fiber

- The shaded region in the bow-tie fiber is highly doped with a material such as boron. Because the *thermal expansion* of this doped region is so different from that of the pure silica cladding, a *nonsymmetrical stress* is exerted on the core. This produces a large *stress-induced birefringence*, which in turn *decouples the two orthogonal modes of the singlemode fiber*.

Field analysis/wave equation of weakly guiding fibers

Key derivations for your own reading

Field analysis of weakly guiding fibers

- Here, we begin the LP mode analysis by assuming field solutions that are *linearly polarized* in the fiber transverse plane.
- These consist of an electric field that can be designated as having x-polarization and a magnetic field that is polarized along y –

the *weak-guidance* character of the fiber results in *nearly plane wave* behavior for the fields, in which ***E*** and ***H*** are orthogonal and exist primarily in the transverse plane (*with very small z components*).

$$\boldsymbol{E} = \mathbf{a_x}\, E_x(r, \phi, z) = \mathbf{a_x}\, E_{x0}(r, \phi) \exp(-i\beta z)$$

$$\boldsymbol{H} = \mathbf{a_y}\, H_y(r, \phi, z) = \mathbf{a_y}\, H_{y0}(r, \phi) \exp(-i\beta z)$$

Field analysis of weakly guiding fibers

- Because *rectangular* components are assumed for the fields, the wave equation

$$\nabla_t^2 \boldsymbol{E_0} + (k^2 - \beta^2)\, \boldsymbol{E_0} = 0$$

is fully separable into the x, y, and z components

$$\nabla_t^2 E_{x1} + (n_1^2 k_0^2 - \beta^2)\, E_{x1} = 0 \quad r \leq a$$

$$\nabla_t^2 E_{x2} + (n_2^2 k_0^2 - \beta^2)\, E_{x2} = 0 \quad r \geq a$$

where $(n_1^2 k_0^2 - \beta^2) = \beta_{t1}^2$ and $(n_2^2 k_0^2 - \beta^2) = \beta_{t2}^2$

Field analysis of weakly guiding fibers

- Assuming transverse variation in both r and ϕ, we find for the *wave equation*, in either region

$$\partial^2 E_x/\partial r^2 + (1/r)\partial E_x/\partial r + (1/r^2)\partial^2 E_x/\partial \phi^2 + \beta_t^2 E_x = 0$$

- We assume that the solution for E_x is a discrete series of modes, each of which has *separated* dependences on r, ϕ, and z in product form:

$$E_x = \sum_i R_i(r)\Phi_i(\phi)\exp(-i\beta_i z)$$

- Each term (mode) in the expansion must itself be a solution of the wave equation. A single mode, $E_x = R\Phi \exp(-i\beta z)$ can be substituted into the wave equation to obtain

$$(r^2/R)\, d^2R/dr^2 + (r/R)\, dR/dr + r^2\beta_t^2 = -(1/\Phi)\, d^2\Phi/d\phi^2$$

Field analysis of weakly guiding fibers

- The left-hand side depends only on r, whereas the right-hand side depends only on ϕ.

- Because r and ϕ vary *independently*, it follows that each side of the equation must be equal to a constant.

- Defining this constant as l^2, we can separate the equation into two equations

$$d^2\Phi/d\phi^2 + l^2\Phi = 0$$

$$d^2R/dr^2 + (1/r)\, dR/dr + (\beta_t^2 - l^2/r^2)R = 0$$

- We identify the term l/r as β_ϕ for LP modes.
- The bracketed term therefore becomes $\beta_t^2 - l^2/r^2 = \beta_r^2$

Solving the Φ wave equation

- We can now readily obtain solutions to the Φ equation:

$$\Phi(\phi) = \cos(l\phi + \alpha) \text{ or } \sin(l\phi + \alpha)$$

  where α is a constant phase shift.

- l must be an *integer* because the field must be *self-consistent* on each rotation of ϕ through 2π.

- The quantity l is known as the angular or *azimuthal* mode number for LP modes.

Solving the R wave equation

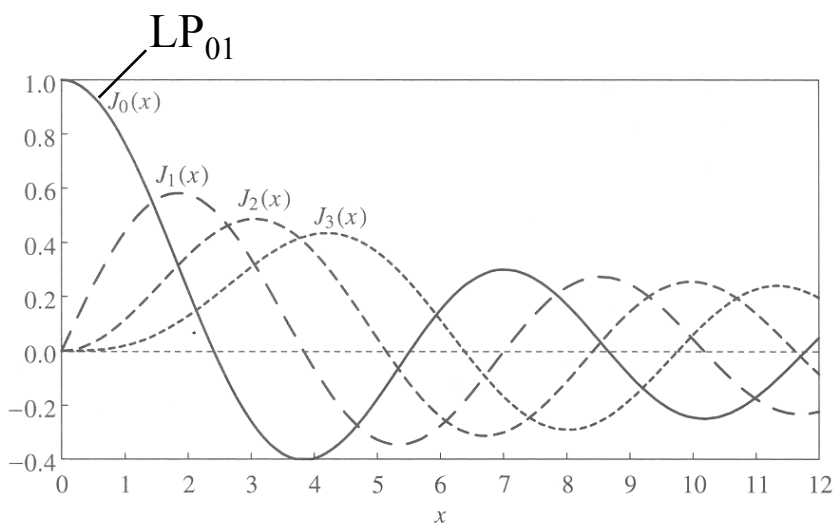
- The R-equation is a form of ***Bessel's equation***. Its solution is in terms of ***Bessel functions*** and assumes the form

$$R(r) = A\, J_l(\beta_t r) \qquad \beta_t \text{ real}$$

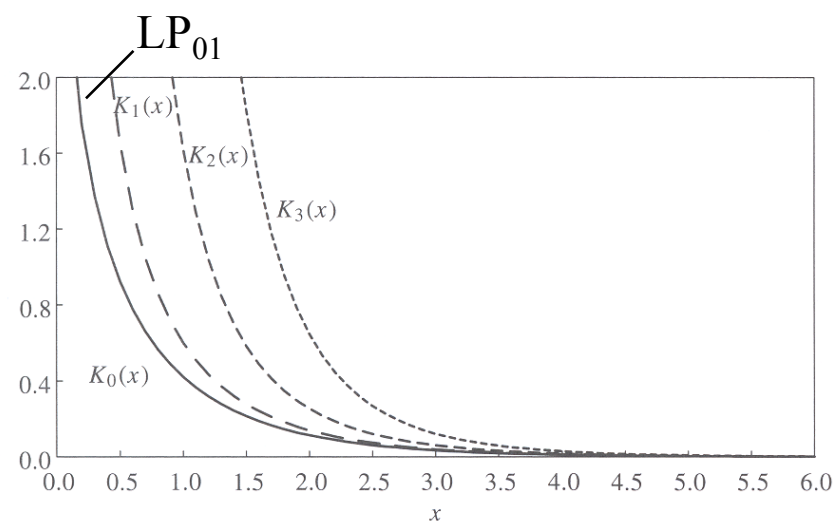
$$= C\, K_l(|\beta_t| r) \qquad \beta_t \text{ imaginary}$$

where J_l are <u>ordinary Bessel functions of the first kind</u> of order l, which apply to cases of *real* β_t. If β_t is *imaginary*, then the solution consists of <u>modified Bessel functions</u> K_l.

Bessel functions



Ordinary Bessel functions of the first kind



Modified Bessel functions of the second kind

- The *ordinary* Bessel function J_l is *oscillatory*, exhibiting no singularities (*appropriate for the field within the core*).

- The *modified* Bessel function K_l resembles an *exponential decay* (*appropriate for the field in the cladding*).

Complete solution for E_x and H_y

- Define *normalized transverse phase / attenuation constants*,

$$u = \beta_{t1} a = a(n_1^2 k_0^2 - \beta^2)^{1/2}$$
$$w = |\beta_{t2}| a = a(\beta^2 - n_2^2 k_0^2)^{1/2}$$

- Using the $\cos(l\phi)$ dependence (with constant phase shift $\alpha = 0$), we obtain the complete solution for E_x:

$$E_x = A\, J_l(ur/a) \cos(l\phi) \exp(-i\beta z) \qquad r \le a$$
$$E_x = C\, K_l(wr/a) \cos(l\phi) \exp(-i\beta z) \qquad r \ge a$$

- Similarly, we can solve the wave equation for H_y

$$H_y = B\, J_l(ur/a) \cos(l\phi) \exp(-i\beta z) \qquad r \le a$$
$$H_y = D\, K_l(wr/a) \cos(l\phi) \exp(-i\beta z) \qquad r \ge a$$

where $A \approx Z\,B$ and $C \approx Z\,D$ in the *quasi-plane-wave* approximation, and $Z \approx Z_0/n_1 \approx Z_0/n_2$

Electric field for LP_{lm} modes

- Applying the field boundary conditions at the core-cladding interface:

$$E_{\phi 1}|_{r=a} = E_{\phi 2}|_{r=a} \qquad n_1^2 E_{r1}|_{r=a} = n_2^2 E_{r2}|_{r=a}$$

$$H_{\phi 1}|_{r=a} = H_{\phi 2}|_{r=a} \qquad \mu_1 H_{r1}|_{r=a} = \mu_2 H_{r2}|_{r=a}$$

where $\mu_1 = \mu_2 = \mu_0$, $H_{r1}|_{r=a} = H_{r2}|_{r=a}$.

- In the *weak-guidance approximation*, $n_1 \approx n_2$, so $E_{r1}|_{r=a} \approx E_{r2}|_{r=a}$

$$\Rightarrow E_{x1}|_{r=a} \approx E_{x2}|_{r=a} \qquad H_{y1}|_{r=a} \approx H_{y2}|_{r=a}$$

- Suppose $A = E_0$,

$$E_x = E_0\, J_l(ur/a) \cos(l\phi) \exp(-i\beta z) \qquad (r \le a)$$

$$E_x = E_0\, [J_l(u)/K_l(w)]\, K_l(wr/a) \cos(l\phi) \exp(-i\beta z) \quad (r \ge a)$$

Electric fields of the fundamental mode

- The fundamental mode LP_{01} has $l = 0$ (assumed x-polarized)

$$E_x = E_0\, J_0(ur/a) \exp(-i\beta z) \qquad (r \le a)$$

$$E_x = E_0\, [J_0(u)/K_0(w)]\, K_0(wr/a) \exp(-i\beta z) \quad (r \ge a)$$

These fields are cylindrically symmetrical, i.e. there is no variation of the field in the angular direction.

They approximate a *Gaussian* distribution. (see the $J_0(x)$ distribution on p. 40)

Intensity patterns

- The LP modes are observed as intensity patterns.

- Analytically we evaluate the *time-average Poynting vector*

$$|\langle S\rangle| = (1/2Z)\,|E_x|^2$$

Defining the peak intensity $I_0 = (1/2Z)\,|E_0|^2$, we find the intensity functions in the core and cladding for any LP mode

$$I_{lm} = I_0\, J_l^2(ur/a)\cos^2(l\phi) \qquad r \le a$$

$$I_{lm} = I_0\, (J_l(u)/K_l(w))^2\, K_l^2(wr/a)\cos^2(l\phi) \qquad r \ge a$$

Eigenvalue equation for LP modes

- We use the requirement for continuity of the z components of the fields at r = a

$$H_z = (i/\omega\mu)\,(\nabla \times E)_z$$

$$\Rightarrow \quad (\nabla \times E_1)_z|_{r=a} = (\nabla \times E_2)_z|_{r=a}$$

- Convert **E** into cylindrical components

$$\mathbf{E_1} = E_0\, J_l(ur/a)\cos(l\phi)\,(\mathbf{a_r}\cos\phi - \mathbf{a_\phi}\sin\phi)\exp(-i\beta z)$$

$$\mathbf{E_2} = E_0\,[J_l(u)/K_l(w)]\,K_l(wr/a)\cos(l\phi)\,(\mathbf{a_r}\cos\phi - \mathbf{a_\phi}\sin\phi)\exp(-i\beta z)$$

Eigenvalue equation for LP modes

- Taking the curl of E_1 and E_2 in cylindrical coordinates:

$$(\nabla \times E_1)_z = (E_0/r)\,\{[lJ_l(ur/a) - (ur/a)J_{l-1}(ur/a)]\cos(l\phi)\sin\phi + lJ_l(ur/a)\sin(l\phi)\cos\phi\}$$

$$(\nabla \times E_2)_z = (E_0/r)(J_l(u)/K_l(u))\{[lK_l(wr/a) - (wr/a)K_{l-1}(wr/a)]\cos l\phi\sin\phi + lK_l(wr/a)\sin(l\phi)\cos\phi\}$$

where we have used the derivative forms of Bessel functions.

- Using $(\nabla \times E_1)_z|_{r=a} = (\nabla \times E_2)_z|_{r=a}$

$$uJ_{l-1}(u)/J_l(u) = -w\,K_{l-1}(w)/K_l(w)$$

This is the *eigenvalue equation for LP modes in the step-index fiber*.

Cutoff condition

- ***Cutoff*** for a given mode can be determined directly from the eigenvalue equation by setting w = 0 (see p.41),

$$u = V = V_c \qquad \text{(Recall from p.21 } V^2 = u^2 + w^2\text{)}$$

where V_c is the cutoff (or *minimum*) value of V for the mode of interest.

- The cutoff condition according to the eigenvalue equation is

$$V_c J_{l-1}(V_c)/J_l(V_c) = 0$$

When $V_c \neq 0$, $J_{l-1}(V_c) = 0$

e.g. $V_c = 2.405$ as the cutoff value of V for the LP_{11} mode.

Attenuation in fibers

Transmission characteristics of optical fibers

- The transmission characteristics of most interest: **attenuation (*loss*)** and **bandwidth**.

- Now, *silica-based* glass fibers have losses less than 0.2 dB/km (i.e. 95 % launched power remains after 1 km of fiber transmission). This is essentially the *fundamental lower limit* for attenuation in silica-based glass fibers.

- **Fiber bandwidth** is limited by the *signal dispersion* within the fiber. Bandwidth determines the number of bits of information transmitted in a given time period. Now, fiber bandwidth has reached many 10's Gbit over many km's per wavelength channel.

Attenuation

- Signal attenuation within optical fibers is usually expressed in the logarithmic unit of the decibel.

The decibel, which is used for comparing two *power* levels, may be defined for a particular optical wavelength as the *ratio* of the output optical power P_o from the fiber to the input optical power P_i.

$$\text{Loss (dB)} = -10 \log_{10} (P_o/P_i) = 10 \log_{10} (P_i/P_o)$$

$$(P_o \leq P_i)$$

*In *electronics*, $\text{dB} = 20 \log_{10} (V_o/V_i)$

Attenuation in dB/km

*The logarithmic unit has the advantage that the operations of *multiplication* (*and division*) reduce to *addition* (*and subtraction*).

In numerical values: $P_o/P_i = 10^{[-\text{Loss(dB)}/10]}$

The attenuation is usually expressed in decibels per unit length (i.e. dB/km):

$$\gamma L = -10 \log_{10} (P_o/P_i)$$

γ (dB/km): signal attenuation per unit length in decibels

L (km): fiber length

dBm

- dBm is a specific unit of power in decibels when the reference power is 1 mW:

$$\text{dBm} = 10 \log_{10} (\text{Power}/1\ \text{mW})$$

e.g. 1 mW = 0 dBm; 10 mW = 10 dBm; 100 μW = - 10 dBm

=> Loss (dB) = input power (dBm) - output power (dBm)

e.g. Input power = 1 mW (0 dBm), output power = 100 μW (-10 dBm)

$$\Rightarrow \text{loss} = -10 \log_{10} (100\ \mu\text{W}/1\ \text{mW}) = 10\ \text{dB}$$

OR 0 dBm – (-10 dBm) = 10 dB

Fiber attenuation mechanisms

1. Material absorption
2. Scattering loss
3. Nonlinear loss
4. Bending loss
5. Mode coupling loss

- ***Material absorption*** is a loss mechanism related to both *the material composition* and the *fabrication process* for the fiber. The optical power is lost as *heat* in the fiber.

- The light absorption can be *intrinsic* (*due to the material components of the glass*) or *extrinsic* (*due to impurities introduced into the glass during fabrication*).

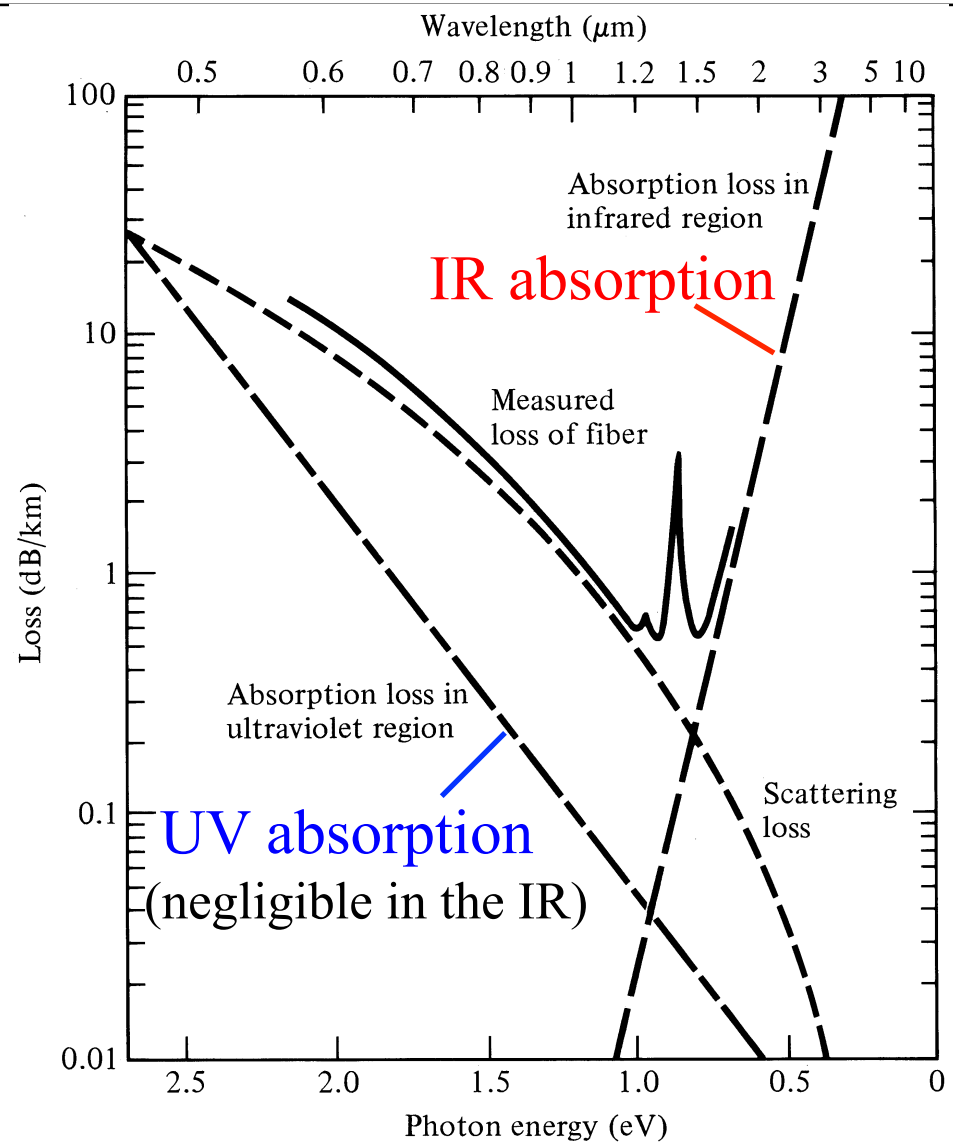
Intrinsic absorption

- Pure silica-based glass has *two* major intrinsic absorption mechanisms at optical wavelengths:

(1) a *fundamental UV absorption* edge, the peaks are centered in the *ultraviolet wavelength region*. This is due to the *electron transitions* within the glass molecules. The tail of this peak may extend into the the shorter wavelengths of the fiber transmission spectral window.

(2) A fundamental *infrared and far-infrared absorption edge*, due to *molecular vibrations* (such as Si-O). The tail of these absorption peaks may extend into the longer wavelengths of the fiber transmission spectral window.

Fundamental fiber attenuation characteristics



Electronic and molecular absorption

- ***Electronic absorption***: the bandgap of fused silica is about 8.9 eV (~140 nm). This causes strong absorption of light in the UV spectral region due to electronic transitions across the bandgap.

  In practice, the bandgap of a material is not sharply defined but usually has ***bandtails*** extending from the conduction and valence bands into the bandgap due to a variety of reasons, such as *thermal vibrations of the lattice ions* and *microscopic imperfections of the material structure*.

  An *amorphous* material like fused silica generally has very long bandtails. These bandtails lead to an absorption tail extending into the visible and infrared regions. Empirically, the absorption tail at photon energies below the bandgap falls off exponentially with photon energy.

Electronic and molecular absorption

- ***Molecular absorption***: in the infrared region, the absorption of photons is accompanied by transitions between different *vibrational modes* of silica molecules.

- The *fundamental vibrational transition* of fused silica causes a very strong absorption peak at about 9 μm wavelength.

- *Nonlinear effects* contribute to important harmonics and combination frequencies corresponding to minor absorption peaks at 4.4, 3.8 and 3.2 μm wavelengths.

  => a long absorption tail extending into the near infrared, causing a sharp rise in absorption at optical wavelengths longer than 1.6 μm.

Extrinsic absorption

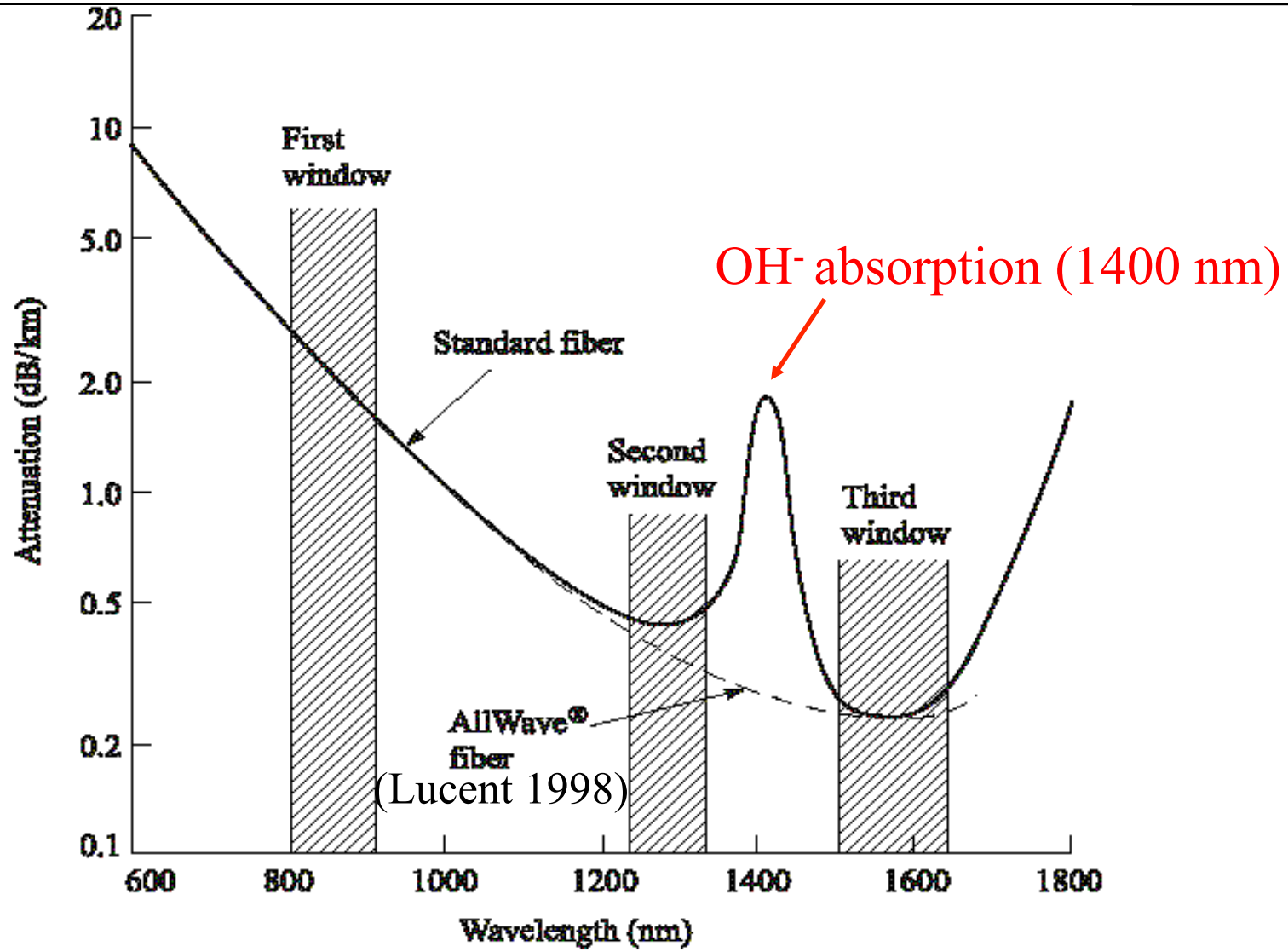
- Major extrinsic loss mechanism is caused by absorption due to water (*as the hydroxyl or OH^- ions*) introduced in the glass fiber during *fiber pulling by means of oxyhydrogen flame*.

- These OH^- ions are bonded into the glass structure and have absorption peaks (due to *molecular vibrations*) at 1.39 μm. The fundamental vibration of the OH^- ions appear at 2.73 μm.

- Since these OH^- absorption peaks are sharply peaked, narrow spectral windows exist around 1.3 μm and 1.55 μm which are essentially unaffected by OH^- absorption.

- The lowest attenuation for typical silica-based fibers occur at wavelength 1.55 μm at about 0.2 dB/km, approaching the *minimum possible attenuation* at this wavelength.

1400nm OH^- absorption peak and spectral windows



Impurity absorption

- ***Impurity absorption***: most impurity ions such as OH^-, Fe^{2+} and Cu^{2+} form absorption bands in the *near infrared* region where both electronic and molecular absorption losses of the host silica glass are very low.
- Near the peaks of the impurity absorption bands, an impurity concentration as low as *one part per billion* can contribute to an absorption loss as high as 1 dB km^{-1}.
- In fact, fiber-optic communications were not considered possible until it was realized in 1966 (Kao) that most losses in earlier fibers were caused by impurity absorption and then ultra-pure fibers were produced in the early 1970s (Corning).
- Today, impurities in fibers have been reduced to levels where losses associated with their absorption are negligible, with the exception of the OH^- radical.

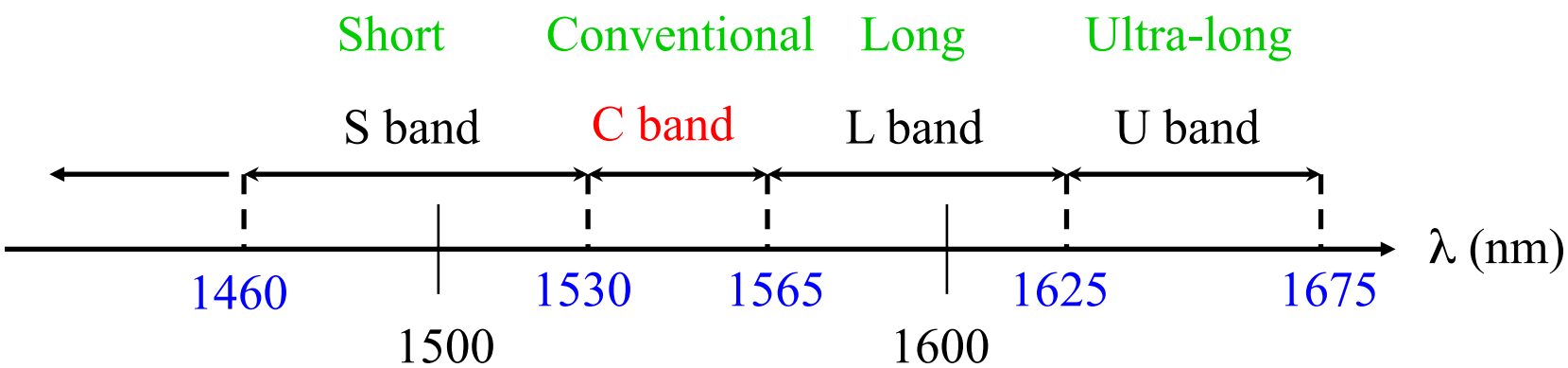
Three major fiber transmission spectral windows

The 1st window: 850 nm, attenuation 4 dB/km

The 2nd window: 1300 nm, attenuation 0.5 dB/km

The 3rd window: 1550 nm, attenuation 0.2 dB/km

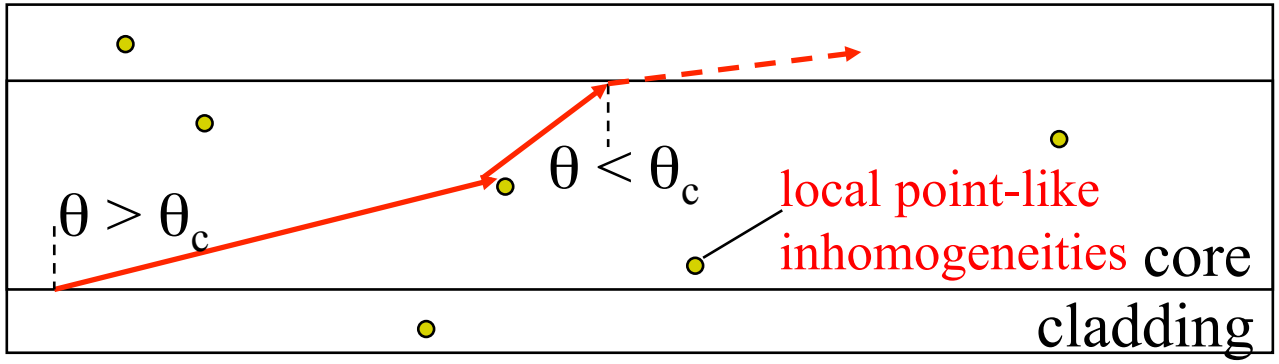
1550 nm window is today's standard **long-haul** communication wavelengths.



Scattering loss

Scattering results in attenuation (*in the form of radiation*) as the scattered light may not continue to satisfy the total internal reflection in the fiber core.

One major type of scattering is known as *Rayleigh scattering*.



The scattered ray can escape by refraction according to Snell's Law.

Rayleigh scattering

- *Rayleigh scattering* results from random *inhomogeneities* that are small in size compared with the wavelength.

$$\circ \quad << \quad \lambda$$

- These inhomogeneities exist in the form of *refractive index fluctuations* which are frozen into the *amorphous* glass fiber upon fiber pulling. Such fluctuations *always exist and cannot be avoided* !

Rayleigh scattering results in an attenuation (dB/km) $\propto 1/\lambda^4$

Where else do we see Rayleigh scattering?

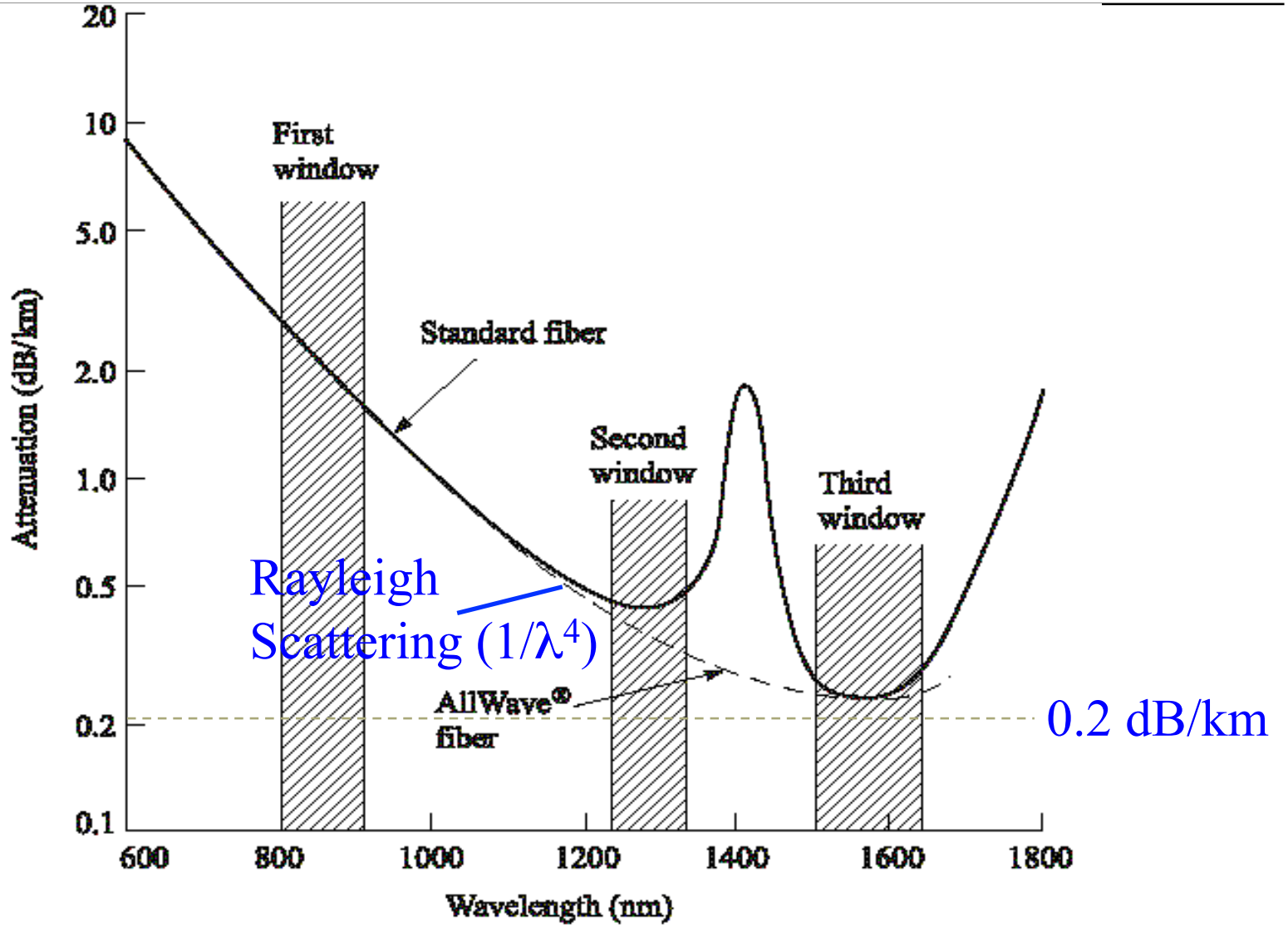
Rayleigh scattering

- The intrinsic Rayleigh scattering in a fiber is caused by *variations in density* and *composition* that are built into the fiber during the manufacturing process. They are primarily a result of ***thermal fluctuations*** in the *density of silica glass* and *variations in the concentration of dopants* before silica passes its glass transition point to become a solid.
- These variations are a ***fundamental thermodynamic phenomenon*** and *cannot be completely removed*. They create microscopic fluctuations in the index of refraction, which scatter light in the same manner as microscopic fluctuations of the density of air scatter sunlight.
- This elastic Rayleigh scattering process creates a loss given by

$$\alpha_R = \frac{8\pi^2}{3\lambda^4}(n^2 - 1)\beta k_B T$$

n: Index of refraction
k_B: Boltzmann constant
T: glass transition temperature
β: isothermal compressibility

Rayleigh scattering is the dominant loss in today's fibers



Waveguide scattering

- ***Imperfections in the waveguide structure*** of a fiber, such as nonuniformity in the size and shape of the core, perturbations in the core-cladding boundary, and defects in the core or cladding, can be generated in the manufacturing process.

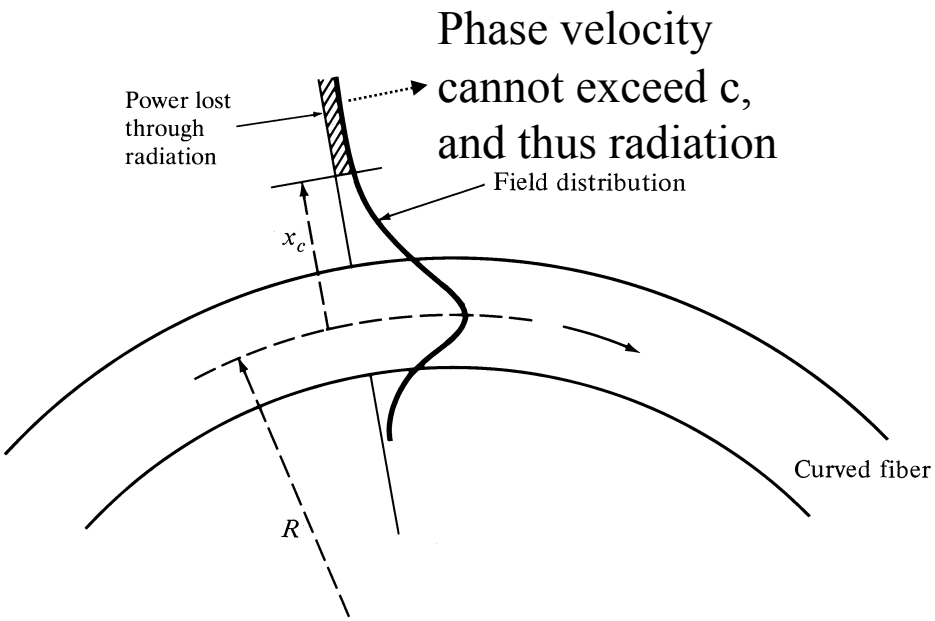
- Environmentally induced effects, such as stress and temperature variations, also cause imperfections.

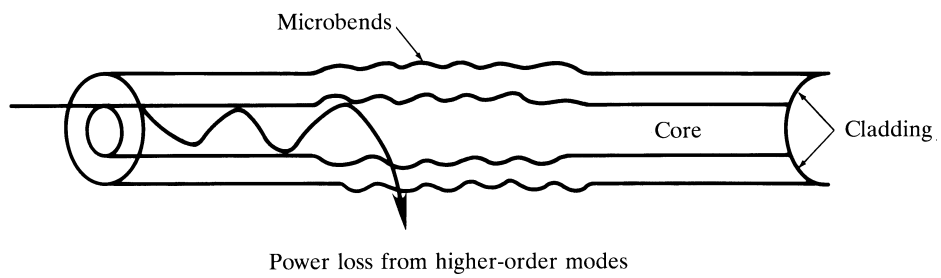
- The imperfections in a fiber waveguide result in additional scattering losses. They can also induce coupling between different guided modes.

Nonlinear losses

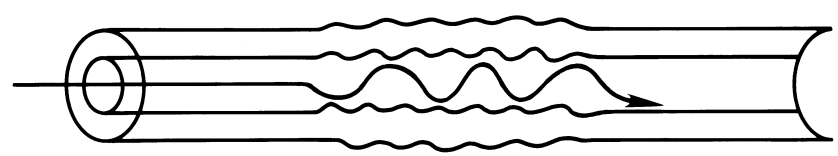
- Because light is confined over long distances in an optical fiber, ***nonlinear optical effects*** can become important even at a relatively moderate optical power.
- Nonlinear optical processes such as *stimulated Brillouin scattering* and *stimulated Raman scattering* can cause significant attenuation in the power of an optical signal.
- Other nonlinear processes can induce *mode mixing* or *frequency shift*, all contributing to the loss of a particular guided mode at a particular frequency.
- Nonlinear effects are *intensity dependent*, and thus they can become very important at high optical powers.

Fiber bending loss and mode-coupling to higher-order modes





Power loss from higher-order modes



Power coupling to higher-order modes

"macrobending"

(*how do we measure bending loss*?)

"microbending" – power coupling to higher-order modes that are more lossy.

Dispersion in fibers

Dispersion in fibers

- Dispersion is the primary cause of limitation on the optical signal transmission bandwidth through an optical fiber.
- Recall from Lecture 4 that there are *waveguide* and *modal dispersions* in an optical waveguide in addition to *material dispersion*.
- Both material dispersion and waveguide dispersion are examples of *chromatic dispersion* because both are frequency dependent.
- *Waveguide dispersion* is caused by frequency dependence of the propagation constant β of a specific mode due to the waveguiding effect. (*recall the b vs. V plot of a specific mode*)
- The combined effect of material and waveguide dispersions for a particular mode alone is called *intramode dispersion*.

Modal dispersion

- ***Modal dispersion*** is caused by the variation in propagation constant between different modes; it is also called *intermode dispersion*. (*recall the b vs. V plot at a fixed V*)
- Modal dispersion appears only when *more than one* mode is excited in a multimode fiber. It exists even when chromatic dispersion disappears.
- *If only one mode is excited in a fiber*, only intramode chromatic dispersion has to be considered even when the fiber is a multimode fiber.

Material dispersion

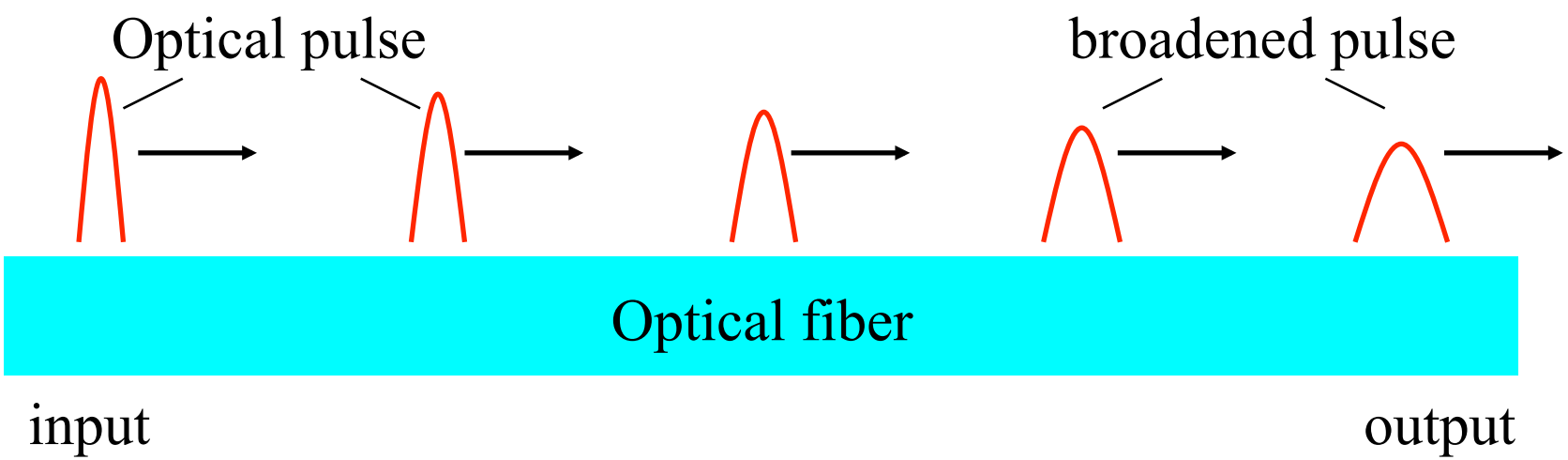
- For optical fibers, the materials of interest are pure silica and doped silica.
- The parameters of interest are the refractive index n, the group index n_g and the group velocity dispersion D.
- The index of refraction of pure silica in the wavelength range between 200 nm and 4 μm is given by the following empirically fitted Sellmeier equation:

$$n^2 = 1 + \frac{0.6961663\lambda^2}{\lambda^2 - (0.0684043)^2} + \frac{0.4079426\lambda^2}{\lambda^2 - (0.1162414)^2} + \frac{0.8974794\lambda^2}{\lambda^2 - (9.896161)^2}$$

  where λ is in micrometers.
- The index of refraction can be changed by adding dopants to silica, thus facilitating the means to control the index profile of a fiber. Doping with *germania* (GeO_2) or *alumina* increases the index of refraction.

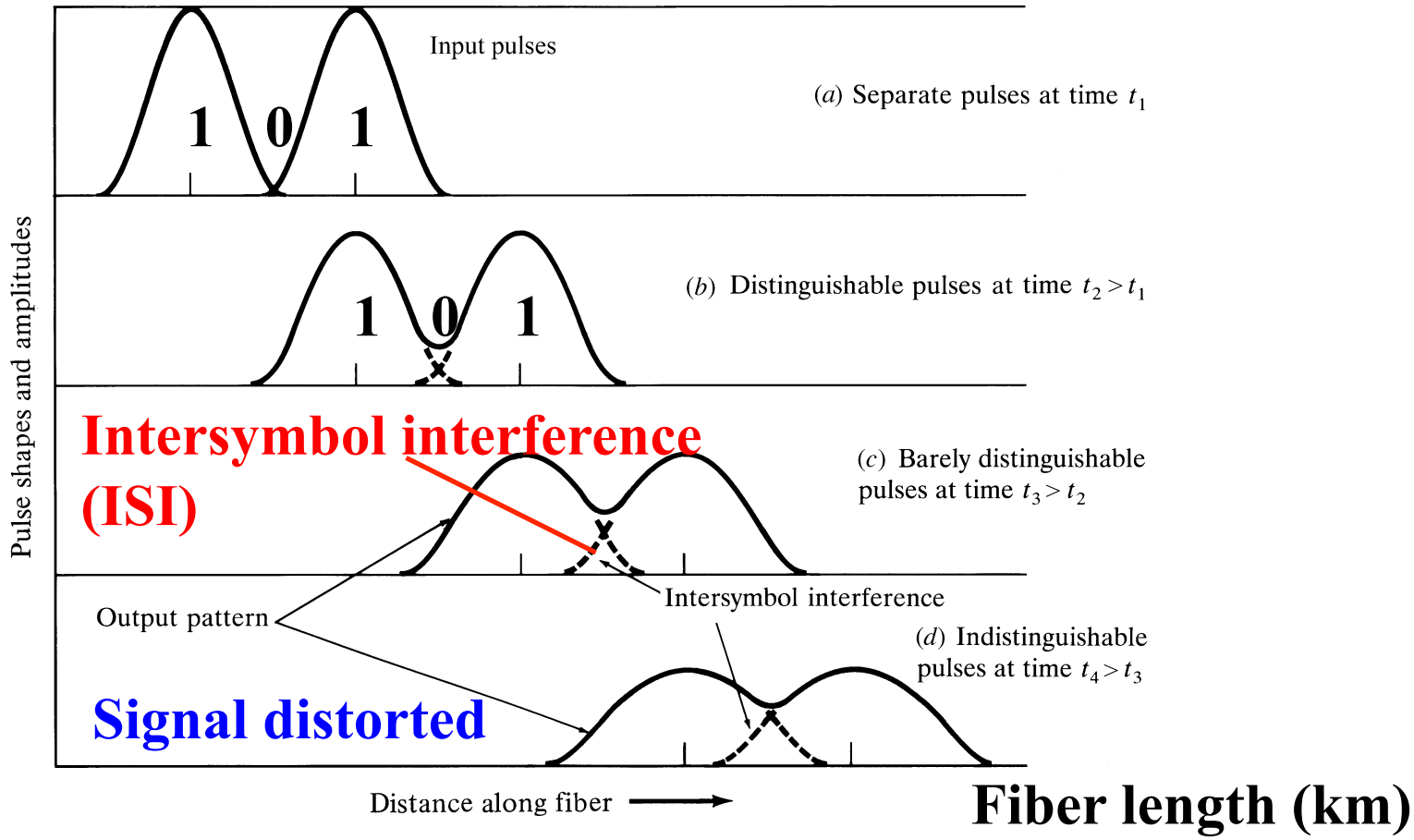
Fiber dispersion

- Fiber dispersion results in *optical pulse broadening* and hence *digital signal degradation*.



Dispersion mechanisms: 1. Modal (or *intermodal*) dispersion
2. Chromatic dispersion (CD)
3. Polarization mode dispersion (PMD)

Pulse broadening limits fiber bandwidth (data rate)



- An *increasing number of errors* may be encountered on the digital optical channel as the ISI becomes more pronounced.

Modal dispersion

- When numerous waveguide modes are propagating, they all travel with different velocities with respect to the waveguide axis.

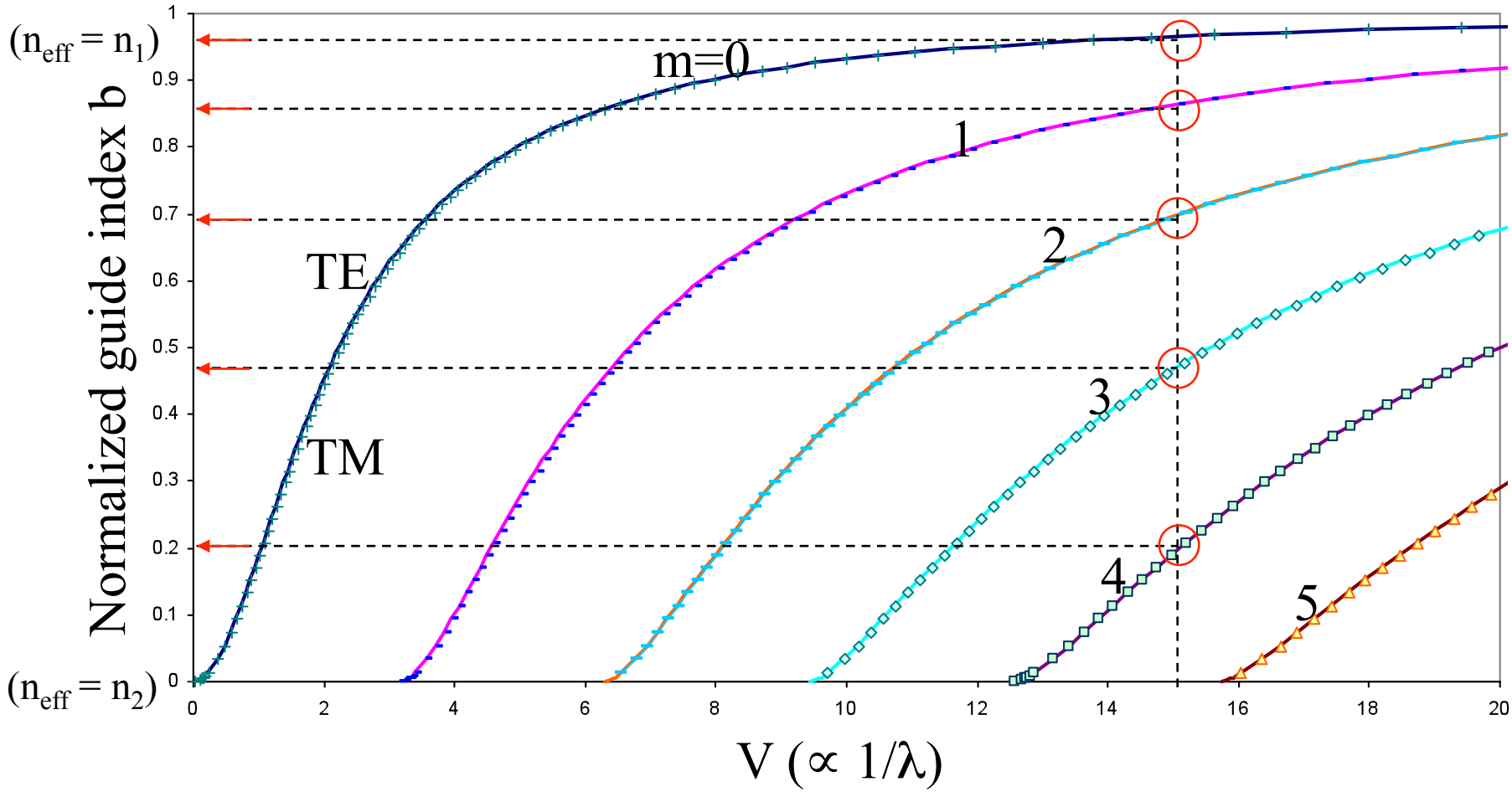
- An input waveform distorts during propagation because its energy is distributed among several modes, each traveling at a different speed.

- Parts of the wave arrive at the output before other parts, spreading out the waveform. This is thus known as multimode (modal) dispersion.

- Multimode dispersion does *not* depend on the source linewidth (even a *single* wavelength can be simultaneously carried by *multiple modes* in a waveguide).

- Multimode dispersion would *not* occur if the waveguide allows *only* one mode to propagate - the advantage of *single*-mode waveguides!

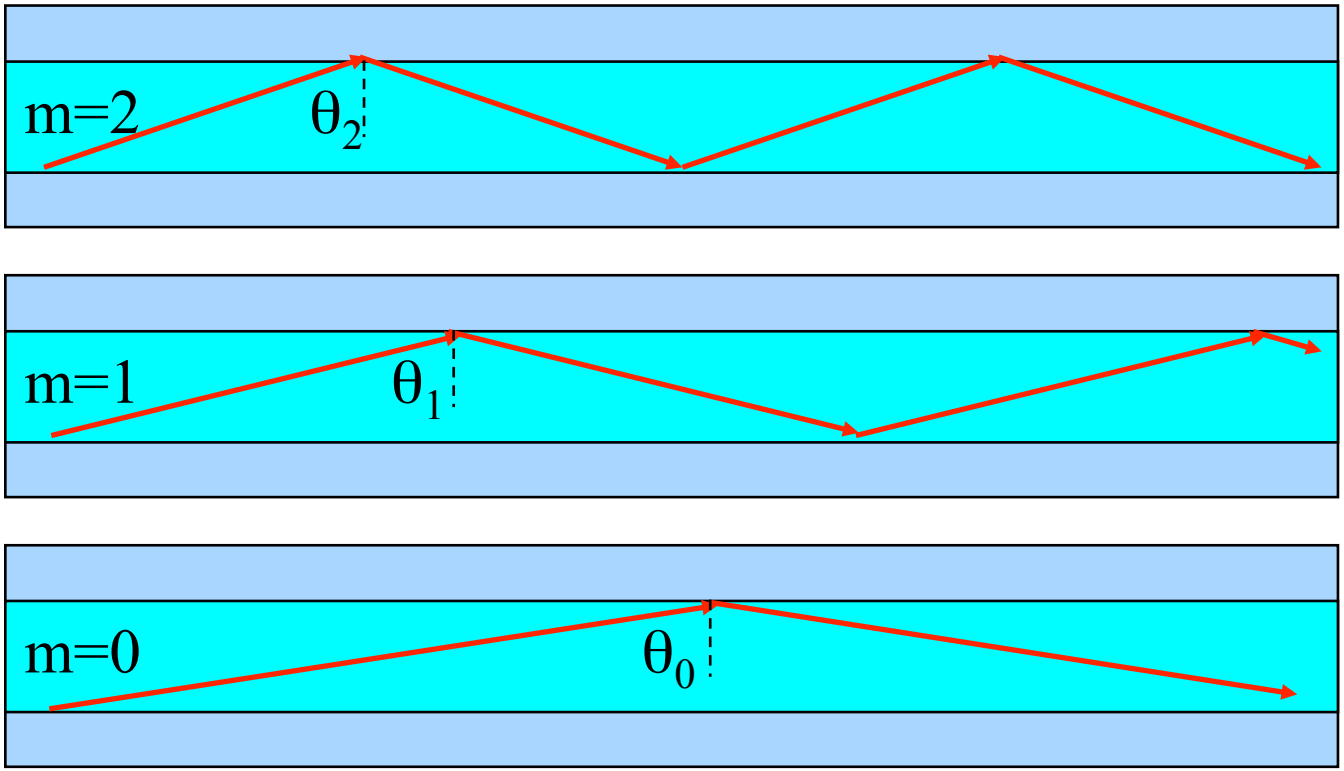
Modal dispersion as shown from the mode chart of a symmetric slab waveguide



- Phase velocity for mode m = $\omega/\beta_m = \omega/(n_{eff}(m)\, k_0)$

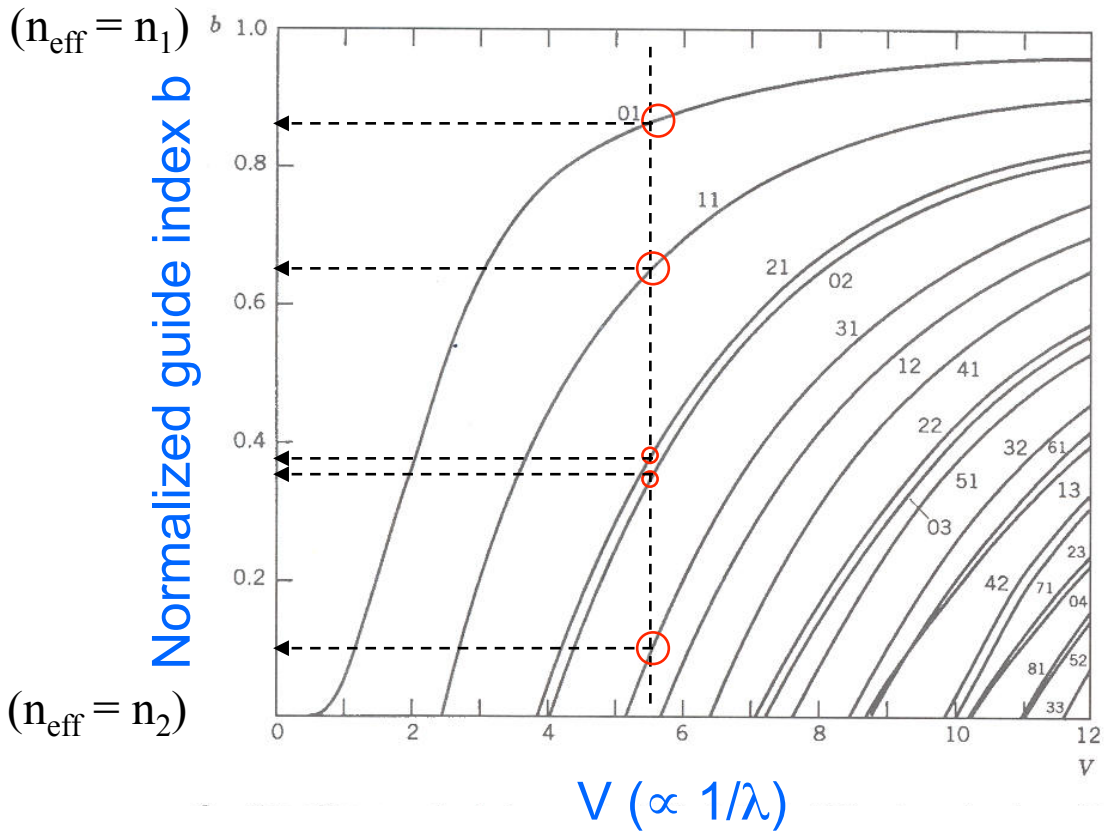
(note that m = 0 mode is the *slowest* mode)

Modal dispersion in multimode waveguides



The carrier wave can propagate along all these different "zig-zag" ray paths of *different path lengths*.

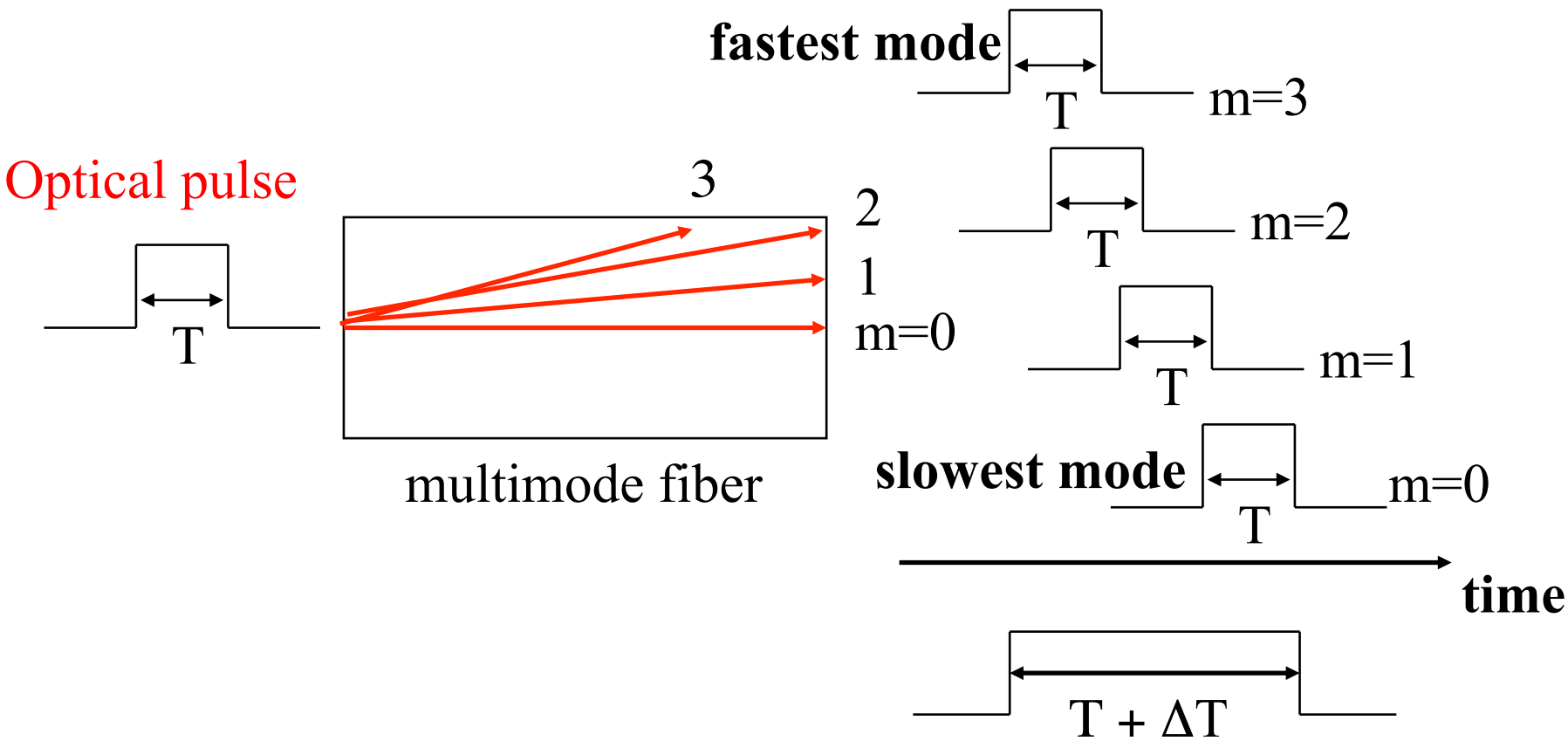
Modal dispersion as shown from the LP mode chart of a silica optical fiber



- Phase velocity for LP mode = $\omega/\beta_{lm} = \omega/(n_{eff}(lm)\, k_0)$

(note that LP_{01} mode is the *slowest* mode)

Modal dispersion results in pulse broadening

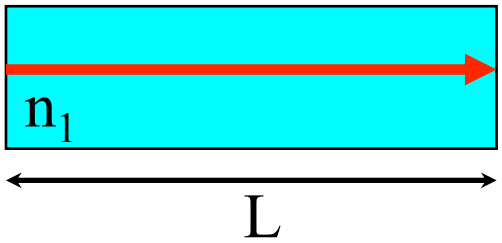


modal dispersion: different modes arrive at the receiver with different delays => pulse broadening

Estimate modal dispersion pulse broadening using phase velocity

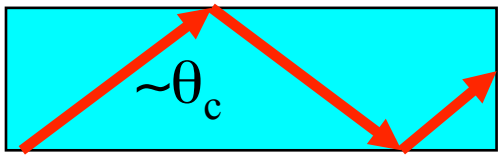
- A zero-order mode traveling near the waveguide axis needs time:

$t_0 = L/v_{m=0} \approx Ln_1/c$ $\quad (v_{m=0} \approx c/n_1)$



- The highest-order mode traveling near the critical angle needs time:

$t_m = L/v_m \approx Ln_2/c$ $\quad (v_m \approx c/n_2)$



=> the *pulse broadening* due to modal dispersion:

$$\Delta T \approx t_0 - t_m \approx (L/c)\,(n_1 - n_2)$$

$$\approx (L/2cn_1)\,NA^2 \qquad (n_1 \sim n_2)$$

How does modal dispersion restricts fiber bit rate?

e.g. How much will a light pulse spread after traveling along 1 km of a step-index fiber whose NA = 0.275 and n_{core} = 1.487?

Suppose we transmit at a low bit rate of 10 Mb/s

=> Pulse duration = $1 / 10^7$ s = 100 ns

Using the above e.g., each pulse will spread up to ≈ 100 ns (i.e. ≈ pulse duration !) every km

⇒The broadened pulses overlap! (**Intersymbol interference** (ISI))

*Modal dispersion limits the bit rate of a km-length fiber-optic link to ~ 10 Mb/s. (a coaxial cable supports this bit rate easily!)

Bit-rate distance product

- We can relate the pulse broadening ΔT to the *information-carrying capacity* of the fiber measured through the bit rate B.

- Although a precise relation between B and ΔT depends on many details, such as the pulse shape, it is intuitively clear that ΔT *should be less than the allocated bit time slot* given by 1/B.

⇒An *order-of-magnitude* estimate of the supported bit rate is obtained from the condition $B\Delta T < 1$.
⇒***Bit-rate distance product*** (limited by modal dispersion)

$$BL < 2c\, n_{core} / NA^2$$

This condition provides a rough estimate of a fundamental limitation of step-index multimode fibers. (*smaller the NA larger the bit-rate distance product*)

Bit-rate distance product

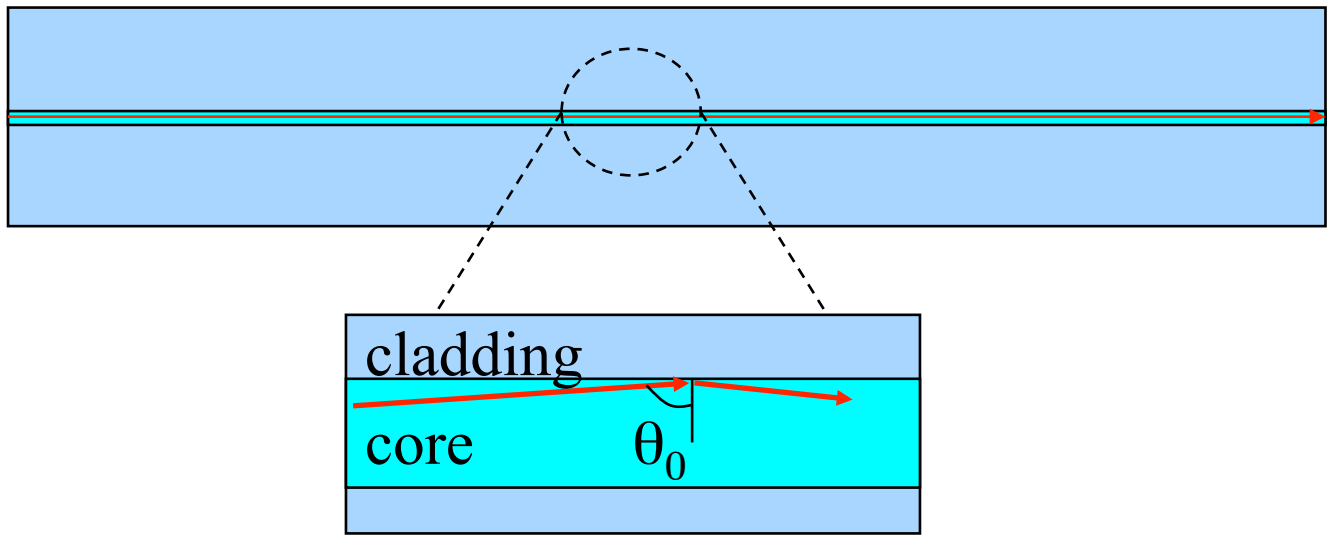
The capacity of optical communications systems is frequently measured in terms of the **bit rate-distance product**.

e.g. If a system is capable of transmitting 100 Mb/s over a distance of 1 km, it is said to have a *bit rate-distance* product of 100 (Mb/s)-km.

This may be suitable for some *local-area networks (LANs)*.

Note that the same system can transmit 1 Gb/s along 100 m, or 10 Gb/s along 10 m, or 100 Gb/s along 1 m, or 1 Tb/s along 10 cm, ...

Single-mode fiber eliminates modal dispersion

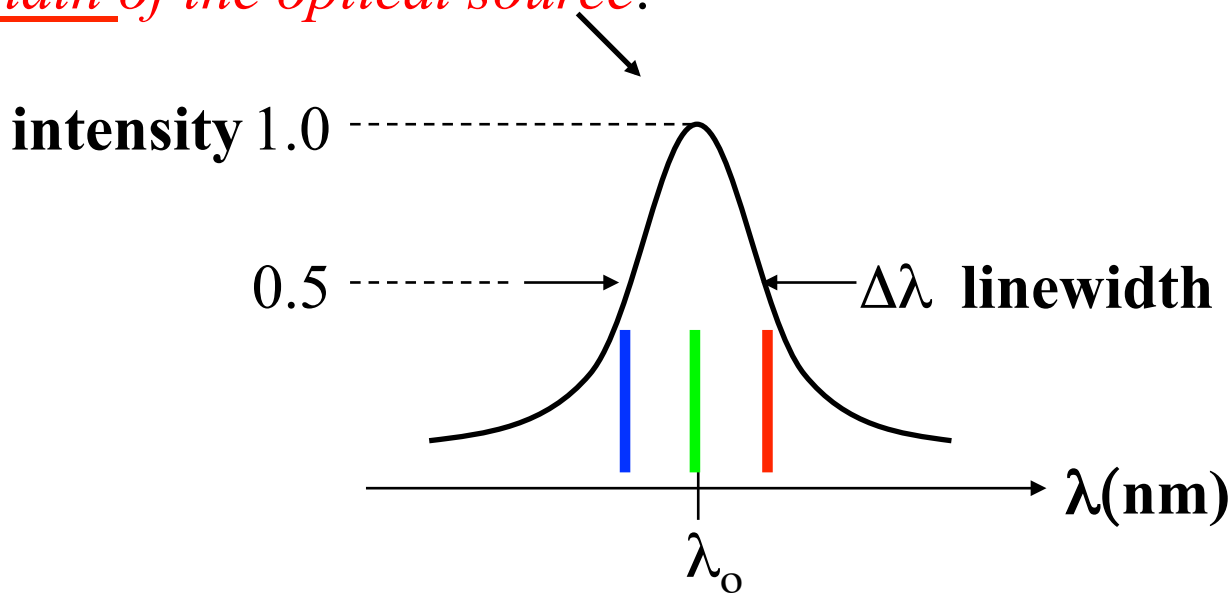


- The main advantage of *single*-mode fibers is to propagate *only one mode* so that *modal dispersion is absent*.

- However, *pulse broadening does not disappear altogether*. The *group velocity* associated with the fundamental mode is *frequency dependent* within the pulse *spectral linewidth* because of chromatic dispersion.

Chromatic dispersion

- Chromatic dispersion (CD) may occur in *all* types of optical fiber. The optical pulse broadening results from the *finite spectral linewidth of the optical source*.



*In the case of the semiconductor laser $\Delta\lambda$ corresponds to only a fraction of % of the centre wavelength λ_o. For LEDs, $\Delta\lambda$ is likely to be a significant percentage of λ_o.

Spectral linewidth

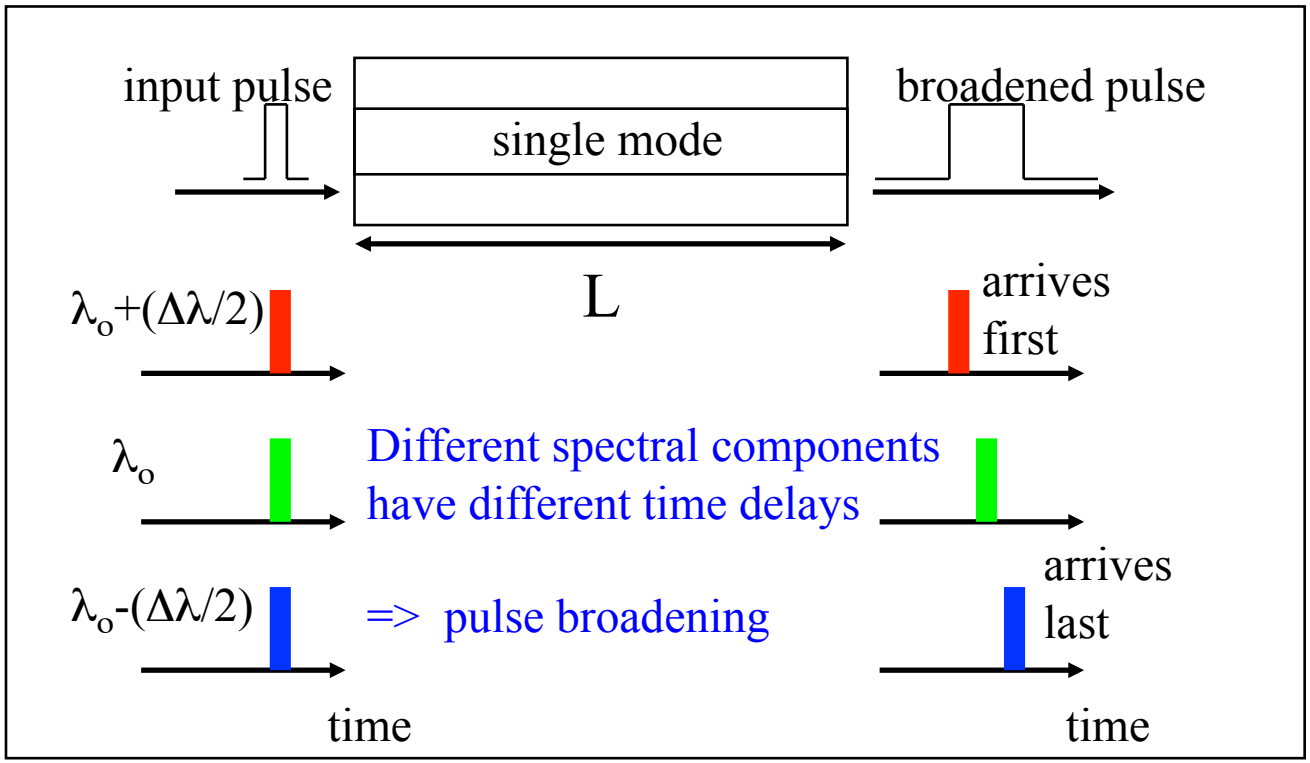
- Real sources emit over a range of wavelengths. This range is the *source linewidth* or *spectral width*.

- The smaller is the linewidth, the smaller the spread in wavelengths or frequencies, the more *coherent* is the source.

- A perfectly coherent source emits light at a *single* wavelength. It has *zero* linewidth and is *perfectly* monochromatic.

Light sources	**Linewidth (nm)**
Light-emitting diodes	20 nm – 100 nm
Semiconductor laser diodes	1 nm – 5 nm
Nd:YAG solid-state lasers	0.1 nm
HeNe gas lasers	0.002 nm

Chromatic dispersion



- Pulse broadening occurs because there may be *propagation delay differences* among the *spectral components* of the transmitted signal.
- **Chromatic dispersion (CD):** Different spectral components of a *pulse* travel at different *group velocities*. This is also known as *group velocity dispersion* (GVD).

Light pulse in a dispersive medium

When a *light pulse* with <u>a spread in frequency $\delta\omega$</u> and <u>a spread in propagation constant δk</u> propagates in a *dispersive* medium $n(\lambda)$, the group velocity:

$$v_g = (d\omega/dk) = (d\lambda/dk)\,(d\omega/d\lambda)$$

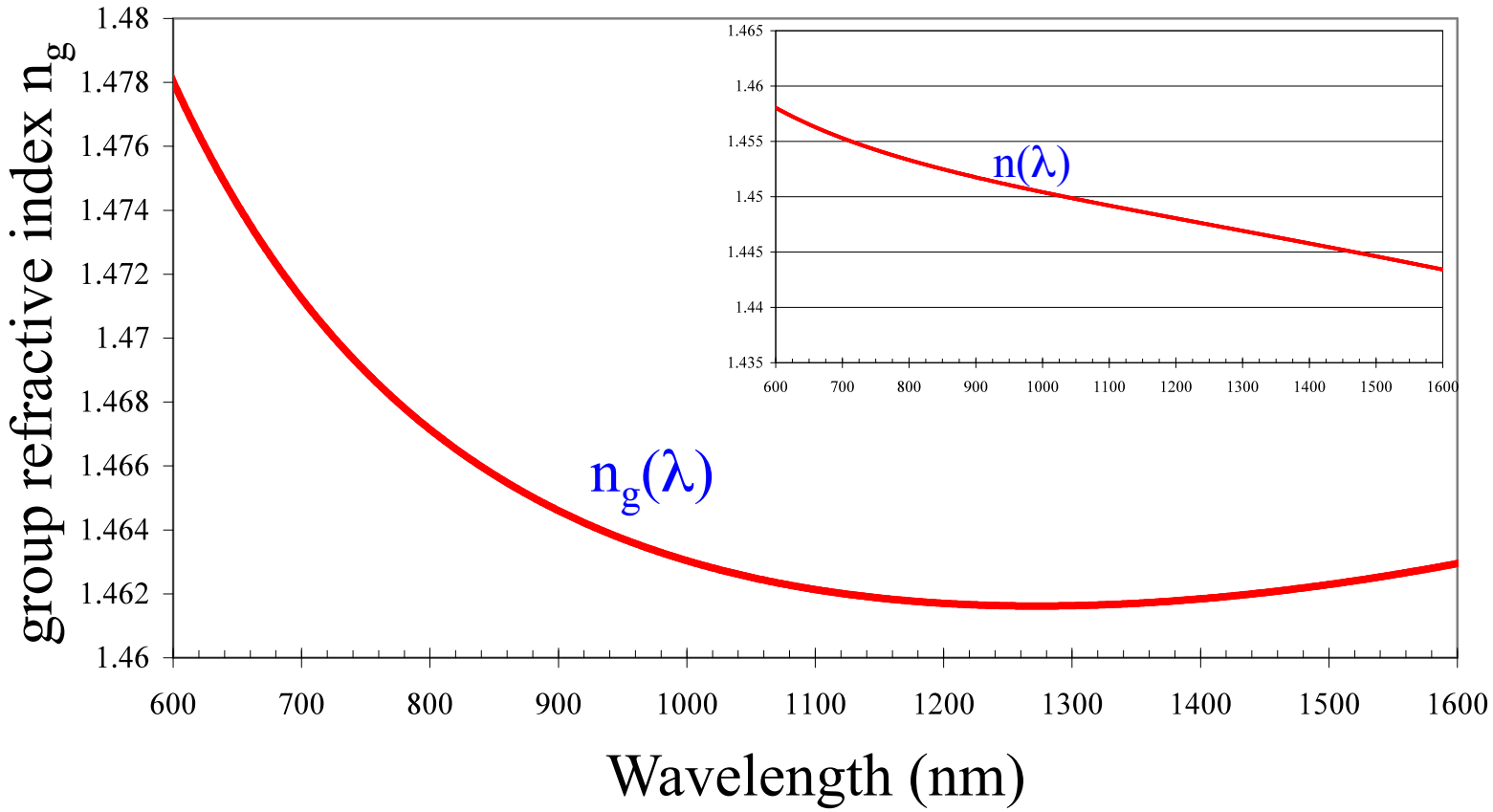
$$k = n(\lambda)\,2\pi/\lambda \quad \Rightarrow \quad dk/d\lambda = (2\pi/\lambda)\,[(dn/d\lambda) - (n/\lambda)]$$

$$\omega = 2\pi c/\lambda \quad \Rightarrow \quad d\omega/d\lambda = -2\pi c/\lambda^2$$

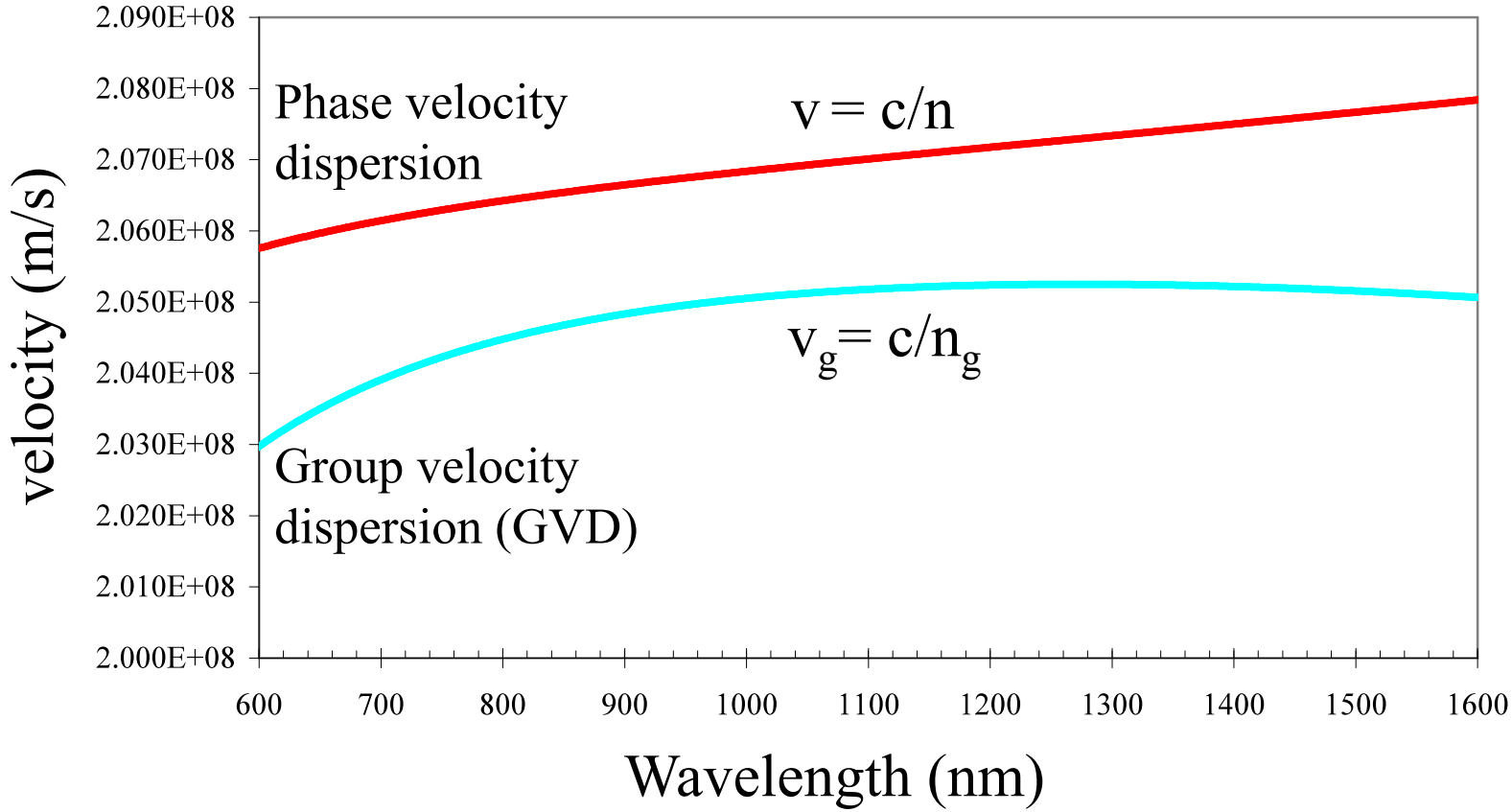
Hence $\quad v_g = c\,/\,[n - \lambda(dn/d\lambda)] = c\,/\,n_g$

Define the **group refractive index** $n_g = n - \lambda(dn/d\lambda)$

Group refractive index n_g vs. λ for fused silica



Phase velocity c/n and group velocity c/n_g vs. λ for fused silica



Group-Velocity Dispersion (GVD)

Consider a single mode fiber of length L

- A specific spectral component at the frequency ω (or wavelength λ) would arrive at the output end of the fiber after a *time delay*:

$$T = L/v_g$$

- If $\Delta\lambda$ is the *spectral width* of an optical pulse, the extent of *pulse broadening* for a fiber of length L is given by

$$\Delta T = (dT/d\lambda)\,\Delta\lambda = [d(L/v_g)/d\lambda]\,\Delta\lambda$$

$$= L\,[d(1/v_g)/d\lambda]\,\Delta\lambda$$

Group-Velocity Dispersion (GVD)

Hence the pulse broadening due to a *differential time delay*:

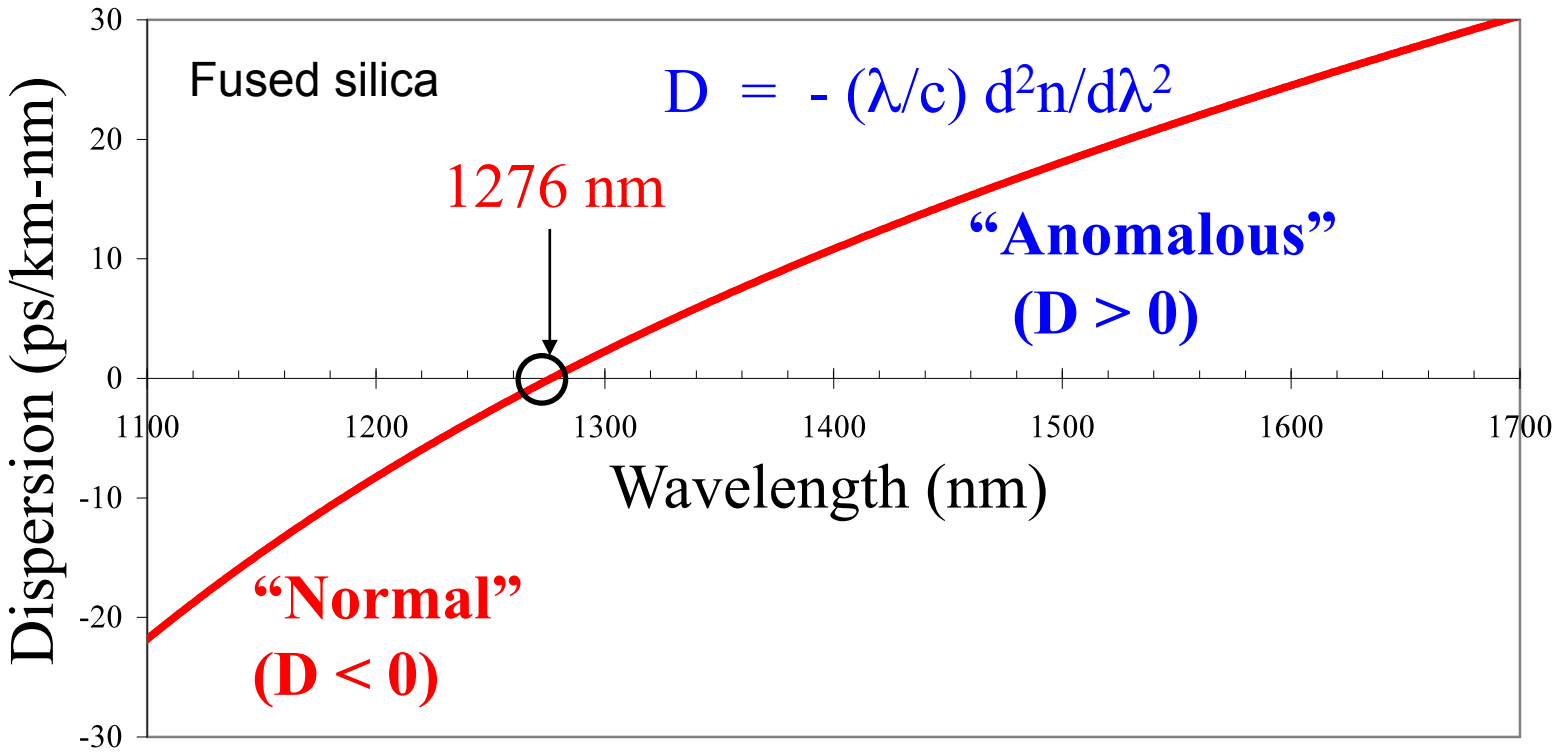
$$\Delta T = L\, D\, \Delta\lambda$$

where $D = d(1/v_g)/d\lambda$ is called the *dispersion parameter* and is expressed in units of ps/(km-nm).

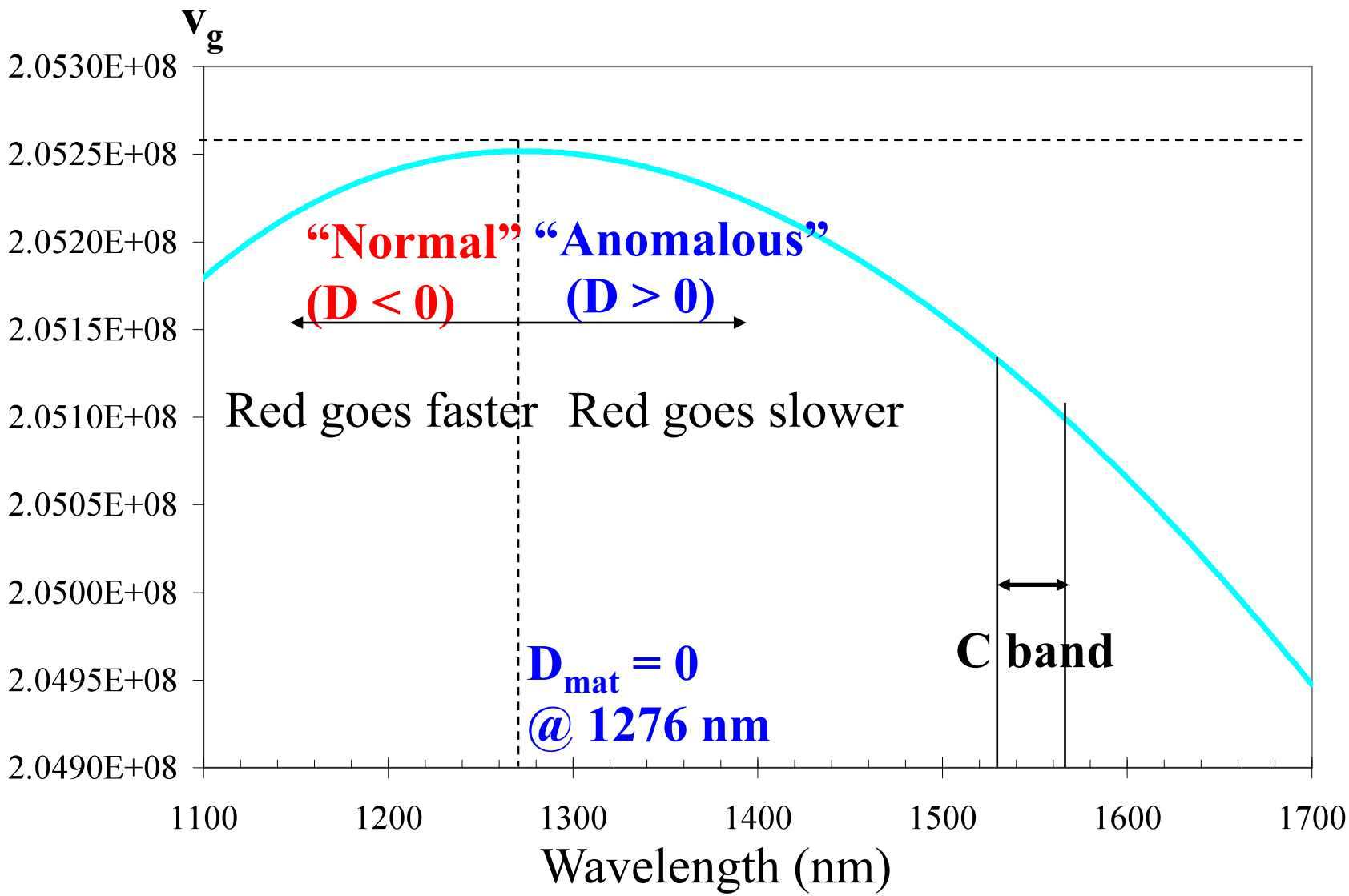
$$D = d(1/v_g)/d\lambda = c^{-1}\, dn_g/d\lambda = c^{-1}\, d[n - \lambda(dn/d\lambda)]/d\lambda$$

$$= -c^{-1}\, \lambda\, d^2n/d\lambda^2$$

Dispersion parameter



Variation of v_g with wavelength for fused silica



Zero-dispersion wavelength

Material dispersion $D_{mat} = 0$ at $\lambda \sim 1276$ nm for fused silica.

This λ is referred to as the ***zero-dispersion wavelength*** $\boldsymbol{\lambda_{ZD}}$.

Chromatic (or *material*) dispersion $D(\lambda)$ can be zero;
or
negative => longer wavelengths travel *faster* than shorter wavelengths;
or
positive => shorter wavelengths travel *faster* than longer wavelengths.

Waveguide dispersion

In fact there are <u>two</u> mechanisms for chromatic dispersion:

(a) Silica refractive index *n is wavelength dependent* (i.e. $n = n(\lambda)$)

=> different wavelength components travel at different speeds in silica

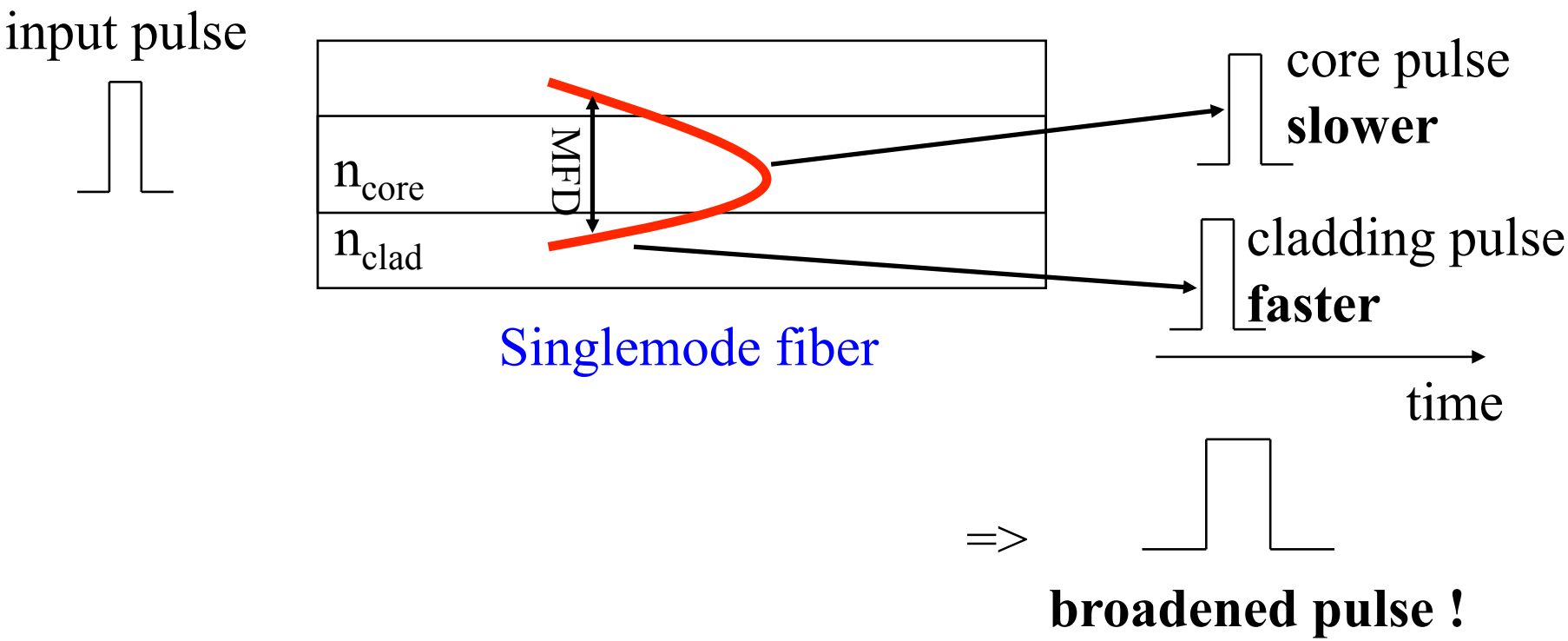
This is known as **<u>material dispersion</u>**.

(b) Light energy of a mode propagates *partly in the core and partly in the cladding*. The mode power distribution between the core and the cladding *depends on* λ. (*Recall the mode field diameter*)

This is known as **<u>waveguide dispersion</u>.**

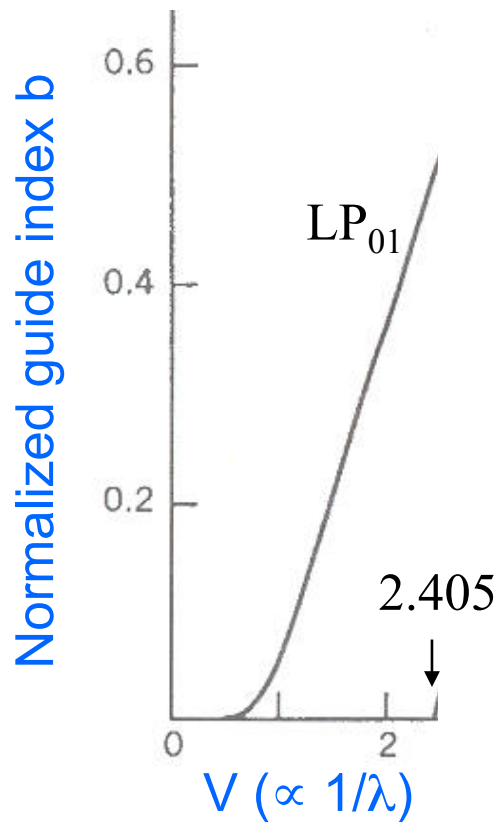
$$\Rightarrow D(\lambda) = D_{mat}(\lambda) + D_{wg}(\lambda)$$

Waveguide dispersion in a single-mode fiber



Waveguide dispersion depends on the *mode field distribution in the core and the cladding*. (i.e. the fiber V number)

Waveguide dispersion of the LP_{01} mode



- Different wavelength components λ of the LP_{01} mode see different effective indices n_{eff}

Waveguide group velocity and time delay

- Consider an optical pulse of linewidth $\Delta\lambda$ ($\Delta\omega$) and a corresponding spread of propagation constant $\Delta\beta$ propagating in a waveguide

Group velocity $v_{g,eff} = d\omega/d\beta$

or $v_{g,eff}^{-1} = d\beta/d\omega$

$$= d/d\omega\,(c^{-1}\,\omega\,n_{eff})$$

$$= c^{-1}\,(n_{eff} + \omega\,dn_{eff}/d\omega)$$

$$= c^{-1}\,(n_{eff} - \lambda\,dn_{eff}/d\lambda) = c^{-1}\,n_{g,eff}$$

Time delay after a waveguide of length L: $\tau = L/v_{g,eff}$

Or *time delay per unit length*: $\tau/L = v_{g,eff}^{-1}$

Waveguide dispersion parameter

- If $\Delta\lambda$ is the spectral width of an optical pulse, the extent of *pulse broadening* for a waveguide of length L is given by

$$\Delta\tau = (d\tau/d\lambda)\,\Delta\lambda = [d(L/v_{g,eff})/d\lambda]\,\Delta\lambda$$

$$= L\,[d(1/v_{g,eff})/d\lambda]\,\Delta\lambda$$

$$= L\,D_{wg}\,\Delta\lambda$$

- $D_{wg} = d(1/v_{g,eff})/d\lambda$ is called the *waveguide dispersion parameter* and is expressed in units of ps/(km-nm).

$$D_{wg} = d(1/v_{g,eff})/d\lambda = c^{-1}\,dn_{g,eff}/d\lambda = c^{-1}\,d[n_{eff} - \lambda\,dn_{eff}/d\lambda]/d\lambda$$

$$= -c^{-1}\,\lambda\,d^2n_{eff}/d\lambda^2$$

Waveguide dispersion parameter

- Recall $v_{g,eff} = (d\beta/d\omega)^{-1}$ and note that the propagation constant β is a *nonlinear* function of the V number, $V = (2\pi a/\lambda)\ NA = a\ (\omega/c)\ NA$
- *In the absence of material dispersion* (i.e. when NA is independent of ω), V is directly proportional to ω, so that

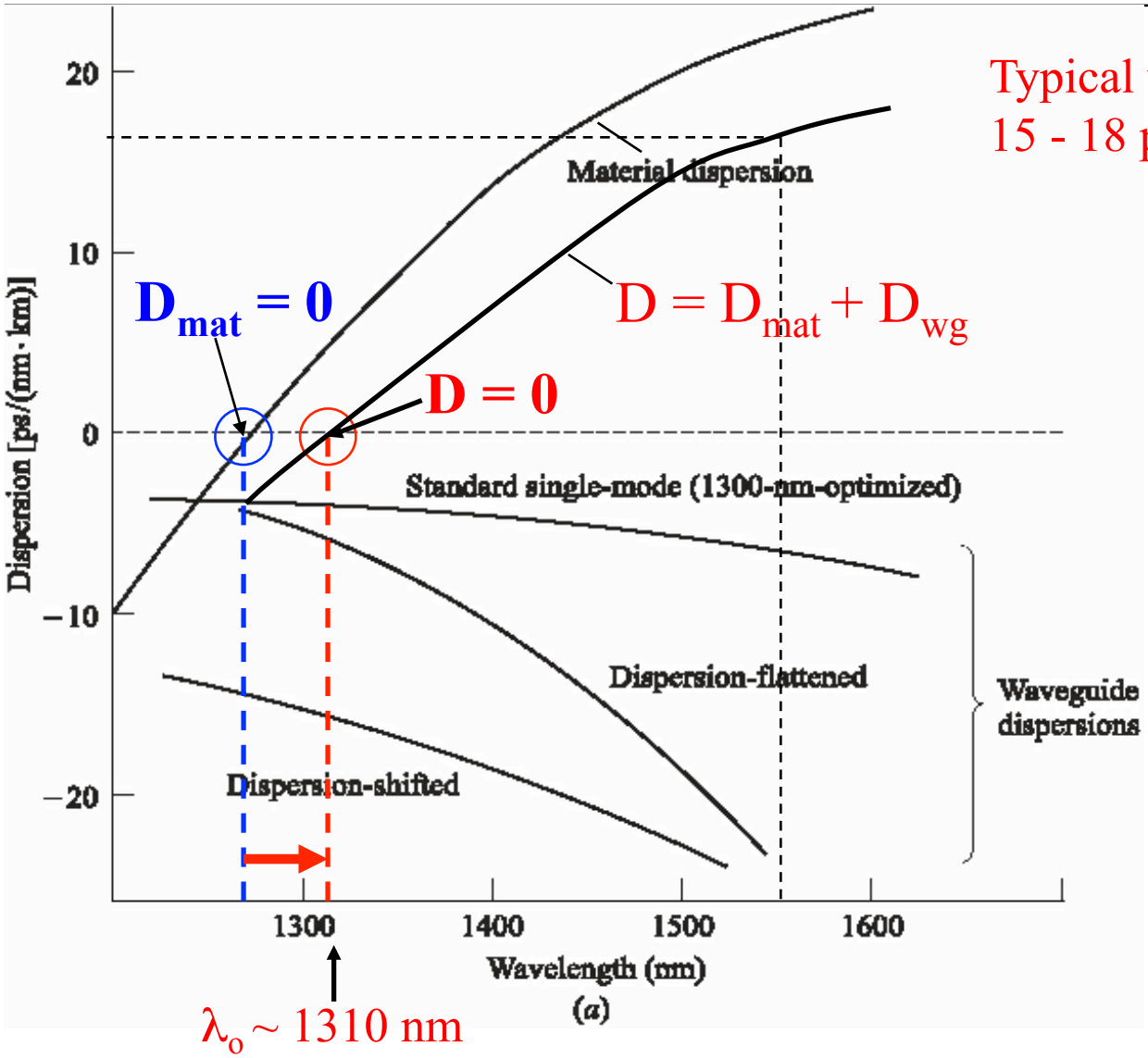
$$1/v_{g,eff} = d\beta/d\omega = (d\beta/dV)\ (dV/d\omega) = (d\beta/dV)\ (a\ NA/c)$$

- The pulse broadening associated with a source of spectral width $\Delta\lambda$ is related to the time delay $L/v_{g,eff}$ by $\Delta T = L\ |D_{wg}|\ \Delta\lambda$. The waveguide dispersion parameter D_{wg} is given by

$$D_{wg} = d/d\lambda\ (1/v_{g,eff}) = -(\omega/\lambda)\ d/d\omega\ (1/v_{g,eff}) = -(1/(2\pi c))\ V^2\ d^2\beta/dV^2$$

⇒*The dependence of D_{wg} on λ may be controlled by altering the <u>core radius,</u> the <u>NA,</u> or the <u>V number.</u>*

Silica fiber dispersion



Typical values of D are about 15 - 18 ps/(km-nm) near 1.55 μm.

- $D_{wg}(\lambda)$ compensate some of the $D_{mat}(\lambda)$ and shifts the λ_{ZD} from about 1276 nm to a *longer* wavelength of ~1310 nm.

Chromatic dispersion in low-bit-rate systems

- Recall broadening of the light pulse due to chromatic dispersion:

$$\Delta T = D\, L\, \Delta\lambda$$

Consider the maximum pulse broadening equals to the bit time period 1/B, then the *dispersion-limited distance*:

$$L_D = 1 / (D\, B\, \Delta\lambda)$$

e.g. For D = 17 ps/(km•nm), B = <u>2.5 Gb/s</u> and $\Delta\lambda$ = 0.03 nm

$$\Rightarrow L_D = 784 \text{ km}$$

(It is known that dispersion limits a 2.5 Gbit/s channel to roughly 900 km! Therefore, chromatic dispersion is *not* much of an issue in low-bit-rate systems deployed in the early 90's!)

Chromatic dispersion scales with B^2

- **When upgrading** from *2.5- to 10-Gbit/s* systems, most technical challenges are less than four times as complicated and the cost of components is usually much less than four times as expensive.
- However, *when increasing the bit rate by a factor of 4, the effect of chromatic dispersion increases by a factor of 16!*
- Consider again the dispersion-limited distance:

$$L_D = 1 / (D\, B\, \Delta\lambda)$$

Note that spectral width $\Delta\lambda$ is proportional to the modulation of the lightwave!

i.e. Faster the modulation, more the frequency content, and therefore wider the spectral bandwidth => $\Delta\lambda \propto B$

=> $L_D \propto 1 / B^2$

Chromatic dispersion in high-bit-rate systems

e.g. In standard single-mode fibers for which D = 17 ps/(nm•km) at a signal wavelength of 1550 nm (assuming from the same light source as the earlier example of 2.5 Gbit/s systems), the maximum transmission distance before significant pulse broadening occurs for 10 Gbit/s data is:

$$L_D \sim 784 \text{ km} / 16 \sim 50 \text{ km!}$$

(A more exact calculation shows that 10-Gbit/s (40-Gbit/s) would be limited to approximately 60 km (4 km!).)

This is why chromatic dispersion compensation must be employed for systems operating at 10 Gbit/s (now at 40 Gbit/s and beyond.)

Zero-dispersion slope

If $D(\lambda)$ is *zero* at a specific $\lambda = \lambda_{ZD}$, can we eliminate pulse broadening caused by chromatic dispersion?

There are higher order effects! The derivative

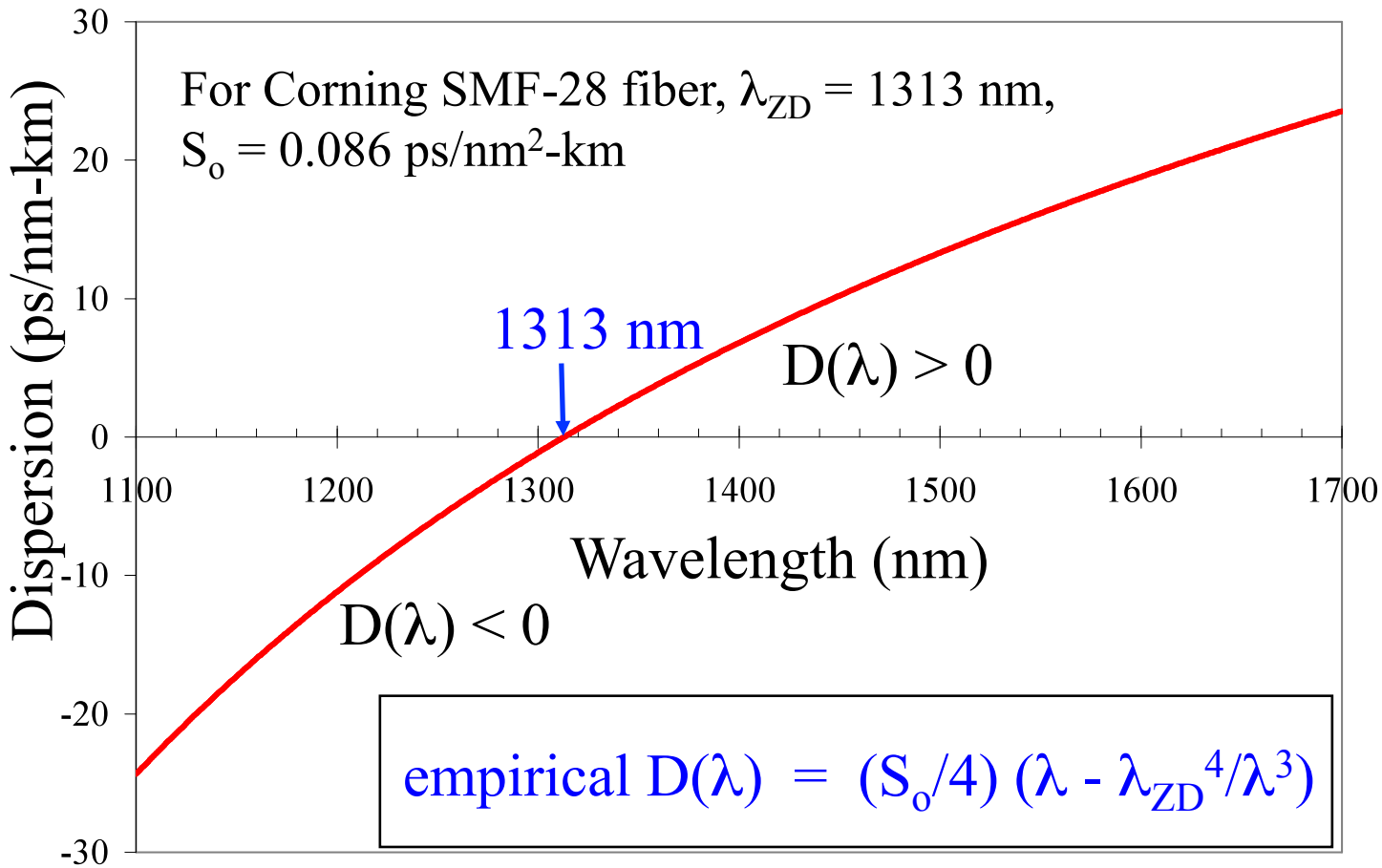
$$dD(\lambda)/d\lambda = S_o$$

needs to be accounted for when the first order effect is zero (i.e. $D(\lambda_{ZD}) = 0$) .

S_o is known as the zero-dispersion slope measured in **ps/(km-nm^2)**.

Pulse broadening near zero-dispersion wavelength

The chromatic pulse broadening near λ_{ZD}: $\Delta T = L\, S_o\, |\lambda - \lambda_{ZD}|\, \Delta\lambda$



Limiting bit rate near zero-dispersion wavelength

- Now it becomes clear that at $\lambda = \lambda_{ZD}$, the dispersion slope S_o becomes the bit rate limiting factor. We can estimate the limiting bit rate by noting that for a source of spectral width $\Delta\lambda$, the effective value of dispersion parameter becomes

$$D = S_o \Delta\lambda$$

=> The limiting *bit rate-distance product* can be given as

$$BL\,|S_o|\,(\Delta\lambda)^2 < 1 \qquad (B\,\Delta T < 1)$$

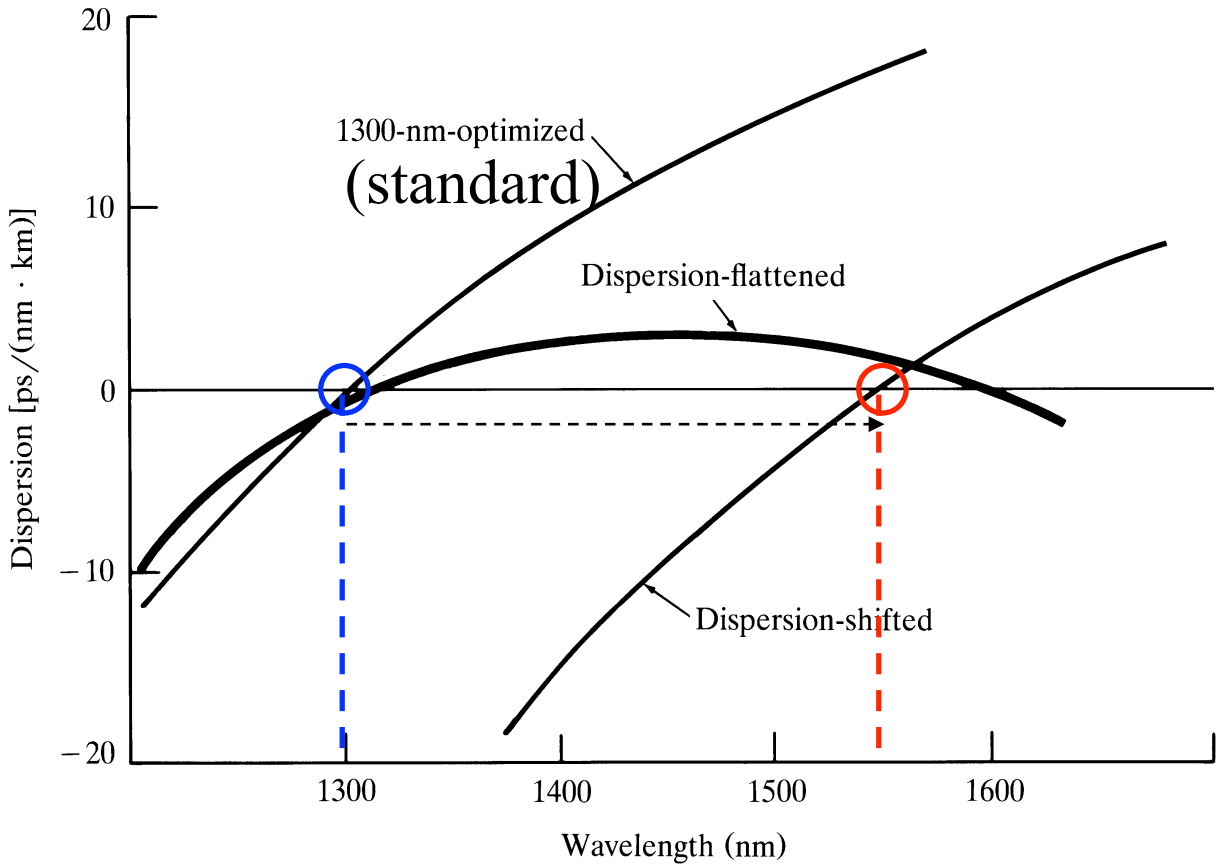
*For a *multimode* semiconductor laser with $\Delta\lambda = 2$ nm and a *dispersion-shifted fiber* with $S_o = 0.05$ ps/(km-nm^2) at $\lambda = 1.55$ μm, the BL product approaches 5 (Tb/s)-km. Further improvement is possible by using *single-mode* semiconductor lasers.

Dispersion tailored fibers

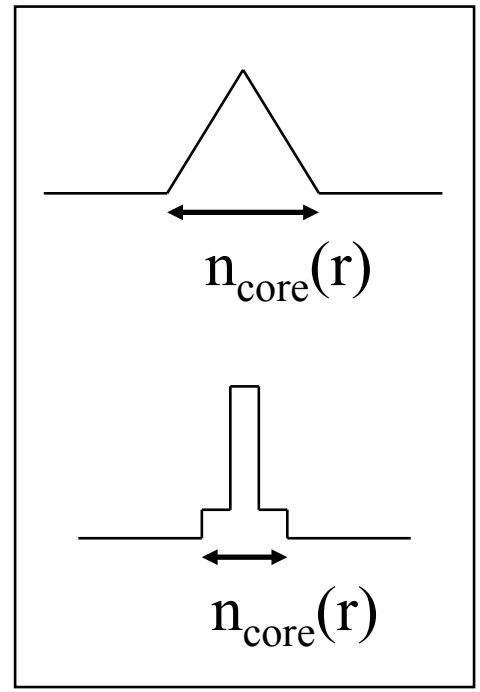
1. Since the waveguide contribution D_{wg} depends on the fiber parameters such as the core radius *a* and the index difference Δ, it is *possible to design the fiber such that λ_{ZD} is shifted* into the neighborhood of 1.55 μm. Such fibers are called ***dispersion-shifted fibers***.

2. It is also possible to tailor the waveguide contribution such that the *total dispersion D is relatively small over a wide wavelength range* extending from 1.3 to 1.6 μm. Such fibers are called ***dispersion-flattened fibers***.

Dispersion-shifted and flattened fibers



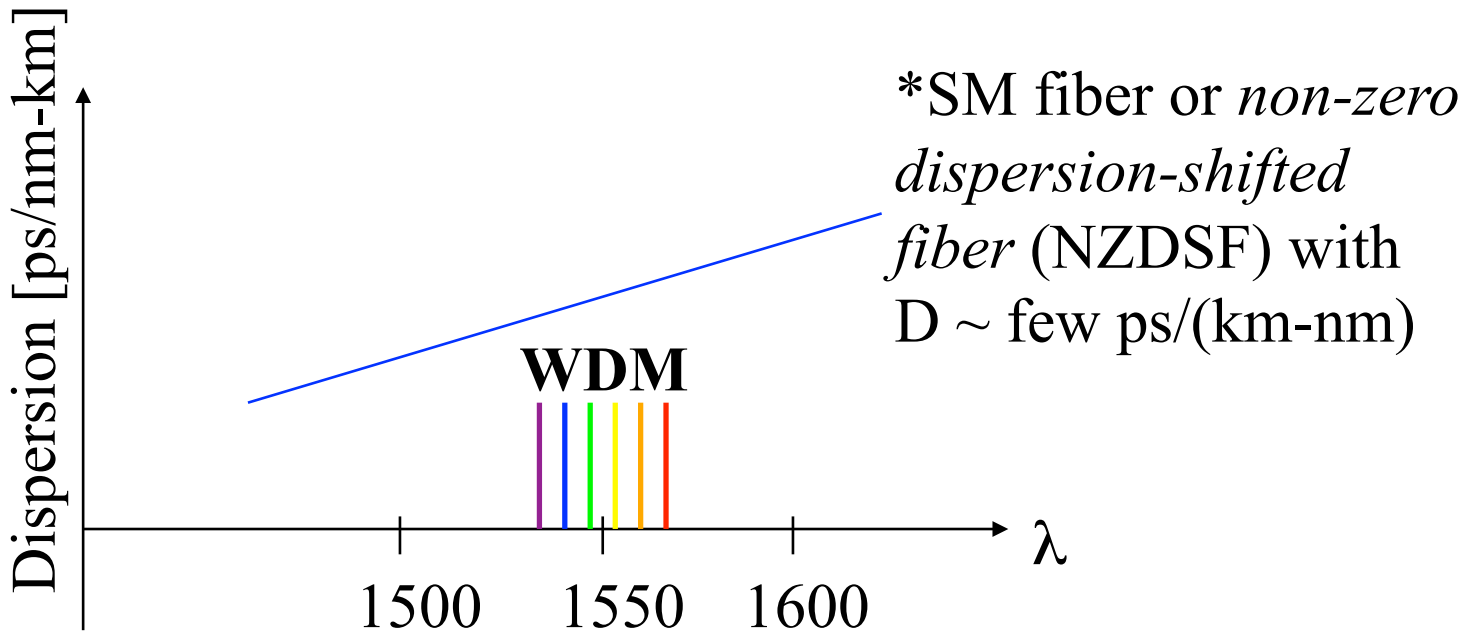
(*b*)



- The design of dispersion-modified fibers often involves the use of multiple cladding layers and a <u>tailoring of the refractive index profile</u>.

Non-zero dispersion shifted fibers

- Since ***dispersion slope*** S > 0 for singlemode fibers => different ***wavelength-division multiplexed (WDM)*** channels have different *dispersion values*.



*In fact, for WDM systems, *small amount of chromatic dispersion is desirable* in order to prevent the impairment of **fiber nonlinearity** (i.e. power-dependent interaction between wavelength channels.)

Chromatic Dispersion Compensation

- Chromatic dispersion is ***time independent*** in a *passive* optical link

⇒allow compensation along the entire fiber span

(**Note** that recent developments focus on *reconfigurable* optical links, which makes chromatic dispersion *time dependent*!)

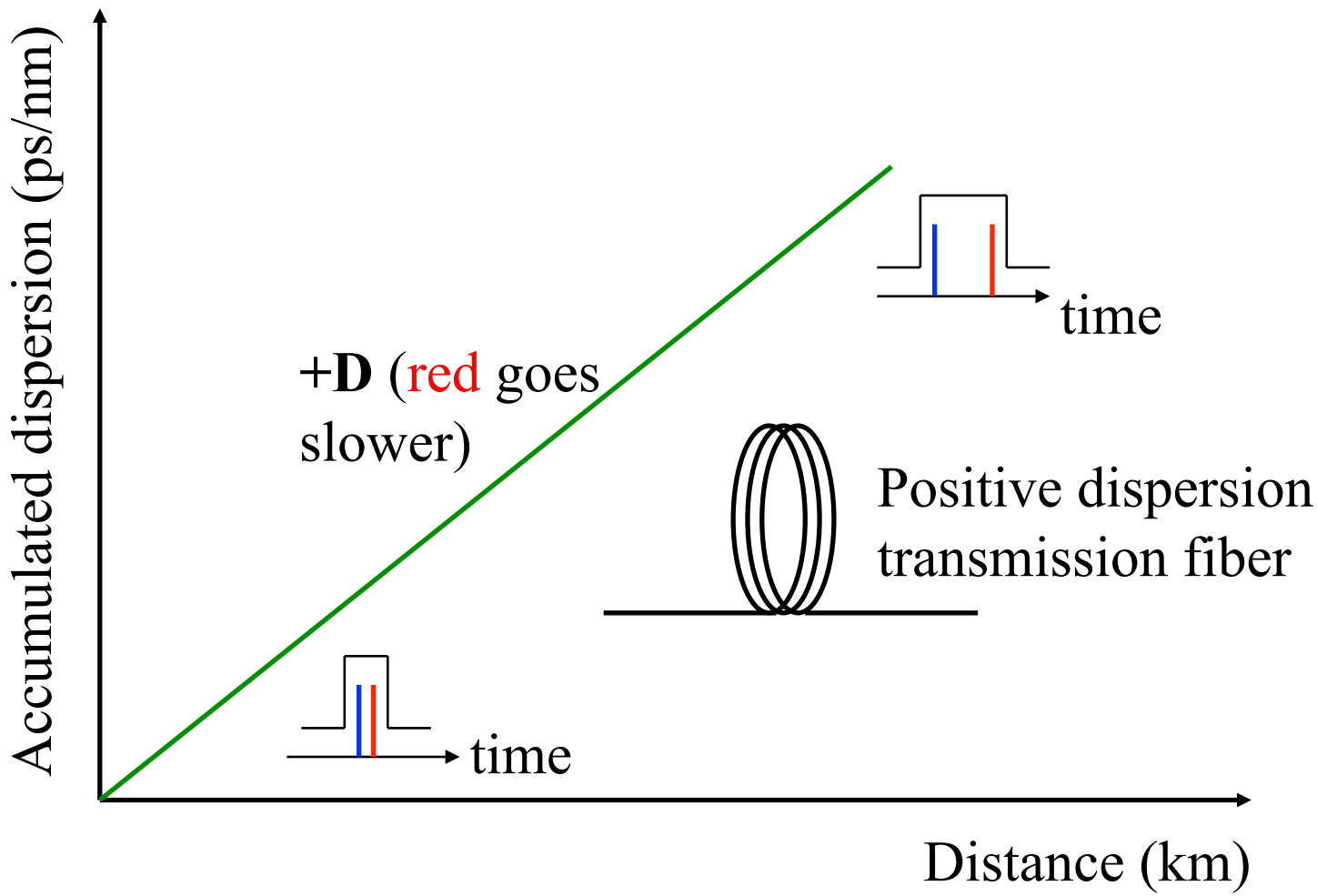
Two basic techniques: (1) **dispersion-compensating fiber DCF**

(2) dispersion-compensating **fiber grating**

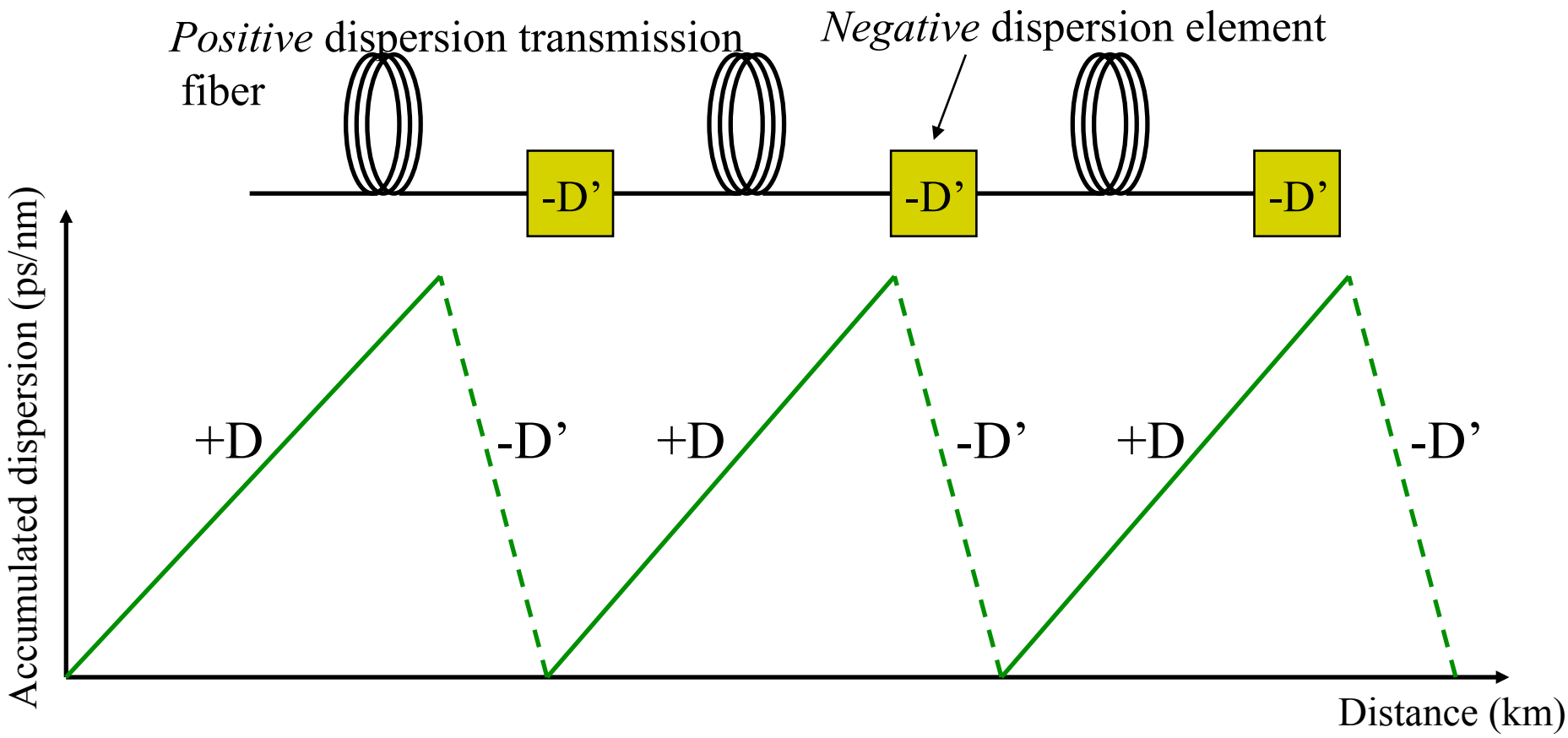
- **The basic idea for DCF**: the ***positive dispersion*** in a conventional fiber (say ~ 17 ps/(km-nm) in the 1550 nm window) can be compensated for by inserting a ***fiber with negative dispersion*** (*i.e. with large -ve* D_{wg}).

Chromatic dispersion accumulates linearly over distance

(recall $\Delta T = D\,L\,\Delta\lambda$)

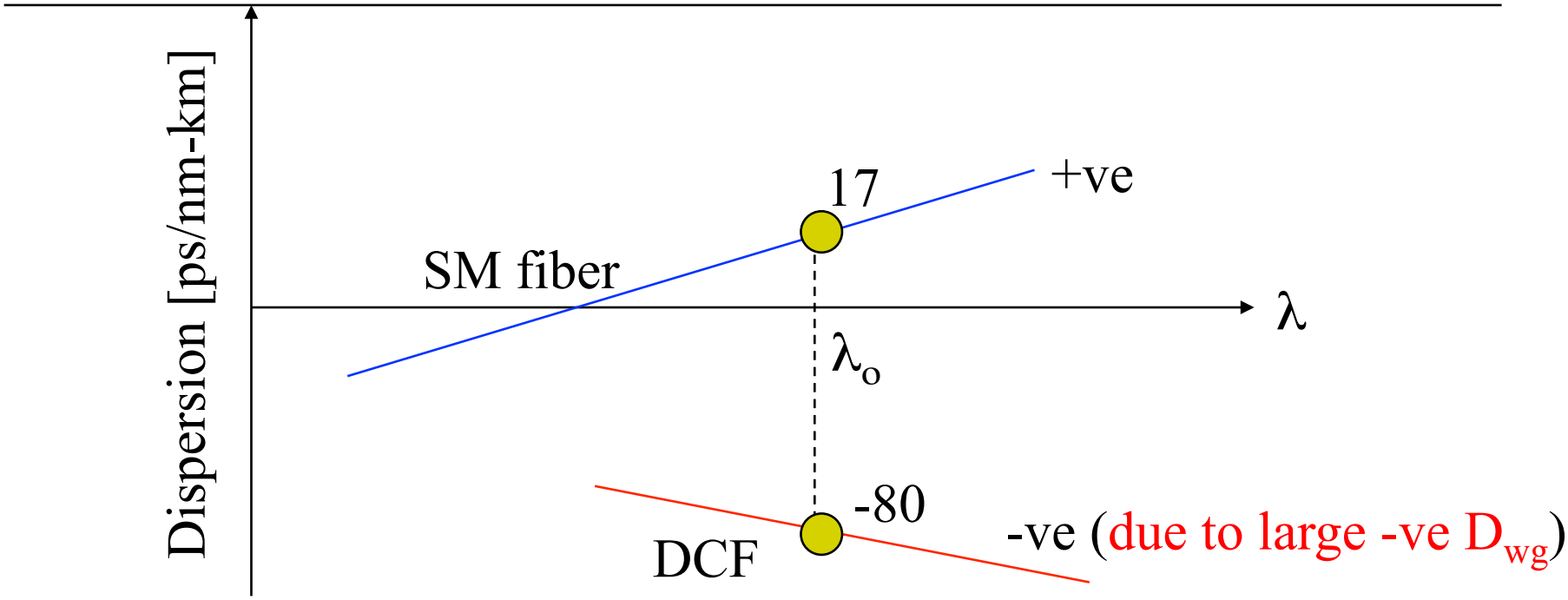


Chromatic Dispersion Compensation



- In a *dispersion-managed* system, positive dispersion transmission fiber *alternates with negative dispersion compensation elements*, such that the <u>*total* dispersion is *zero* end-to-end</u>.

Fixed (passive) dispersion compensation

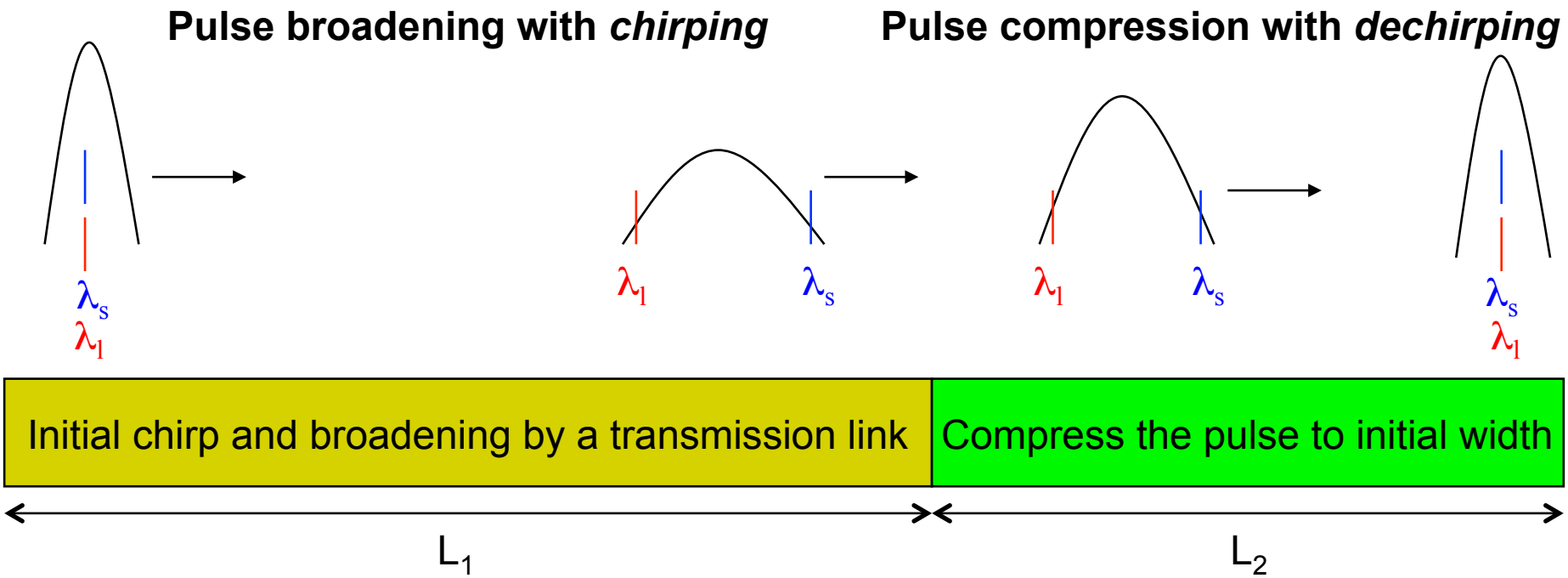


SM	DCF	SM	DCF	SM	DCF
+ve	-ve	+ve	-ve	+ve	-ve

*DCF is a length of fiber producing -ve dispersion *four to five times* as large as that produced by conventional SMF.

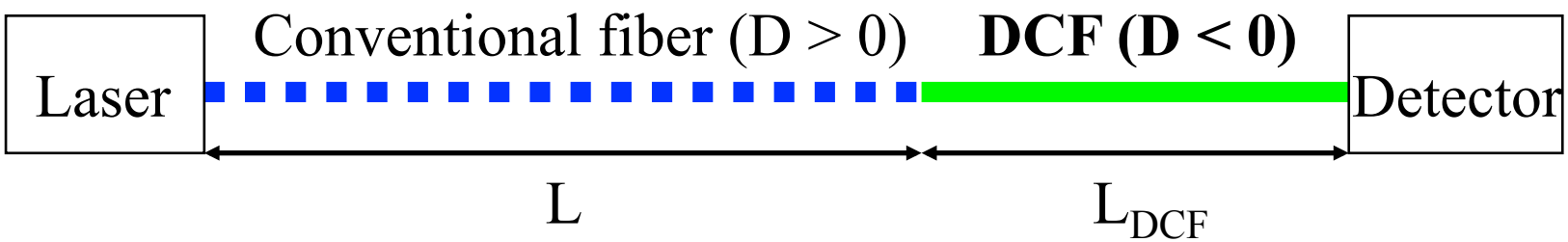
Dispersion-Compensating Fiber

The concept: using a span of fiber to *compress* an initially chirped pulse.



Dispersion compensated channel: $D_2 L_2 = - D_1 L_1$

Dispersion-Compensating Fiber



e.g. What DCF is needed in order to compensate for dispersion in a conventional single-mode fiber link of 100 km?

Suppose we are using Corning SMF-28 fiber,

=> the dispersion parameter D(1550 nm) ~ 17 ps/(km-nm)

$\Rightarrow$ Pulse broadening $\Delta T_{chrom} = D(\lambda)\, \Delta\lambda\, L \sim 17 \times 1 \times 100 = 1700$ ps.

assume the semiconductor (diode) laser linewidth $\Delta\lambda \sim 1$ nm.

Dispersion-Compensating Fiber

⇒The DCF needed to compensate for 1700 ps with a large *negative-dispersion* parameter

i.e. we need $\Delta T_{chrom} + \Delta T_{DCF} = 0$

$$\Rightarrow \Delta T_{DCF} = D_{DCF}(\lambda)\, \Delta\lambda\, L_{DCF}$$

suppose typical ratio of $L/L_{DCF} \sim 6 - 7$, we assume $L_{DCF} = 15$ km

$$\Rightarrow D_{DCF}(\lambda) \sim -113 \text{ ps/(km-nm)}$$

*Typically, **only one wavelength can be compensated exactly.** Better CD compensation requires **both *dispersion* and *dispersion slope* compensation**.

Dispersion slope compensation

Compensating the dispersion slope produces the additional requirement:

$$L_2\, dD_2/d\lambda = -\, L_1\, dD_1/d\lambda$$

⇒ The compensating fiber must have a ***negative*** dispersion slope, and that the *dispersion* and *slope* values need to be compensated for a given length.

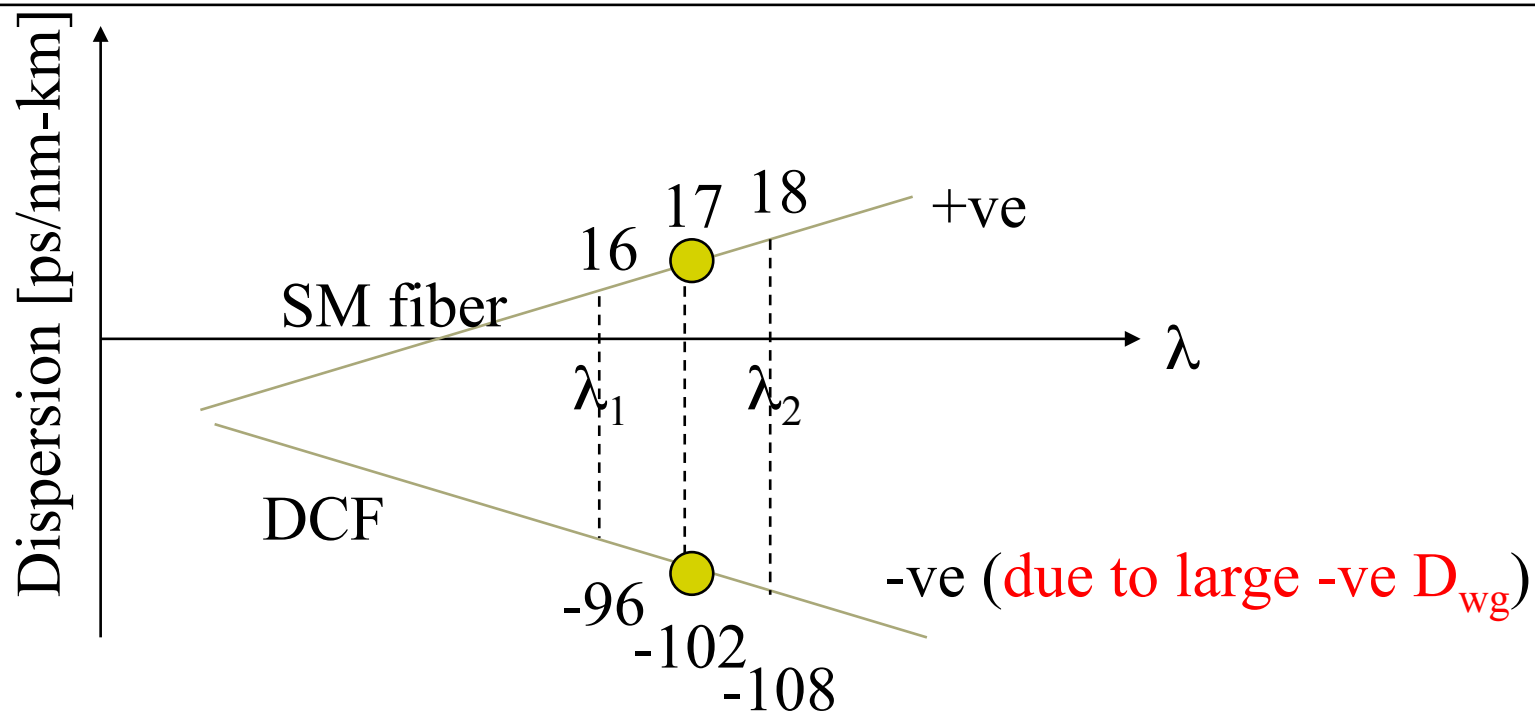
$$D_2\, L_2 = -\, D_1\, L_1$$

$$L_2\, dD_2/d\lambda = -\, L_1\, dD_1/d\lambda$$

=> Dispersion and slope compensation: $D_2 / (dD_2/d\lambda) = D_1 / (dD_1/d\lambda)$

(In practice, two fibers are used, one of which has negative slope, in which the pulse wavelength is at zero-dispersion wavelength λ_{zD}.)

Dispersion slope compensation



Within the spectral window (λ_1, λ_2), $D_{DCF}/D_{SM} = -6$

$S_{DCF} = -12/(\lambda_2 - \lambda_1)$; $S_{SM} = 2/(\lambda_2 - \lambda_1)$ => $S_{DCF}/S_{SM} = -6$

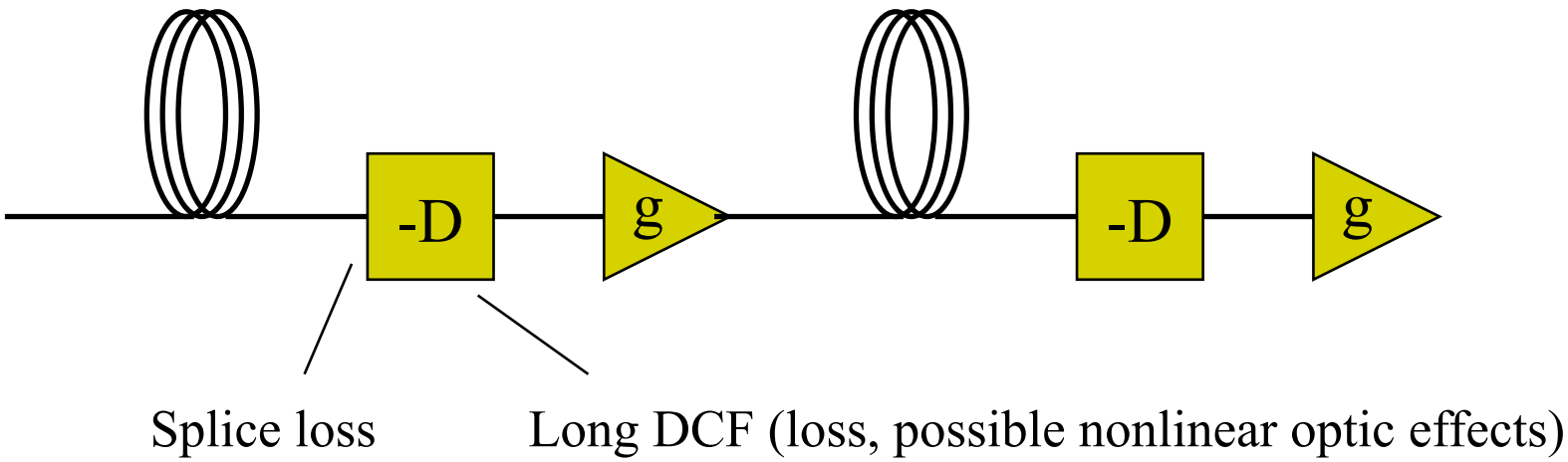
=> Dispersion slope compensation: $(D_{SM}/S_{SM}) / (D_{DCF}/S_{DCF}) = 1$

Disadvantages in using DCF

- *Added loss* associated with the increased fiber span

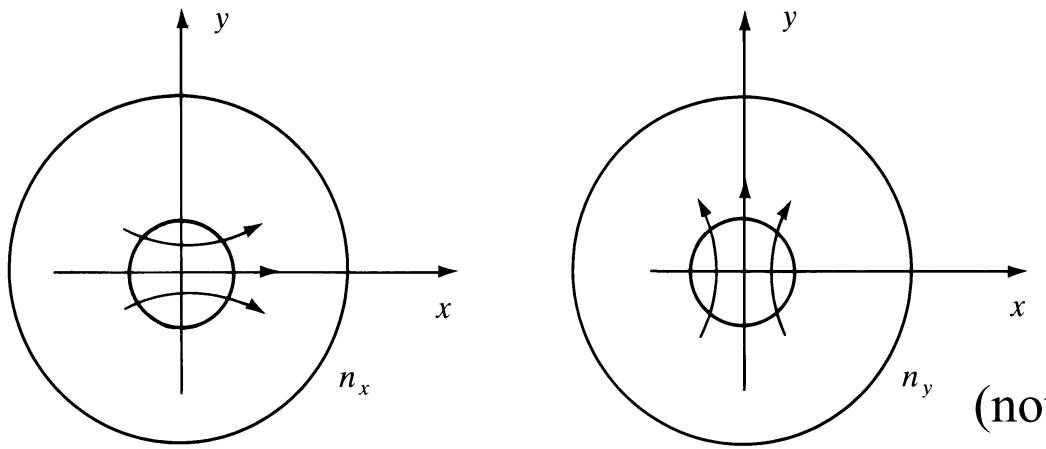
- *Nonlinear optic effects* may degrade the signal over the long length of the fiber if the signal is of sufficient intensity.

- Links that use DCF often require an *amplifier* stage to compensate the added loss.



Polarization Mode Dispersion (PMD)

- In a single-mode optical fiber, the optical signal is carried by the *linearly polarized* "fundamental mode" LP_{01}, which has ***two polarization components that are orthogonal***.



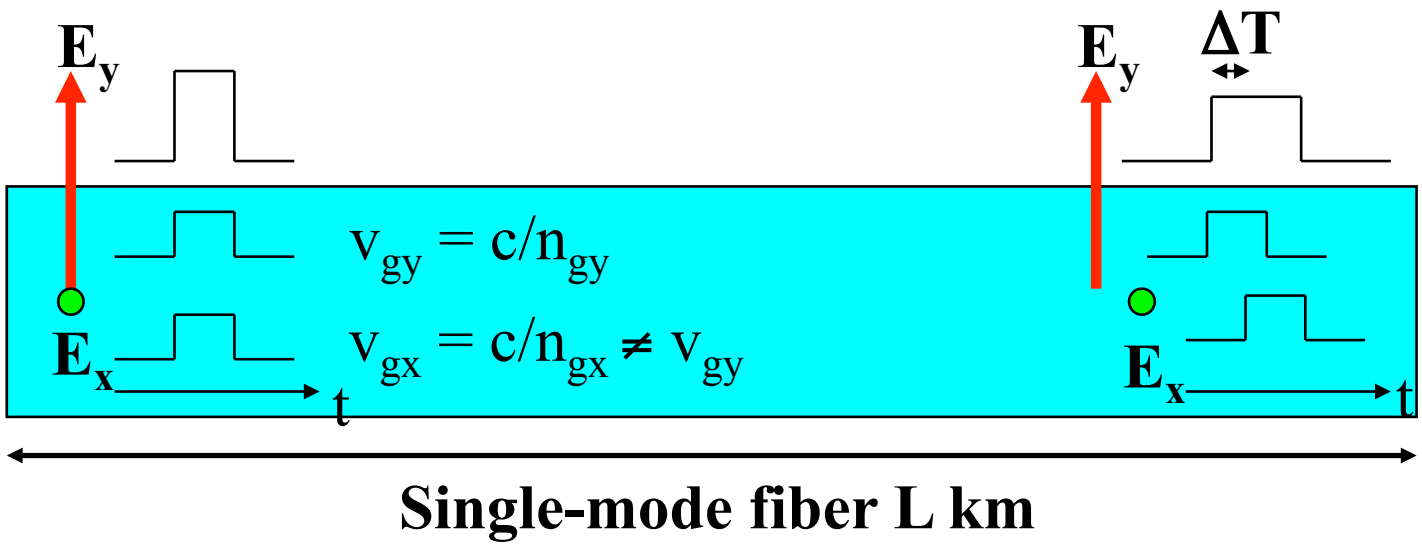
Horizontal mode

Vertical mode

(note that x and y are chosen arbitrarily)

- In a *real* fiber (i.e. $n_{gx} \neq n_{gy}$), the two orthogonal polarization modes propagate at *different group velocities*, resulting in pulse broadening – polarization mode dispersion.

Polarization Mode Dispersion (PMD)



1. ***Pulse broadening*** due to the orthogonal polarization modes (*The time delay between the two polarization components is characterized as the* ***differential group delay (DGD).***)

2. ***Polarization varies*** along the fiber length

Polarization Mode Dispersion (PMD)

- The *refractive index difference* is known as *birefringence*.

$$B = n_x - n_y$$

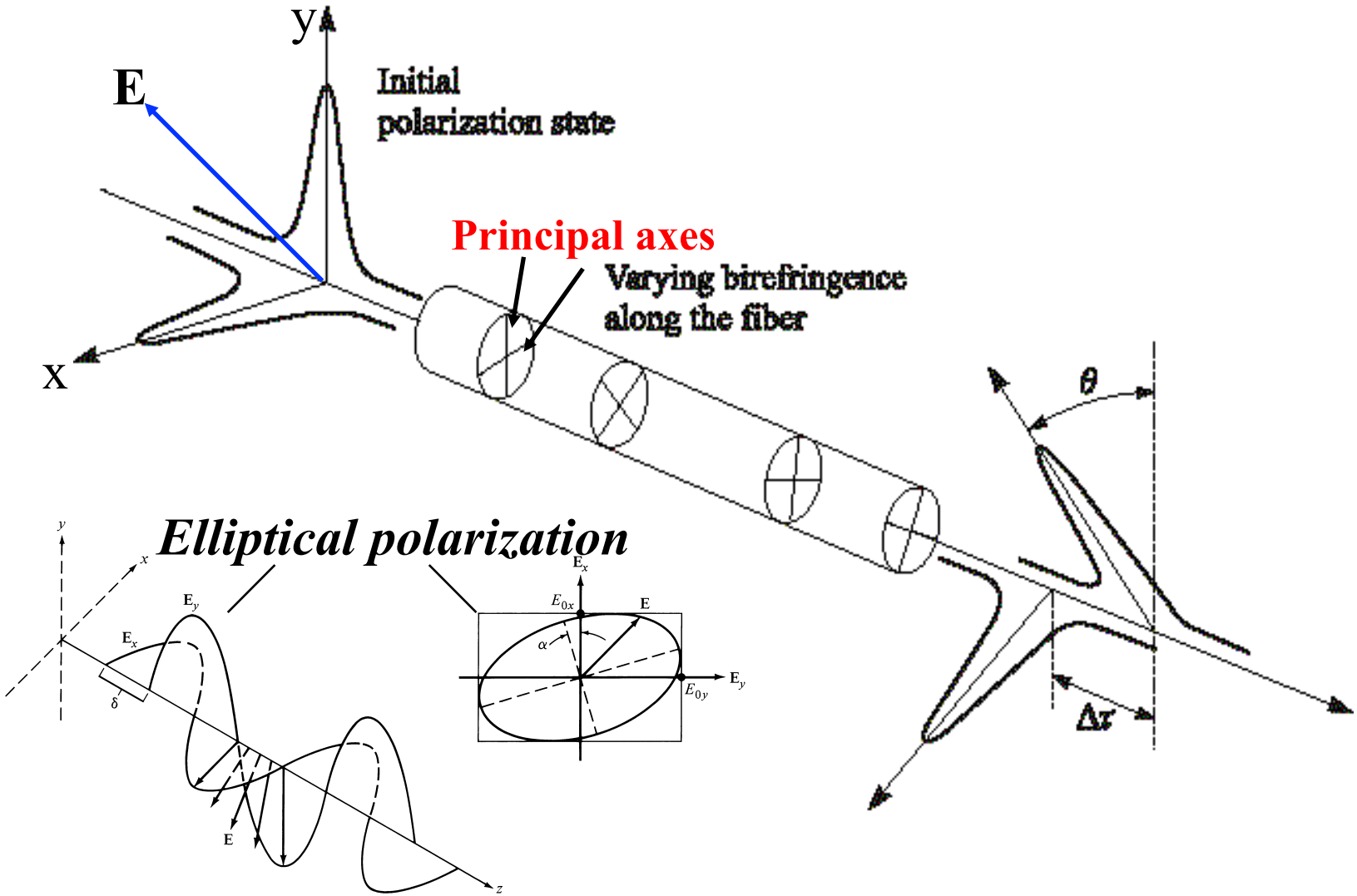
(~ 10^{-6} - 10^{-5} for single-mode fibers)

assuming $n_x > n_y$ => y is the *fast axis*, x is the *slow axis*.

*B varies ***randomly*** because of *thermal and mechanical stresses over time* (due to *randomly varying* environmental factors in submarine, terrestrial, aerial, and buried fiber cables).

=> ***PMD is a statistical process*** **!**

Randomly varying birefringence along the fiber



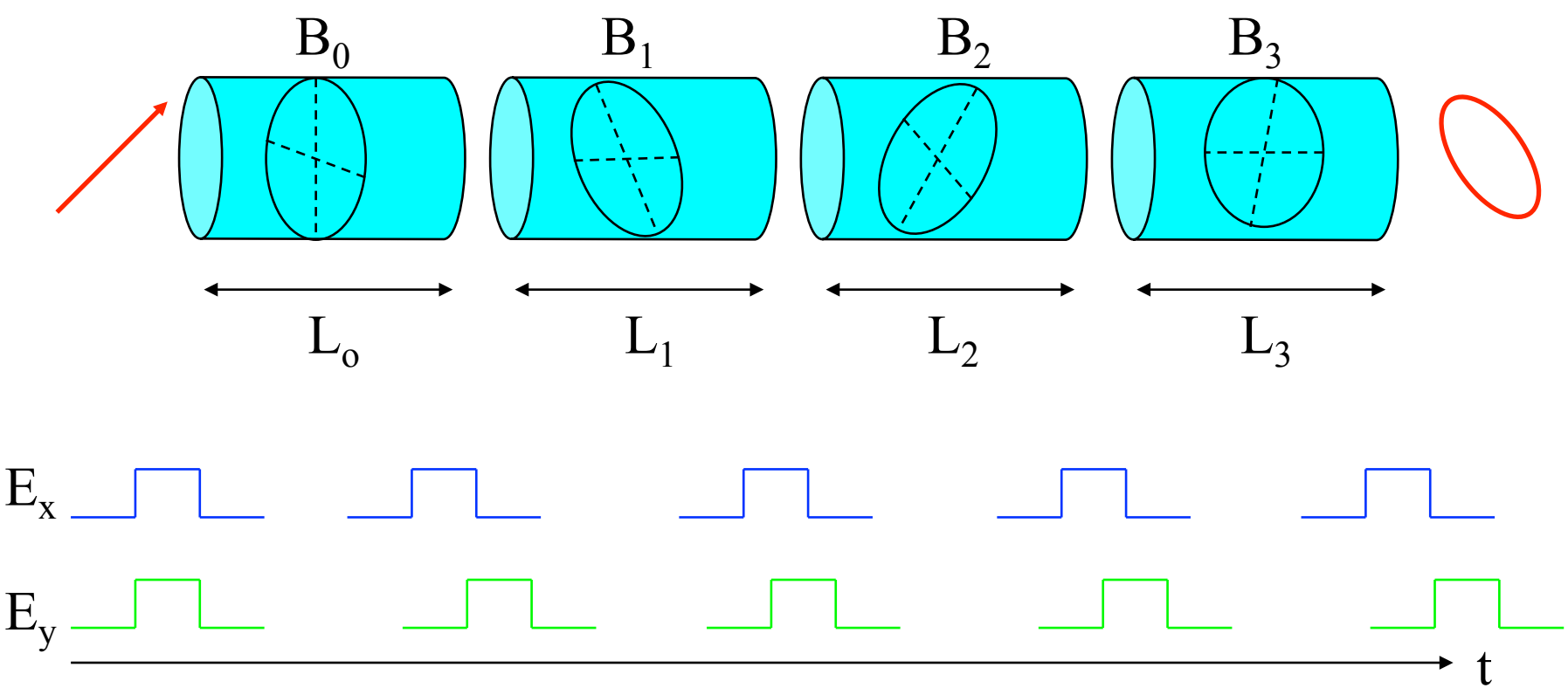
Randomly varying birefringence along the fiber

- The polarization state of light propagating in fibers with ***randomly varying birefringence*** will generally be ***elliptical and would quickly reach a state of arbitrary polarization***.

*However, the final polarization state is *not* of concern for most lightwave systems as ***photodetectors are insensitive to the state of polarization***.
(**Note**: the recent revival of technology developments in "*Coherent Optical Communications*" do require polarization state to be analyzed!)

- A <u>simple model</u> of PMD divides the fiber into a large number of segments. Both the *magnitude of birefringence B* and the *orientation of the principal axes* remain constant in each section but *changes randomly from section to section*.

A simple model of PMD



Randomly changing differential group delay (DGD)

PMD pulse broadening

- Pulse broadening caused by a *random* change of fiber polarization properties is known as <u>polarization mode dispersion</u> (PMD).

PMD pulse broadening $\Delta T_{PMD} = D_{PMD} \sqrt{L}$

D_{PMD} is the PMD parameter (coefficient) measured in **ps/√km**.

√L models the "random" nature (like "random walk")

* D_{PMD} does not depend on wavelength (first order) ;

*Today's fiber (since 90's) PMD parameter is 0.1 - 0.5 ps/√km.
(*Legacy fibers deployed in the 80's have D_{PMD} > 0.8 ps/√km.*)

PMD pulse broadening

e.g. Calculate the pulse broadening caused by PMD for a singlemode fiber with a PMD parameter $D_{PMD} \sim 0.5$ ps/√km and a fiber length of 100 km. (i.e. $\Delta T_{PMD} = 5$ ps)

Recall that pulse broadening due to chromatic dispersion for a 1 nm linewidth light source was ~ 15 ps/km, which resulted in 1500 ps for 100 km of fiber length.

=> **PMD** pulse broadening is ***two orders of magnitude less*** than chromatic dispersion !

*PMD is relatively small compared with chromatic dispersion. But when one operates at zero-dispersion wavelength (or *dispersion compensated wavelengths*) with narrow spectral width, PMD can become a significant component of the total dispersion.

So why do we care about PMD?

Recall that chromatic dispersion can be compensated to ~ 0, (at least for single wavelengths, namely, by designing proper -ve waveguide dispersion)

but there is no simple way to eliminate PMD completely.

=> *It is PMD that limits the fiber bandwidth after chromatic dispersion is compensated*!

PMD in 40Gbit/s systems

- PMD is of lesser concern in lower data rate systems. At lower transmission speeds (***up to and including 10 Gb/s***), networks have higher tolerances to all types of dispersion, including PMD.

As data rates increase, this dispersion tolerance reduces significantly, creating a need to control PMD as much as possible at the current **40 Gb/s** system.

e.g. The pulse broadening caused by PMD for a singlemode fiber with a PMD parameter of 0.5 ps/√km and a fiber length of 100 km => **5 ps**.

However, this is comparable to **the 40G bit period = 25 ps !**