

3. Gas Detectors

3.1. General introduction

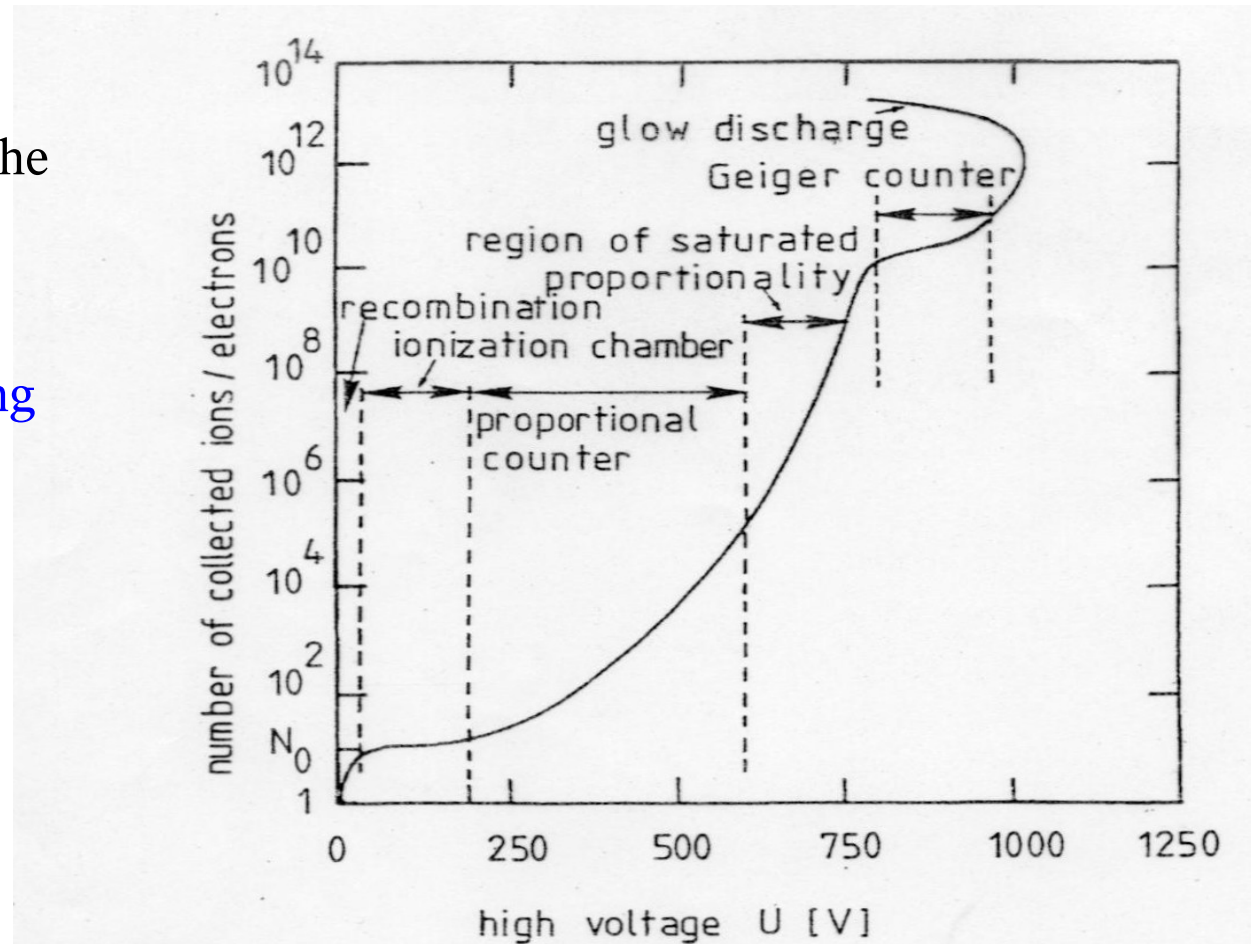
principle

- ionizing particle creates primary and secondary charges via energy loss by ionization (Bethe-Bloch, chapter 2)

N_0 electrons and ions

- charges drift in electric field
- generally gas amplification in the vicinity of an anode wire
- signal generation

different operational modes depending on strength of electric field



after F. Sauli, 1977 lecture notes

Charge carriers in layer of thickness L for a mean energy W to produce electron-ion pair

- mean number

$$\langle n_t \rangle = \frac{L \cdot \langle dE/dx \rangle_{\text{ion}}}{W}$$

about 2-6 times the primary number (see chapter 2)

important for spatial resolution: **secondary ionization** by δ – electrons happens on length scale **$10 \mu\text{m}$**

e.g. $T_e = 1 \text{ keV}$ in isobutane $R = 20 \mu\text{m}$

- ionization statistics

$\lambda = 1/n_e \cdot \sigma_I$ mean distance between ionization events with cross section σ_I

mean number of ionization events $\langle n \rangle = L/\lambda$

Poisson distribution about mean $\langle n \rangle$

$$P(n, \langle n \rangle) = \frac{\langle n \rangle^n \exp(-\langle n \rangle)}{n!}$$

and specifically probability for no ionization

$$P(0) = \exp(-\langle n \rangle) = \exp(-L/\lambda)$$

efficiency of gas detectors allows determination of λ and hence σ_I

λ (cm)

typical values:	He	0.25	$\rightarrow \sigma_I = 10^{-22} \text{ cm}^2 \text{ or } 100 \text{ b}$
	air	0.053	
	Xe	0.023	

3.2. Charge Transport

3.2.1 Ion mobility

drift along field lines in external E-field plus superimposed random thermal motion

ion transfers in collisions with gas atoms typically half of its energy $\rightarrow$ kinetic energy of ion is approximately thermal energy

$$\langle T_{ion}(\vec{E}) \rangle \approx \langle T_{ion}(thermal) \rangle = \frac{3}{2} kT$$

drift velocity in direction of $\vec{E}$: (thermal velocity has random orientation relative to $\vec{E}$)
assume at $t = 0$, $u_e = 0$ and typical collision time τ



directly prior to collision

$$\vec{u}_e = \vec{a} \cdot \tau = \frac{e\vec{E}}{m} \cdot \tau$$

mean drift velocity of ion

$$\langle v_0 \rangle = \frac{1}{2} u_e = \frac{e|\vec{E}|}{2m} \tau = \mu_+ |\vec{E}|$$

$\mu_+ \equiv$ ion mobility

where $\tau \propto \lambda \propto 1 / \sigma_+ \cong$ constant since $\langle T \rangle$ essentially thermal

$\mu^+ \equiv$ ion mobility

e.g. $C_4H_{10}^+$ in C_4H_{10} $\mu_+ = 0.61 \frac{cm}{s \cdot V/cm}$

at $|\vec{E}| = 1 kV/cm \rightarrow v_{D+} = 0.6 cm/ms$

typical drift distances $cm \rightarrow$ typical ion drift times ms

3.2.2. Electron mobility

electrons drift towards anode of a gas detector in a given field with a constant velocity, measurement of drift time allows to determine point of ionization

$$\Delta t = L / v_D$$

equation of motion of electron in superimposed $\vec{E}$ and $\vec{B}$ – fields (Langevin):

$$m \frac{d\vec{v}}{dt} = e\vec{E} + e(\vec{v} \times \vec{B}) + \vec{Q}(t)$$

with instantaneous velocity $\vec{v}$ and a stochastic, time dependent term $Q(t)$ due to collisions with gas atoms

assume: collision time τ
 $\vec{E}$ and $\vec{B}$ constant between collisions
consider $\Delta t \gg \tau$ (averaging)

drift velocity v_0 is given by

$$\langle m \frac{d\vec{v}}{dt} \rangle = e (\vec{E} + \langle \vec{v} \rangle \times \vec{B}) - \underbrace{\frac{m}{\tau} \vec{v}_D}_{\text{Stokes-type}}$$

- $B = 0$: $\vec{v}_D = \mu_- \vec{E}$
and $\mu_- = \frac{e\tau}{m} \equiv \mu$
- $B \neq 0$: Larmor frequency $\omega = \frac{eB}{m}$

compared to ions $\mu_+ \ll \mu_-$ since $M \gg m$

• 2 types of gases:

- a) **hot gases:** atoms with few low-lying levels, electron loses little energy in a collision with atom $\rightarrow T_e \gg kT$
acceleration in E-field and friction lead to constant v_D for a given $\vec{E}$
“free fall with friction”

but $\lambda(T_e) \approx \lambda(|\vec{E}|)$ and $\mu \propto \tau \propto \frac{1}{\sigma(|\vec{E}|)}$ not constant

typical drift velocity: $v_D = 3\text{-}5 \text{ cm}/\mu\text{s}$ for 90 % Ar / 10 % CH₄ (methane)

b) **cold gases:** many low-lying degrees of freedom

→ electrons lose kinetic energy they gain in between collisions
(similar to ions)

$$T_e \cong kT$$

$$\mu \cong \text{constant}$$

$$V_D \propto |E|$$

examples: Ar/CO₂ or Ne/CO₂

in latter $\mu_- \cong 7 \cdot 10^{-3} \text{ cm}^2/\mu\text{s V}$ at 10% CO₂ or $v_D = 2 \text{ cm}/\mu\text{s}$ at 300 V/cm
 $3.5 \cdot 10^{-3} \text{ cm}^2/\mu\text{s V}$ 20% 1

- drift in combined $\vec{E}$ and $\vec{B}$ - fields

$$\vec{v}_D = \frac{\mu |\vec{E}|}{1 + \omega^2 \tau^2} \left[\hat{E} + \underbrace{\omega \tau \hat{E} \times \hat{B}}_{\text{component in direction } \vec{E} \times \vec{B} \text{ prop. } \omega \tau} + \underbrace{\omega^2 \tau^2 (\hat{E} \cdot \hat{B}) \hat{B}}_{\text{component in direction } \vec{B} \text{ prop. } (\omega \tau)^2 \sigma} \right]$$

$\hat{E}, \hat{B}$:

unit vectors

3.2.3. Electron loss

with some probability a free electron is lost during drift

a) recombination $\text{ion}^+ + \text{e}^-$

decrease in number of negative/positive charge carriers

$$-\frac{dn^-}{dt} = \rho_r \cdot n^+ n^-$$



coefficient of recombination

$$\approx 10^{-7} \text{ cm}^3/\text{s}$$

generally not important

b) electron attachment

on electro-negative molecules, probability could be large



otherwise dissociative attachment



for gases like O_2 , Cl_2 , freon, SF_6 probability per collision order of 10^{-4}

capture coefficient p_c is strongly energy dependent (e.g. in O_2 there is a minimum at 1 eV

“Ramsauer effect”)

per second electron undergoes order of 10^{11} collisions $\rightarrow$ for drift time of 10^{-6} s fraction lost

X_{loss} depends on partial oxygen pressure

$$X_{\text{loss}} = 10^{-4} \cdot 10^{11} / \text{s} \cdot 10^{-6} \text{s} \cdot P_{O_2} / P_{\text{atm}}$$

$$\rightarrow \text{less than 1\% lost for } P_{O_2} / P_{\text{atm}} \leq 10^{-3}$$

remark: in presence of certain quencher gases such as CO_2 the effect of O_2 is enhanced by multistep catalytic reaction

- 10 ppm O_2 can lead to 10 % loss within 10 μs $\rightarrow$ need to keep oxygen out of gas

3.2.4. Diffusion

original ionization trail diffuses (spreads apart) with drift time

↔ effect on space point and momentum resolution, ultimate limit

a) only thermal motion ($|\vec{E}| = |\vec{B}| = 0$)

mean thermal velocity

$$\begin{aligned} \langle v \rangle &= \lambda / \tau \\ \text{time between collisions} \\ \text{mean free path} \\ \langle T \rangle &= \frac{1}{2} m \langle v \rangle^2 \end{aligned}$$

for a pointlike source at time $t = 0$, collisions between electrons and gas atoms (molecules)

→ smearing

spread of charge cloud at time of first collision

$$R^2 = 2 \lambda^2$$

and after $n = t / \tau$ collisions

$$\sigma^2(t) = 2 \lambda^2 t / \tau$$

define **diffusion coefficient**

$$D = \frac{\sigma^2(t)}{2t}$$

for $|\vec{E}| = |\vec{B}| = 0$

$$D = D_0 = \frac{\lambda_0^2}{\tau} = \frac{2 \langle T \rangle}{m} \tau$$

diffusion is **isotropic**

longitudinal diffusion coefficient

transverse diffusion coefficient

$$D_{0L} = \frac{1}{3} \frac{\lambda_0^2}{\tau}$$
$$D_{0T} = \frac{2}{3} \frac{\lambda_0^2}{\tau}$$

→ after time t charge cloud has width

$$\sigma(t) = \sqrt{D \cdot 2t}$$

respectively, in each dimension

$$\sigma_x(t) = \sigma_y(t) = \sigma_z(t) = \sqrt{(1/3) D \cdot 2t}$$

charge distribution

$$N(x) = c \cdot \exp\left(-\frac{x^2}{2\sigma_x^2}\right)$$

diffusion equation:

charge density $\rho(\vec{r}, t)$ defined by

$$\frac{\partial \rho}{\partial t} + \vec{\nabla} \cdot \vec{j} = 0$$

without field $\vec{j} = -D \vec{\nabla} \rho$

$$\leadsto \frac{\partial \rho}{\partial t} = D \Delta \rho$$

solved by

$$\rho(\vec{r}, t) = c \cdot \exp\left(-\frac{\vec{r}^2}{4Dt}\right)$$

hot gases: $\langle T \rangle \gg \frac{3}{2} kT$ D large
 cold gases: $\langle T \rangle \approx \frac{3}{2} kT$ D small

with $D = \frac{2\langle T \rangle}{3m} \tau$ and $\mu = \frac{e}{m} \tau$
 $\langle T \rangle = \frac{3}{2} e \frac{D}{\mu}$

can define a characteristic energy

$$\epsilon_k = \frac{2}{3} \langle T \rangle = e \frac{D}{\mu}$$

for hot gas the same characteristic energy is reached at much lower T

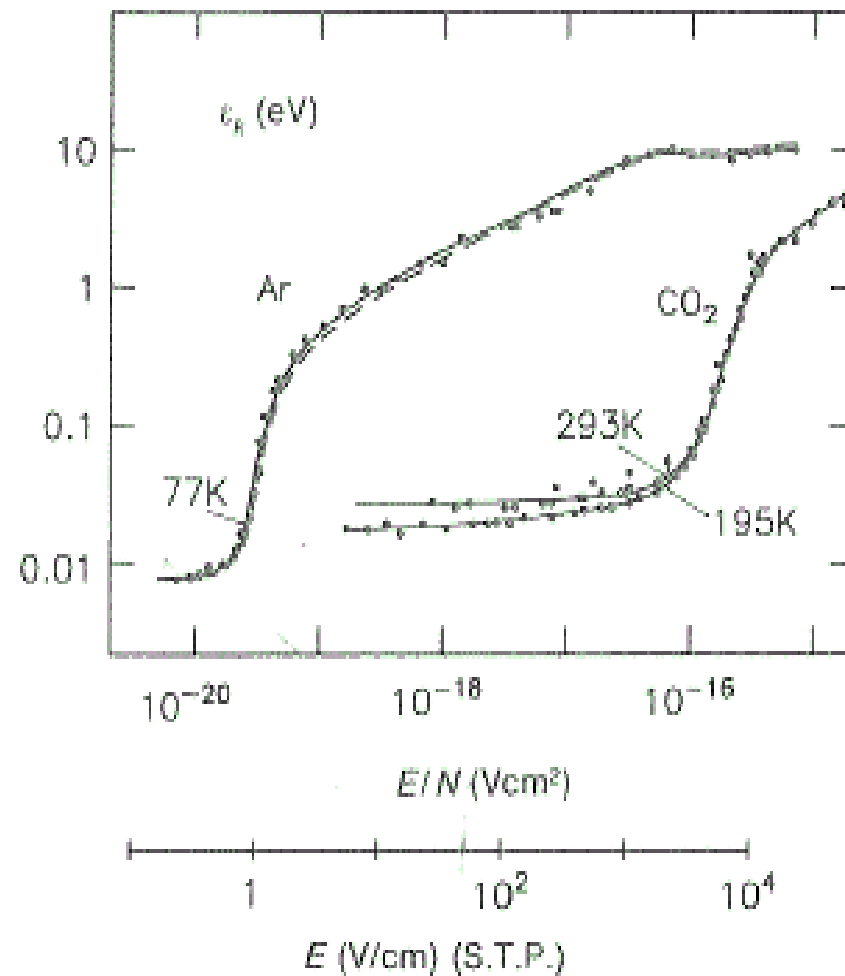


Fig. 2.9. Characteristic energy of electrons in Ar and CO₂ as a function of the reduced E/N . The electric field under normal conditions is also indicated. The parameters refer to temperatures at which the measurements were made [SCH 76]

b) diffusion in B-field

$$\vec{B} = B \vec{e}_z$$

along B no Lorentz force

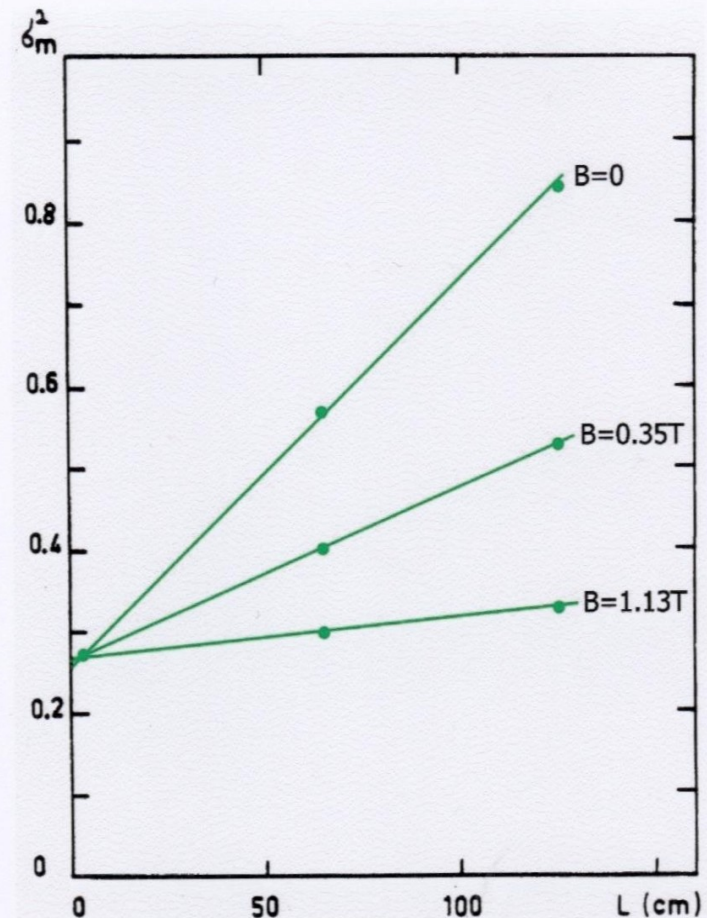
$$D_L(B) = D_{0L} = \frac{1}{3} D_0$$

in transverse direction Lorentz force helps charge cloud together, i.e. it counteracts diffusion

$$D_T(B) = \frac{D_{0T}}{1 + \omega^2 \tau^2}$$

for $\vec{B}$ large $\rightarrow \omega\tau \gg 1$ $D_T(B) \ll D_{0T}$

e.g. Ar/CH₄ B = 1.5 T $D_T(1.5 \text{ T}) \cong 1/50 D_{0T}$



transverse σ^2 as function of L

- c) diffusion in electric field: ordered drift along field superimposed to statistical diffusion
mobility μ is function of $\langle T \rangle$

$$\vec{v}_D = \mu(\langle T \rangle) \cdot \vec{E}$$

→ energy spread leads to longitudinal spreading of electron cloud
statistical transverse diffusion not affected by E-field

$$D_L \neq \frac{1}{2} D_T$$

in hot gases: for large E, $D_L < D_T$

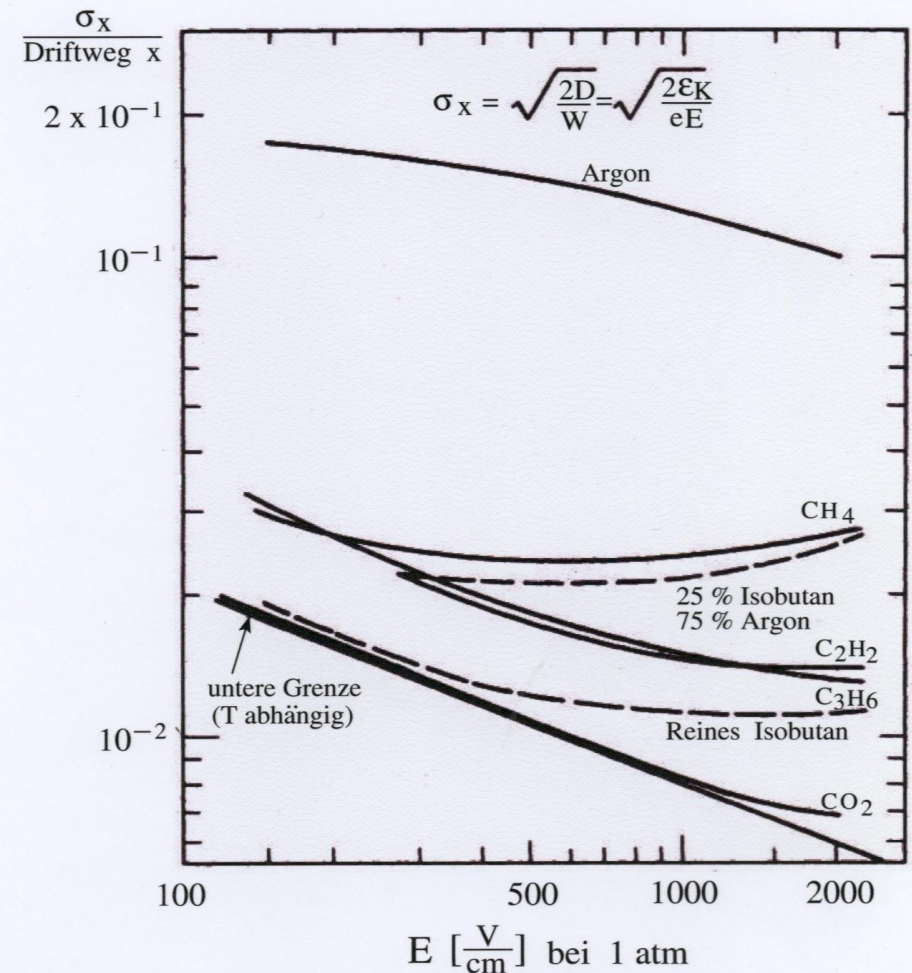
and values are large

in cold gases: $D_L \cong D_T$ small

$$\sigma^2(t) = 2Dt = 2DL_D/v_D = \frac{3kT}{e|\vec{E}|} L_D$$

$$\sigma^2(t)/L_D = \frac{3kT}{e|\vec{E}|}$$

longitudinal diff. in E-field



heisse und kalte Gase

3.2.5 Exact solution

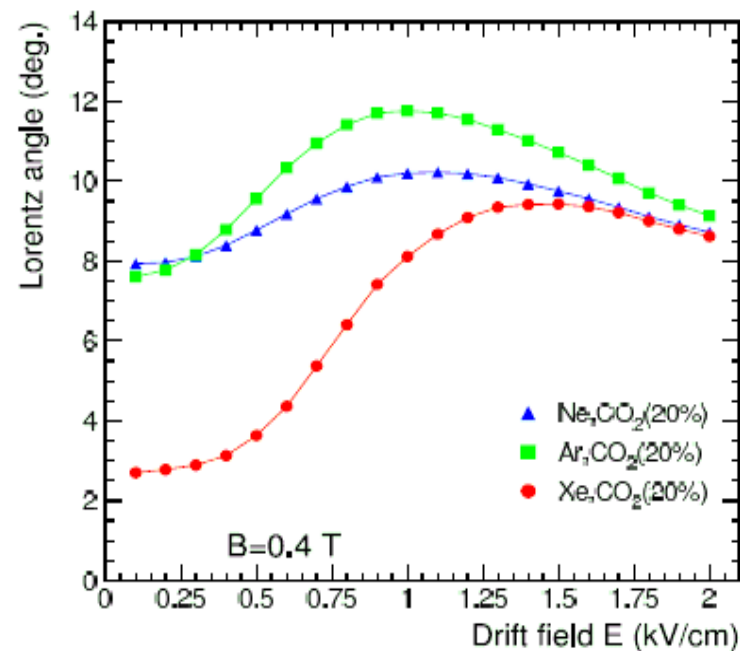
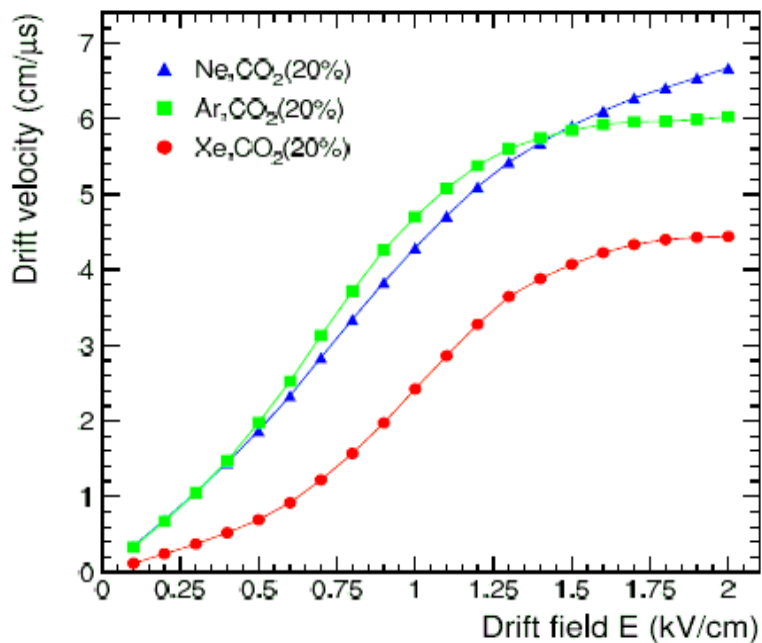
of drift and diffusion by solving a “transport equation”

electron density distribution $f(t, \vec{r}, \vec{v})$

Boltzmann – equation:

$$\underbrace{\frac{\partial f}{\partial t}}_{\text{flow term}} + \underbrace{\vec{v} \frac{\partial}{\partial \vec{r}} f}_{\text{external forces}} + \underbrace{\frac{\partial}{\partial \vec{v}} \vec{g}}_{\text{collision term (stochastic)}} = Q(t)$$
$$\vec{g} = \left(\frac{e\vec{E}}{m} + \vec{\omega} \times \vec{v} \right) f$$

numerical solution with codes such as Magboltz & Garfield



Garfield + Magboltz calculation

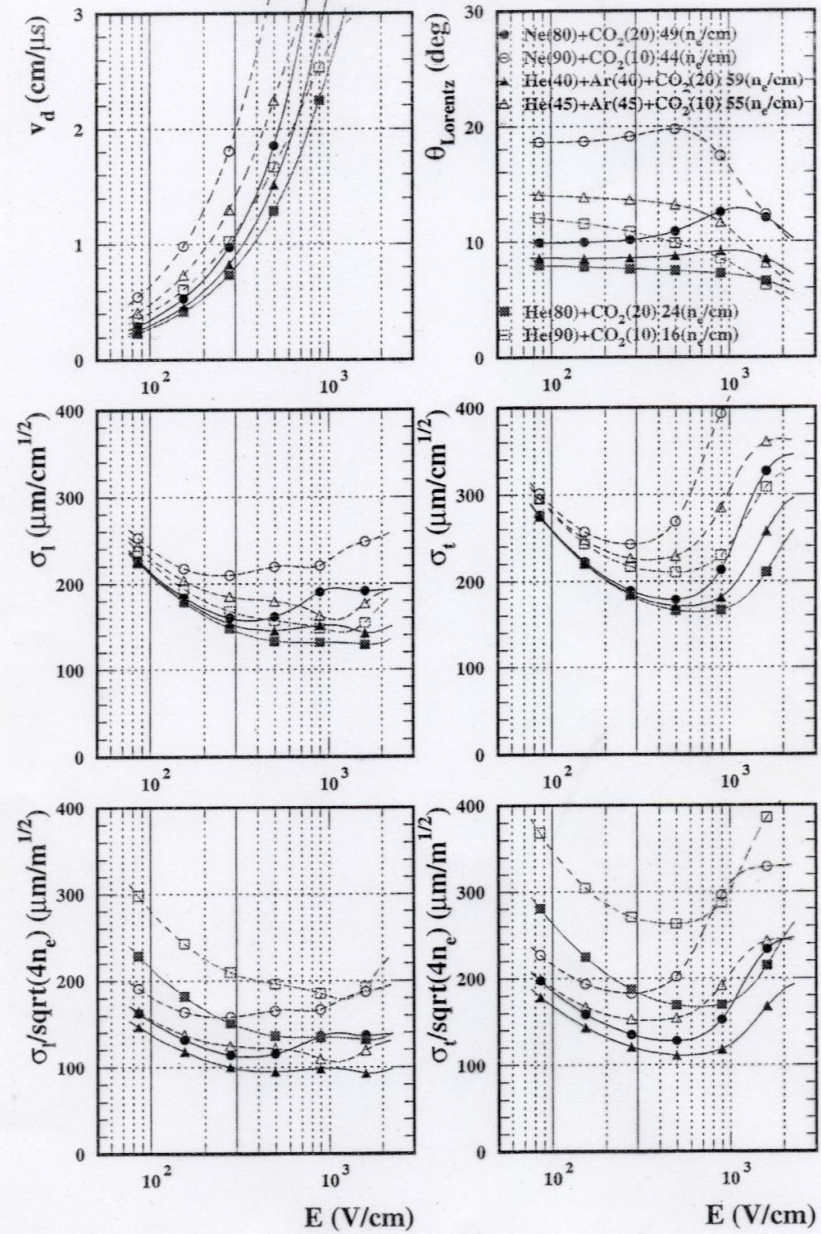
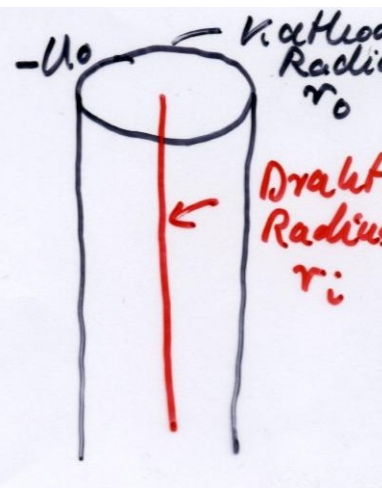


Figure 7: Drift velocity (top left), Lorentz angle (top right), longitudinal and transverse diffusion constants (middle) and longitudinal and transverse diffusion constants normalized to the square root of the number of charge carriers (bottom) for different mixtures of noble gas and CO_2 .

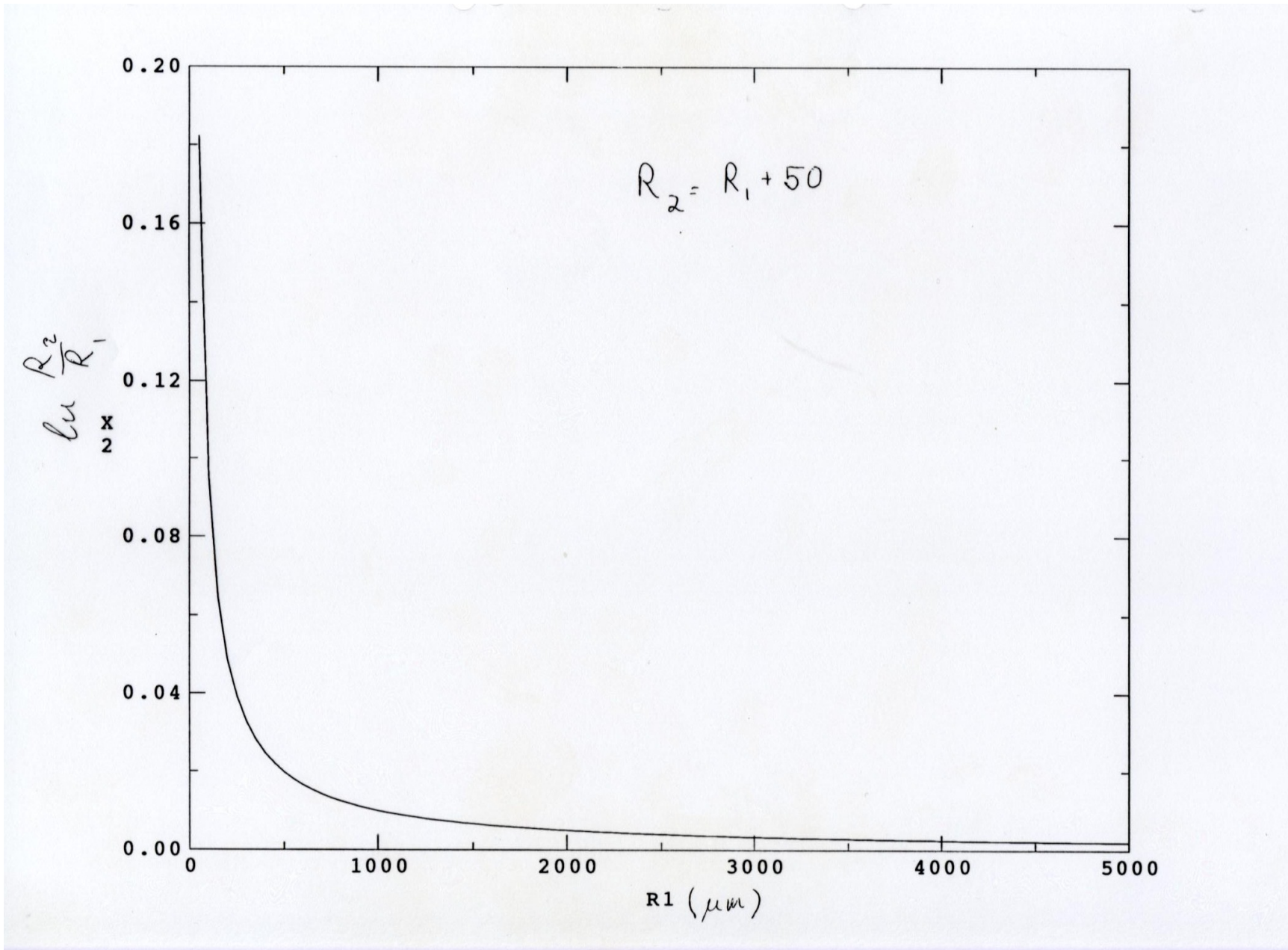
3.3. Gas amplification

in case anode is a (thin) wire, E-field in vicinity of wire very large $E \propto 1/r$
and electron gains large kinetic energy

$$\Delta T = e \Delta U = e \int_{r_i}^{r_2} E(r) dr = \frac{e U_0}{\ln(r_0/r_i)} \int_{r_i}^{r_2} \frac{1}{r} dr$$
$$= e U_0 \frac{\ln(r_2/r_i)}{\ln(r_0/r_i)}$$


The diagram illustrates a cylindrical gas detector. It consists of a central anode wire (red line) and an outer cathode cylinder (black line). The anode wire is labeled "Anode Radius r_i " in red. The cathode cylinder is labeled "Cathode Radius r_0 " in black. The potential difference between the anode and cathode is indicated as $-U_0$ at the top of the cylinder. A red arrow points from the cathode towards the anode, indicating the direction of electron drift.

in order to obtain large E and hence large $\Delta T \rightarrow$ very thin wire ($r_i \cong 10 - 50 \mu\text{m}$)



within a few wire radii $\rightarrow \Delta T$ large enough for secondary ionization

strong increase of $E \rightarrow$ avalanche formation for $r \rightarrow r_i$

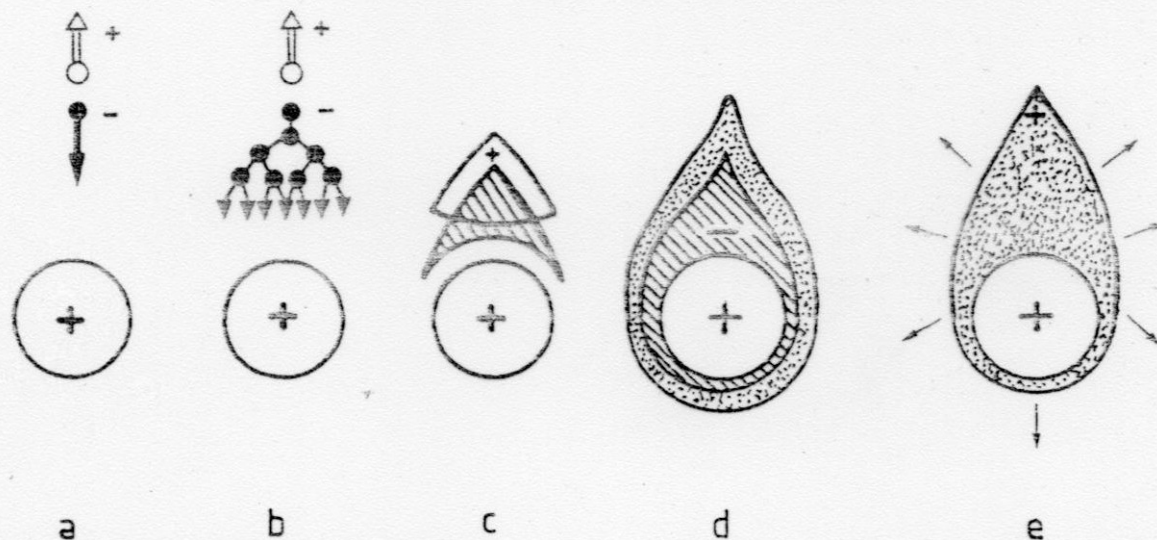


Fig. 4.27. Temporal and spatial development of an electron avalanche [51, 186].

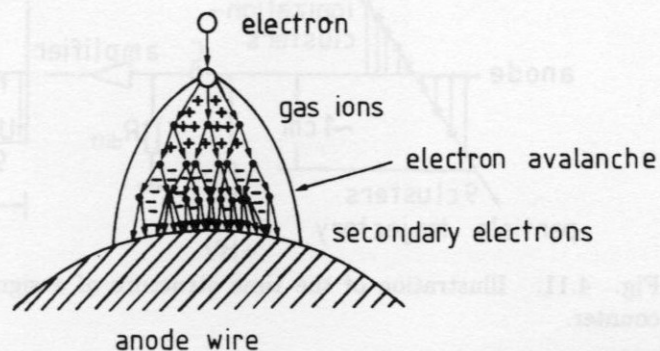


Fig. 4.9. Illustration of the avalanche formation on an anode wire in a proportional counter. By lateral diffusion a drop-shaped avalanche develops.

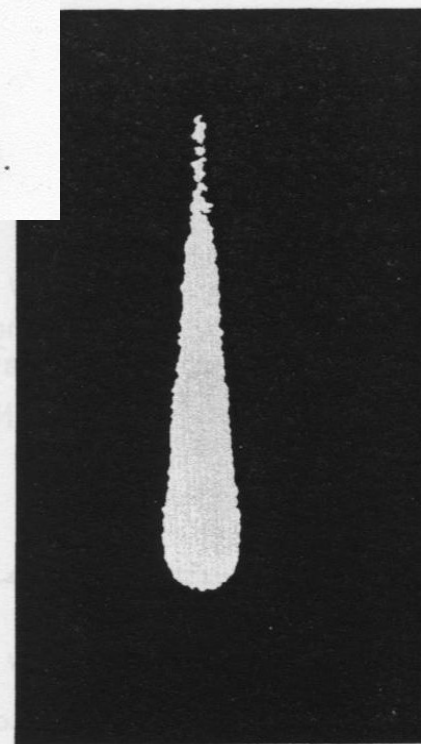


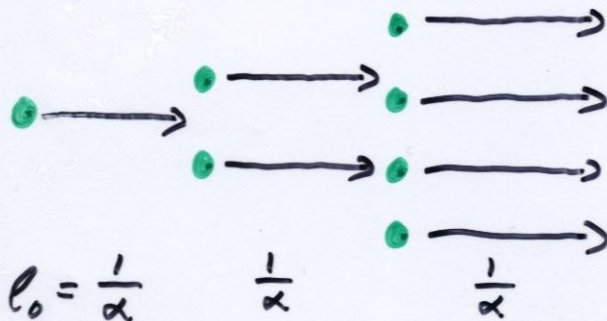
Fig. 4.12. Photographic reproduction of an electron avalanche [51, 155, 156]. The photo shows the form of the avalanche. It was made visible in a cloud chamber (see section 4.12) by droplets which have condensed on the positive ions.

First Townsend coefficient α

number of electrons

$$N(x) = N_0 \exp(\alpha x)$$

gas gain



mean free path

since

$$l_0 = \frac{1}{\alpha} = \frac{1}{N_0 \sigma(T)}$$

and

$$T = T(E)$$

$$\Rightarrow \alpha = \alpha(x)$$

and

gas gain

$$G = \exp\left(\int_{r_1}^{r_2} \alpha(x) dx\right)$$

typically

$$10^4 - 10^5$$

up to 10^6 possible in proportional mode

limit: discharge (spark) at $\alpha x \cong 20$ or $G = 10^8$ "Raether - limit"

gas gain and ionization by collisions

1. Townsend coefficient

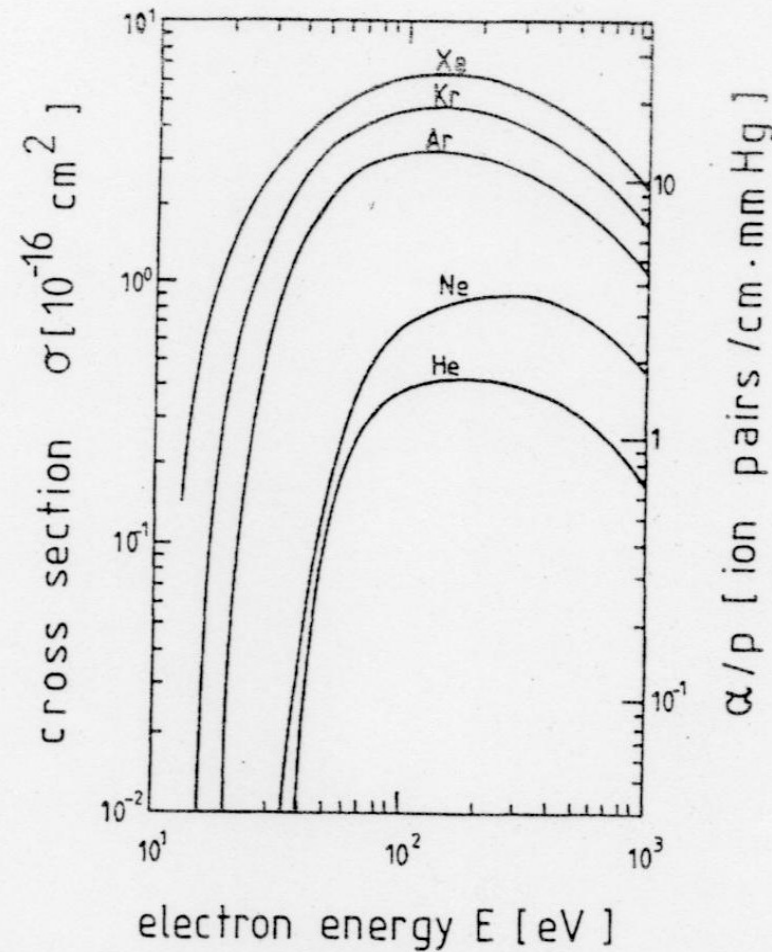


Fig. 4.5. Energy dependence of the cross section for ionization by collision [104, 139, 140].

Second Townsend coefficient γ

excitation of gas generates UV – photons which in turn can lead to photoeffect in gas and on cathode wire contributing thus to avalanche

$$\gamma = \frac{\# \text{ photo effect events}}{\# \text{ avalanche electrons}}$$

gas gain including photoeffect

$$G_{\gamma} = G + \underbrace{G(G\gamma)}_{\substack{\text{no} \\ \text{photo effect events}}} + \underbrace{G(G\gamma)^2}_{\text{one}} + \underbrace{\dots}_{\text{two}} + \dots = \frac{G}{1 - \gamma G}$$

limit: $\gamma G \rightarrow 1$ continuous discharge independent of primary ionization

to prevent this, add to gas so-called **quench-gas** which absorbs UV photons strongly leading to excitation and radiationless transitions

examples: CH_4 , C_4H_{10} , CO_2

gas gain by photo effect and
second Townsend coefficient

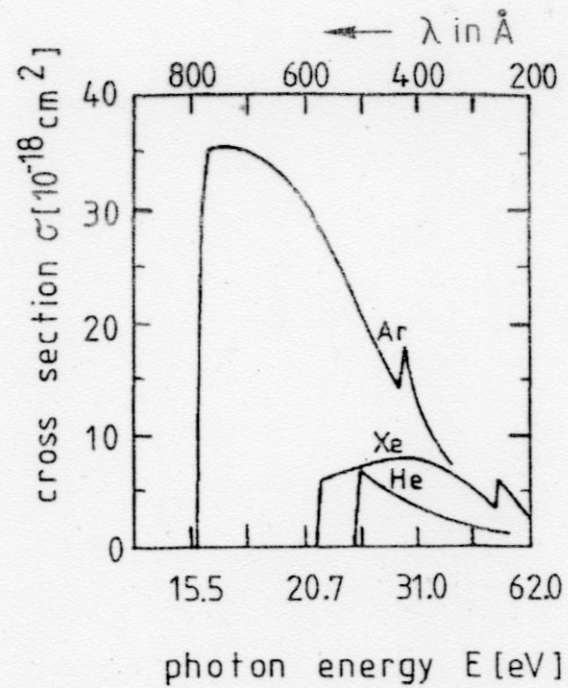


Fig. 4.6. Energy dependence of the cross section for photoionization

3.4. Ionization chamber

no gas gain, charges move in electric field and induce signal in electrodes

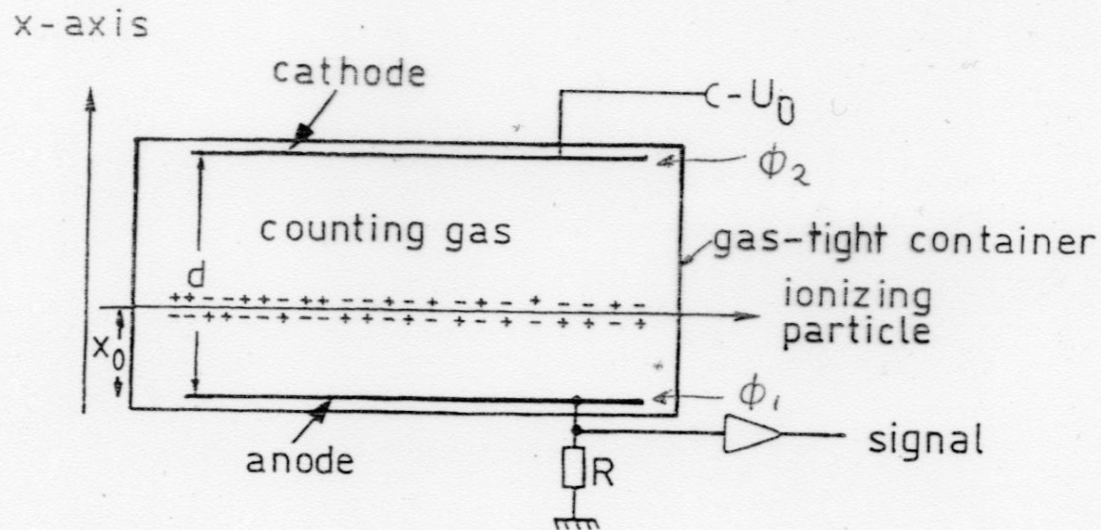


Fig. 4.1. Principle of operation of a planar ionization chamber.

2 electrodes form parallel plate capacitor

consider motion of a free charge q : electric field does work, capacitor is charged (lowering in energy of capacitor)

$$q \vec{\nabla} \varphi \cdot d\vec{x} = dq_i \cdot U_0$$

leads to induced current

$$I_{ind} = \frac{q}{U_0} \vec{\nabla} \varphi \cdot \vec{v}_0$$

with $\vec{E} = - \vec{\nabla} \varphi$ and $U_0 = \varphi_1 - \varphi_2$

- current is constant while charge is drifting
- total induced signal (charge) independent of x_0
- signal induced by electrons
- signal induced by ions

$$\Delta q_- = \frac{N_e}{u_0} (\phi(x_0) - \phi_i)$$

$$\Delta q_+ = -\frac{N_e}{u_0} (\phi(x_0) - \phi_e)$$

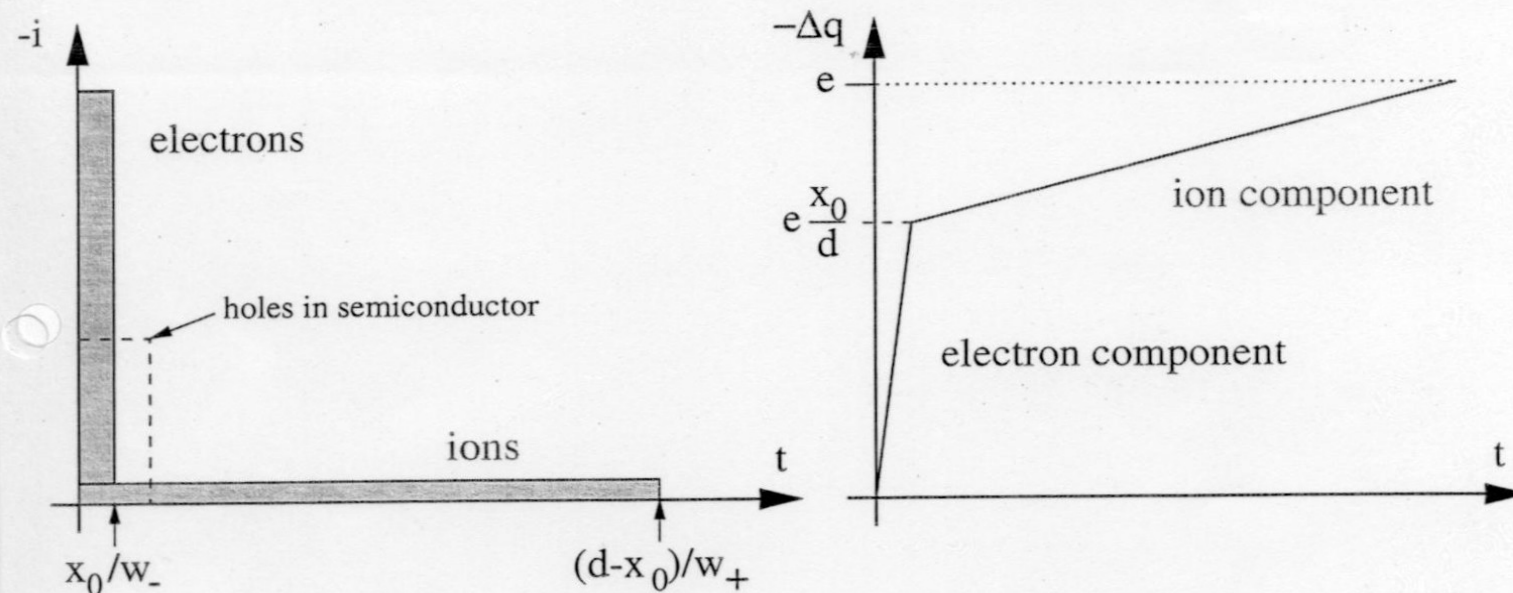
$$|N_{ion}| = |N_e|$$

opposite charge

→ total $\Delta q = N_e$

Practical problem: ion drift comparatively slow

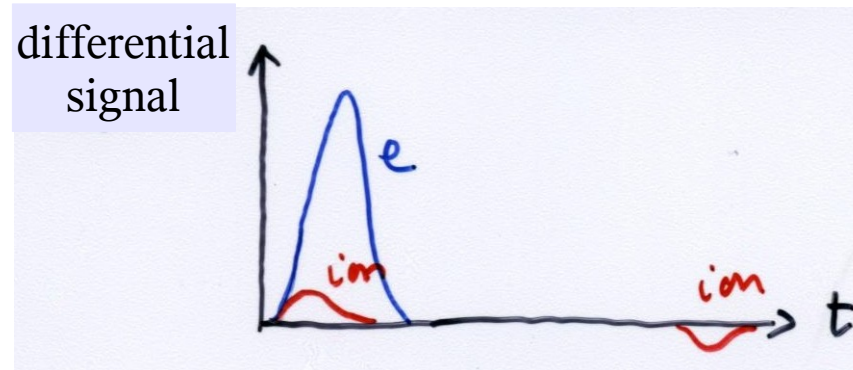
$$w_+ = 10^{-3} \dots 10^{-2} w_- \quad (\text{except for semiconductors: typ. } w_+ \approx 0.5 w_-)$$



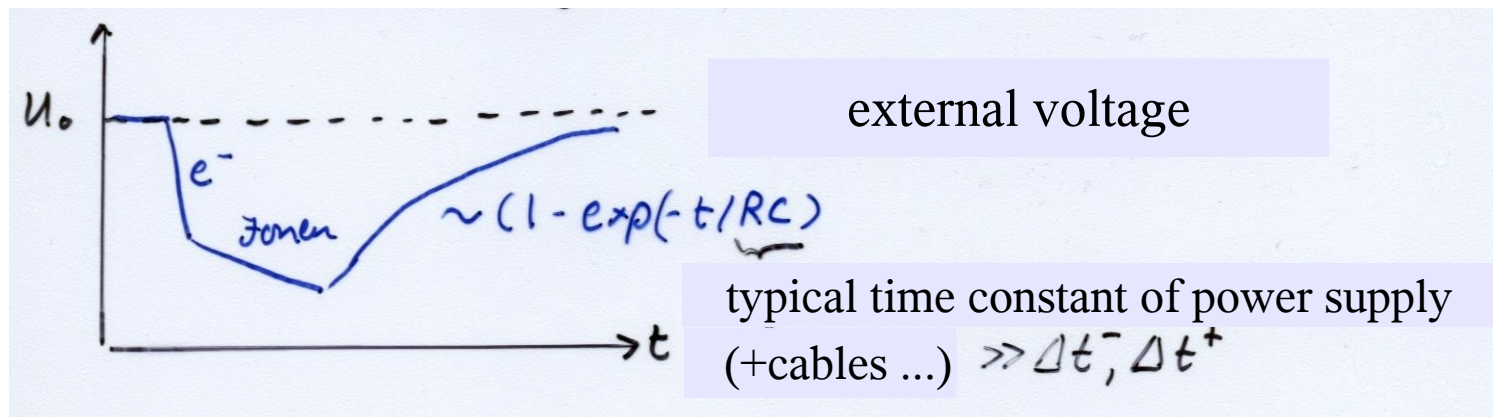
Induced current and charge for parallel plate case, ratio w_-/w_+ decreased for clarity

signal generated during drift of charges

- induced current ends when charges reach electrodes
- induced charge becomes constant (total number N_e)
- signal shaping by differentiation (speed of read-out) $\rightarrow$ suppresses slow ion component

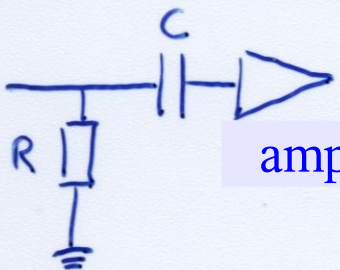


change in potential $dU = dQ/C$



usually electronic signal shaping needed

signal shaping by RC – filter



amplifier

choose e.g. $\Delta t^- \ll RC \ll \Delta t^+$

damps ion component

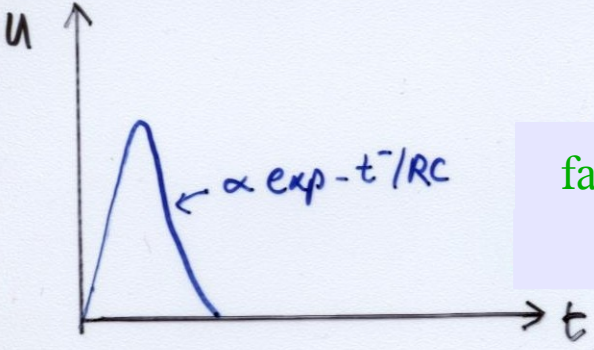
$$\Delta U = \Delta U^- + \Delta U^+$$
$$= \frac{\Delta Q^-}{C} + \frac{\Delta Q^+}{C}$$

where ΔQ^{+-} is charge induced in anode by motion of ions and electrons for total number of ionization events in gas N_e

$$\Delta Q^- = N_e \frac{\phi(x_0) - \phi_1}{U_0} = N_e \frac{x_0}{d}$$
$$\Delta Q^+ = -N_e \frac{\phi(x_0) - \phi_2}{U_0} = N_e \frac{d - x_0}{d}$$

without filter $\Delta Q = N_e \quad \Delta U = \frac{Ne}{C}$

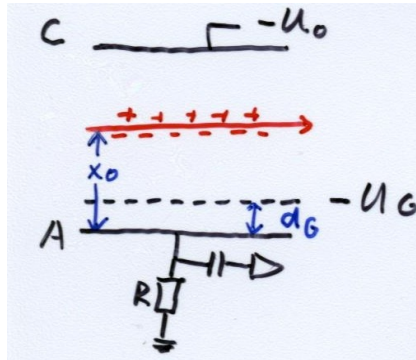
with filter $d - x_0 = v^+ \Delta t^+ \rightarrow v^+ RC (1 - \exp(-\frac{\Delta t^+}{RC}))$



Damping of ion component

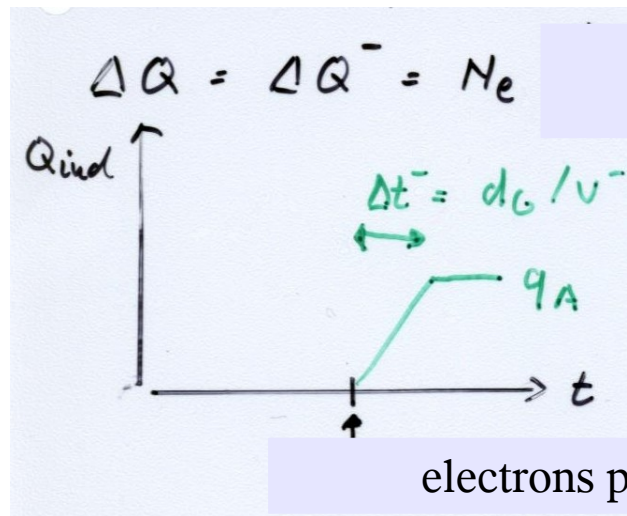
fast rise and decrease of signal
but now pulse height depends on x_0

trick: introduce additional grid “Frisch-grid”



while electrons drift towards Frisch-grid, no induced signal on anode, only on FG

as soon as electrons pass Frisch-grid, signal induced on anode



choose U_G such that
E-field is unchanged

difficulty: small signals

example 1 MeV particle stops in gas

$$\left. \begin{aligned} N_e &\approx \frac{10^6 \text{ eV}}{35 \text{ eV}} \approx 3 \cdot 10^4 \\ C &\approx 100 \text{ pF} \end{aligned} \right\} \Delta U_{\text{max}} = \frac{3 \cdot 10^4 \cdot 1.6 \cdot 10^{-19} \text{ C}}{10^{-10} \text{ F}} \approx 4.6 \cdot 10^{-5} \text{ V}$$

need sensitive, low-noise preamplifier

application: e.g. cylindrical ionization chamber for radiation dosimetry

$$\vec{E}(r) = - \frac{U_0}{\ln(r_a/r_i)} \hat{e}_r$$

ionization at radius r_0 :

$$\begin{aligned} \Delta t^- &= \int_{r_0}^{r_i} \frac{dr}{v^-(r)} = - \int_{r_0}^{r_i} \frac{dr}{\mu^- E} = - \int_{r_0}^{r_i} \frac{dr}{\mu^- U_0} r \ln \frac{r_a}{r_i} \\ &= \frac{\ln \frac{r_a}{r_i}}{2\mu^- U_0} (r_0^2 - r_i^2) \end{aligned}$$

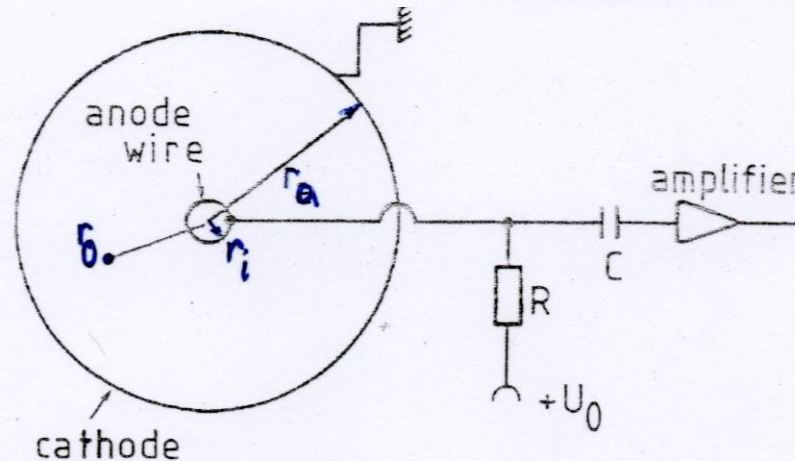


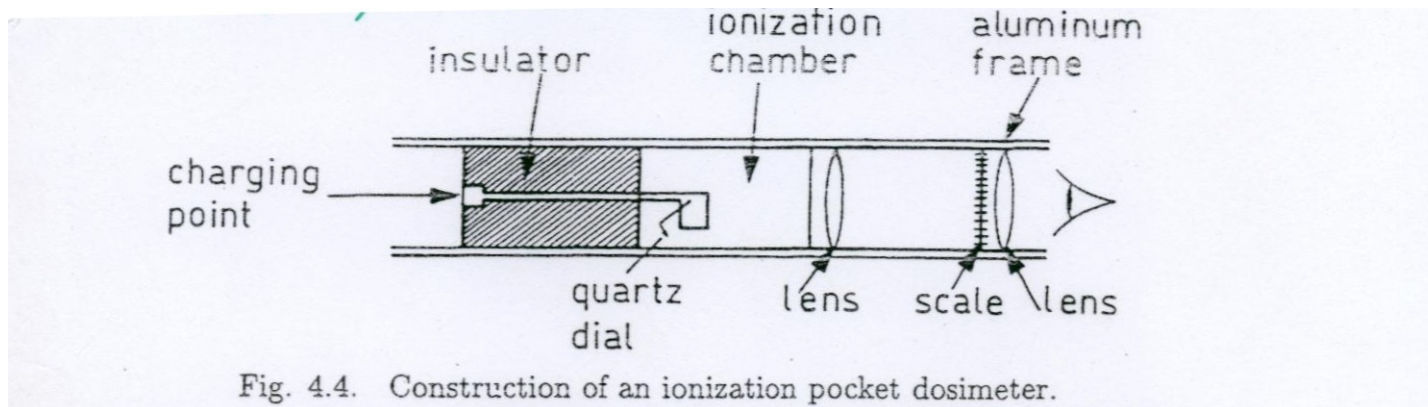
Fig. 4.3. Principle of operation of a cylindrical ionization chamber.

l_0 : typical ionization length, the centroid of the avalanche is this amount away from the wire

$$\Delta Q^- = \frac{N_e}{U_0} \int E(r) dr = \frac{N_e}{\ln(r_a/r_i)} \ln\left(\frac{r_i}{l_0}\right) \quad \Delta U^- = \Delta Q^- / C$$

$$\Delta U^+ / \Delta U^- = \frac{\ln(r_a/l_0)}{\ln(r_i/l_0)} \text{ and since } r_a \gg r_i \rightarrow \Delta U^+ > \Delta U^-$$

in cylindrical geometry, ion signal dominates by typically factor 100



- cylindrical capacitor filled with air
- initially charged until potential U_0
- ionization continuously discharges capacitor
- reduction of potential ΔU is measure for integrated absorbed dose (view via electrometer, e.g.)

other applications: measure energy deposit of charged particle, should be highly ionizing (low energy) or even stop (then measure total kin. energy)
 nuclear physics experiments with energies of 10 to 100 MeV
 combination of ΔE and E measurements $\rightarrow$ particle id (nuclei)