Natural frequency of Mesh Plane:

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mesh frame radius — wire diameter — mesh wire spacing

$R_{mp} := 51\text{cm}$ $\quad d_{mw} := .0012\text{in}$ $\quad s_{mw} := \frac{1}{50}\text{in}$ assume no wire crossing; isotropic properties

open area fraction

$$\eta_a := \frac{(s_{mw} - d_{mw})^2}{s_{mw}^{\;2}} \qquad \eta_a = 88\,\%$$

Assume a homogeneous mesh of thickness equal to wire diameter. Then equivalent mesh elastic modulus will be material modulus * cross section areal fill ratio

$$N_a := \frac{\frac{\pi}{4} d_{mw}^{\;2}}{s_{mw} \cdot d_{mw}} \qquad N_a = 0.047$$

Assume we can stretch mesh well past S.S. Yield Stength of 35 ksi and close to Ultimate (breaking) Strength.

S.S Ultimate Strength

$$S_{u\_ss} := 75000\text{psi} \qquad S_{u\_ss} = 517.1\,\text{MPa}$$

Then let equivalent (cross section area average) mesh prestress be:

$$\sigma_{ps} := 0.75 S_{u\_ss} \cdot N_a \qquad \sigma_{ps} = 18.276\,\text{MPa}$$

Natural frequency:

given :

mass density — Tension per unit length on rim — Areal mass density

$$\rho_{SS} := 8.0\frac{\text{gm}}{\text{cm}^3} \qquad T_m := \sigma_{ps} \cdot d_{mw} \qquad \sigma_m := \rho_{SS} \cdot d_{mw} \cdot 2N_a$$

$$T_m = 557.054\,\frac{\text{N}}{\text{m}} \qquad \sigma_m = 0.023\,\frac{\text{kg}}{\text{m}^2}$$

natural frequency equation and roots, from <u>Dynamics of Smart Structures</u>, Ranjan Vepa

Proceeding exactly in the case of the thin circular clamped plate and considering the case of a circular membrane that is stretched to a uniform tension over a circular frame and clamped at its periphery, the associated frequency equation is

$$J_n(kR) = 0. \qquad (5.80)$$

The roots of the frequency equation (5.80)

$$kR = \lambda_{n,m},\ m = 1,\ 2,\ 3,\ \ldots$$

for each n may be found numerically. The associated natural frequencies then are

$$\omega = \sqrt{\frac{T}{\rho d}}\left(\frac{\lambda_{n,m}}{R}\right). \qquad (5.81)$$

The values of $\lambda_{n,m}$ are tabulated in Table 5.2. Associated with each of the natural frequencies is a pattern of nodal lines, which are almost identical to those illustrated in Figure 5.3 for the case of a thin circular plate clamped along the boundary.

The fundamental vibration of a stretched circular membrane is with the circumference as a node. The ratios of the next two natural frequencies to the fundamental, with circles as the nodal lines, are 2.3 and 3.6. The ratios of the next two natural frequencies to the fundamental, with diameters as the nodal lines, are 1.59 and 2.14. The ratio of the natural frequency of vibration with one nodal circle and one

Table 5.2 Numerically obtained values of $\lambda_{n,m}$

m $\quad n \rightarrow$	0	1	2	3	4
1	2.405	3.832	5.135	6.379	3.586
2	5.520	3.016	8.417	9.760	11.064
3	8.654	10.173	11.620	13.017	14.373

fundamental frequency equation root: — fundamental frequency:

$$\lambda_{01} := 2.405$$

$$\omega_{01} := \frac{\lambda_{01}}{R_{mp}} \cdot \sqrt{\frac{T_m}{\sigma_m}} \qquad \omega_{01} = 734.2\,\text{s}^{-1} \qquad f_{01} := \frac{\omega_{01}}{2\pi} \qquad \boxed{f_{01} = 117\,\text{Hz}}$$