Electric Forces between Charged Plates

Goals of this lab

- **determine the force between charged parallel plates**
- **measure the permittivity of the vacuum (ε_0)**

Overview

In this experiment you will measure the force between the plates of a parallel plate capacitor and use your measurements to determine the value of the vacuum permeability ε_0 that enters into Coulomb's law. Accordingly, we need to develop a formula for the force between the plates in terms of geometrical parameters and the constant ε_0. We conduct the experiment in air, which as a permittivity equal to that of a vacuum to within one part in 10^4.

The capacitor consists of two circular plates, each with area *A*. If a voltage *V* is applied across the capacitor the plates receive a charge ±*Q*. The surface charge density on the plates is $\pm\sigma$ where $\sigma = Q/A$

If the plates were infinite in extent each would produce an electric field of magnitude

$E = \sigma/2\varepsilon_0 = Q/2A\varepsilon_0$, as illustrated in **Figure 1**.

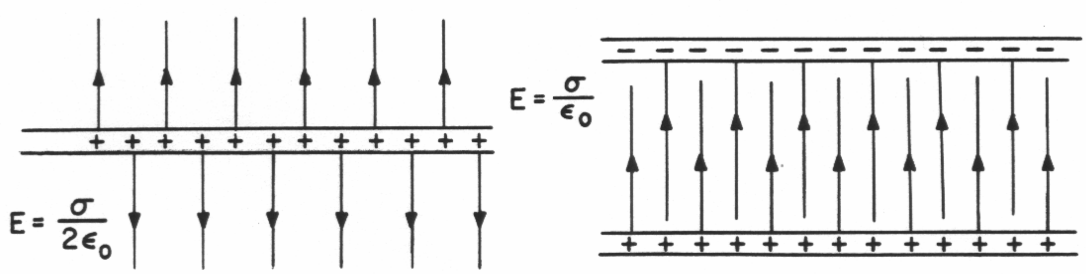

Figure 1: The electric field made by (left) a single charged plate and (right) two charged plates

Since each plate contributes equally, the total electric field between the plates would be

$$E_{total} = Q/A\varepsilon_0$$

The potential difference is $V = E_{total}d = d\dfrac{Q}{A\varepsilon_0}$, where *d* is the plate separation.

Solving for *Q* yields $Q = A\varepsilon_0 \dfrac{V}{d}$ (1)

The plates are oppositely charged, so the attractive force F_{att} between the two plates is equal to the electric field produced by one of the plates times the charge on the other:

$$F_{att} = Q\frac{Q}{2A\varepsilon_0} = \frac{\varepsilon_0 AV^2}{2d^2} \qquad (2)$$

where Equation (1) has been used to express Q in terms of the potential difference V.

Prelab Question 1: The force of attraction, F, between two charged metal plates is proportional to $\frac{\varepsilon_0 AV^2}{2d^2}$. Show that F has the units of Newtons (N).

Prelab Question 2: If you had two charged plates with twice the diameter of the lab apparatus, with the same separation distance and same V, how would the force between the plates change? What would you have to do to the separation distance between the plates to make the force between the plates the same as the lab apparatus?

Equation (2) was derived under the assumption that the plates are infinite in extent. For example, the expression $E = \sigma / 2\varepsilon_0$ for the electric field is rigorous only for an infinite uniformly charged sheet. A small correction factor is needed to account for the finite size of the circular plates. Denoting the diameter of each plate by D, the corrected formula for the attractive force is

$$F_{att} = \frac{\varepsilon_0 AV^2}{2d^2}\left(1 + 2d/D\right) \qquad (3)$$

The final factor is the correction term, which is unity if the diameter of the plates is very large, but has a measurable effect for plates of finite diameter.

The lab apparatus is illustrated in **Figure 2**. The object of the experiment is to measure the attractive force between two circular plates and infer the value of ε_0. Figure 2 shows the diagram of the forces and the torques involved. The lower plate is fixed, but the see-saw formed by the upper plate and the mass pan attached to it and the counter-balance weight rotates about the pivot point, *A*. The distances *R* and R_0 and the mass *m* of the counter-balance have been chosen so that in the absence of a mass *m* on the mass pan and/or an attractive electric force between the plates there is a

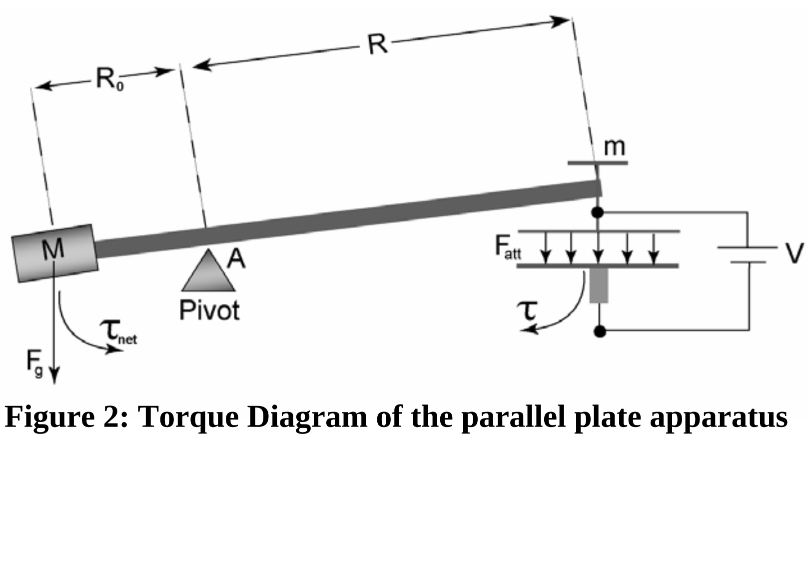

Figure 2: Torque Diagram of the parallel plate apparatus

net counter-clockwise torque τ_{net} on the system.

Rotational equilibrium is established by adding additional mass m to the mass pan and/or applying a voltage between the plates to produce an attractive force between them. Since both the gravitational force F_g and the electrical force F_{att} have a lever arm R with respect to the pivot when the balance arm is level, the equation of rotational equilibrium is

$$\tau_{net} = RF_g + RF_{att} = Rmg + RF_{att}$$

Using (3) for F_{att}

$$\tau_{net} = Rmg + R\frac{\varepsilon_0 A V_m^2}{2d^2}\left(1+\frac{2d}{D}\right) \tag{4}$$

where V_m is the specific value of the voltage that produces rotational equilibrium in the presence of the mass m.

Since it is difficult to determine the precise value of the voltage that will produce equilibrium, we will adopt the opposite tack and find the smallest voltage that will produce a net torque in the clockwise direction, thereby causing the plates to come together. If the voltage is increased in small increments, this voltage will be a good approximation to V_m.

Accordingly we rewrite Equation (4) in the form

$$m = -\frac{\varepsilon_0 A}{2gd^2}\left(1+\frac{2d}{D}\right)V_m^2 + \frac{\tau_{net}}{Rg} \tag{5}$$

Thus, a plot of m *vs* V_m^2 will be a straight line whose slope is equal to the product of measurable and ε_0. The intercept of this line can be analyzed to determine τ_{net} although we will not in fact do this.

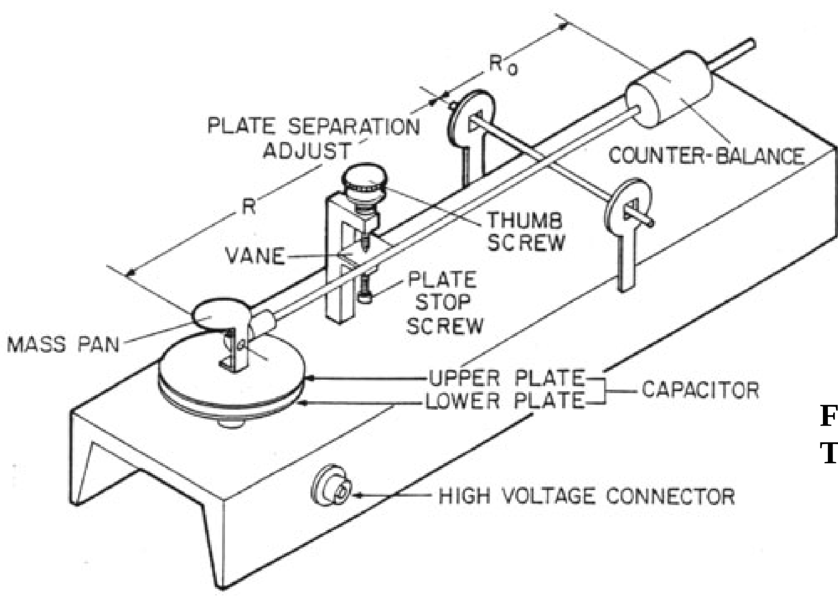

Figure 3:
The parallel plate apparatus

Caution: Although the current available from the high voltage supply is too low to cause any permanent damage, the voltage on the capacitor plates is high enough to cause a distinctly unpleasant sensation if you touch them when the voltage is turned on!

Note: There is a high voltage probe connected to the power supply which reduces the voltage read by the meter by a factor of 1000. Thus, if the meter **indicates** 0.05 volts, the **actual** voltage is $0.05 \times 1000 = 50\ Volts$.

Questions

1. In Part II step 3) why would you <u>not</u> want to plot *V vs. m*?
2. Did your value for ε_0 come within error of the accepted value of $8.85 \times 10^{-12} \frac{C^2}{Nm^2}$?
3. What is the effect of an error in the plate separation distance on your experimental results?
4. What is the reason for the correction factor d/D in Equation (3)? How big of an effect does it have?

Part I Procedure: Calibrating the balance

The balance is sensitive to very small forces, air currents and vibrations. Avoid touching it or the connecting cable while making your measurements. Avoid sudden movements, such as flipping of pages, which may generate air currents. Don't move the balance!

1. <u>With the power supply off,</u> measure the diameter *D* of the plates. This can also be done after your measurements are made.
2. The capacitor plates must be adjusted so that they are concentric and parallel. Adjust the upper plate until it is concentric with the lower (fixed) plate. While doing this make sure that the cross rod that pivots the plates is not touching the corners of the square holes in the supports.
3. Now look at the horizontal space between the plates and check to see whether the plates are parallel to each other when the plate separation distance is approximately 1/16 inch. If they are not, ask the instructor to adjust them. From here on avoid any change in the alignment of the plates.
4. Place 500 mg on the mass pan. The upper plate should not move. Add 50 mg. The upper plate should now move downward. If it does not, adjust the counterbalance weight and try this again.
5. If the capacitor plates come too close together when there is a high voltage between them, an arc will occur, as indicated by a sizzling noise. At the same time, you can get a variable reading or complete blank-out of the voltmeter. Adjust the lower screw

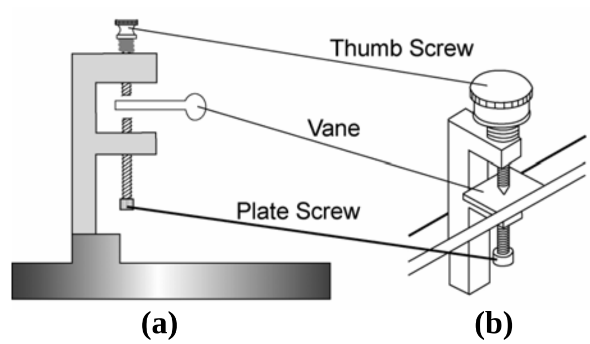

Figure 4: Plate Separation Adjustment with the F bracket (a) showing the side view and (b) showing a perspective view.

in the F bracket to avoid this arcing so that the upper plate cannot come closer than 1 mm to the lower plate by setting the screw to leave a small space between the plates. When the two screws in the F-bracket are properly adjusted the upper plate can only move downward a small distance, about 1/32 inch (1 mm). The voltmeter reading should tell you the correct voltage.

6. To set the plate separation *d*, insert a small copper plate with a precision ball bearing of approximately 1/16 inch diameter fitted in it, between the plates. Position the ball bearing so that it is located about one third of the way in from the outer edge of the plate (avoid the center).

7. Set the digital meter to measure DC volts. Set the power supply voltage to 50 volts (recall the meter reading will be 0.050 volts due to the probe). Adjust the thumb-screw until the upper plate just barely touches the bearing. This is indicated by the voltage going to zero, which means there is electrical contact between the capacitor plates and the ball bearing. Turn the thumb-screw counter-clockwise a small amount and then turn it clockwise again, to find the exact point at which contact is made.

8. Carefully remove the copper plate and bearing. The plate separation has now been set equal to the diameter of the precision ball bearing. Measure the diameter of the bearing with a micrometer. Use the tweezers to discharge any residual charge on the capacitor by touching the lower plate and the base at the same time.

Part II: Acquiring data

1. To a good approximation the voltage which causes the upper plate to move downward equals the voltage V_m in Equation (5). Be sure that the weights are placed at the center of the mass pan. Find the voltages that will move the upper plate with no mass on it and then with 500 mg on it. This will define the range of voltages you will be graphing.

2. Take data for the intermediate masses. For each mass decrease the voltage to zero and then *slowly* increase it until the upper plate moves abruptly toward the lower plate. (You will hear a soft click when this happens.) The reading on the voltmeter when this occurs is an approximate value of V_m. To refine the measurement, lower the voltage and repeat this procedure four or five times. Find the best value of V_m. The uncertainty should not be greater than a few percent.

3. Using the relationship between mass and voltage in Equation (5), make a graph and determine the slope to find ε_0 with its uncertainty.