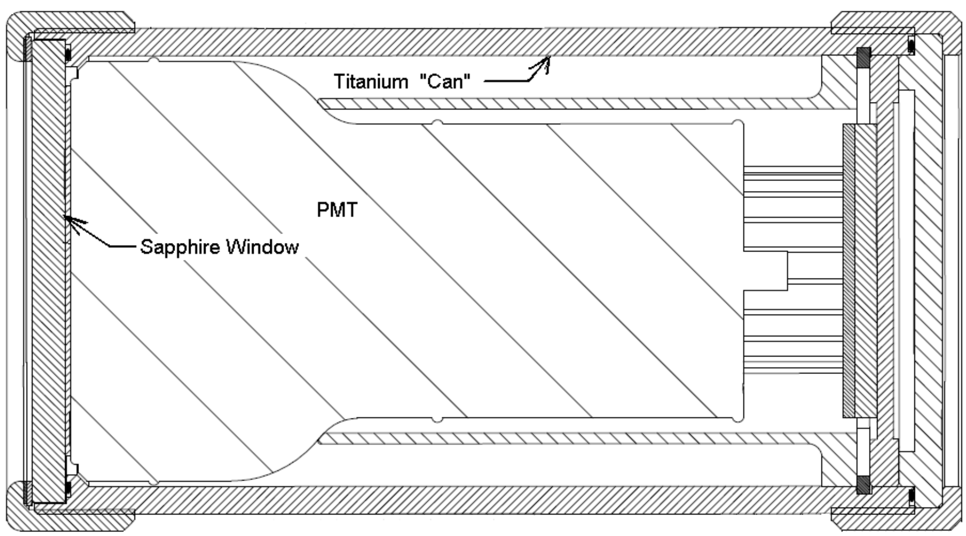
proj: **NEXT-100**

Sapphire window

title: **Sapphire Window Pressure Safety**

DRAFT

The NEXT100 ANGEL design uses 60 photomultiplier tubes (PMTs) mounted inside pressure resistant "cans" having a high strength window. These "canned PMTs" are mounted inside a pressure vessel filled with Xenon gas at 15 bar(d) operating pressure. Single crystal sapphire is the strongest material available, allowing a thin window. We desire to maximize optical transmission and minimize cost, and so the task at hand is to determine an appropriate window thickness. The figure below shows a longitudinal cross section of the assembly. The inside of each can is kept at vacuum by direct unvalved lines (not shown) leading to an active pumping system; no isolation is possible, and the can cannot become pressurized through slow leakage of Xe through seals. Thus the windows do not present a safety hazard and the usual high safety factors are not appropriate. We do however need a high reliability against failure in operation. To assure this, we will pressure test each window (both sides) in a hydrostatic test chamber beforehand to eliminate any weak windows.



Two questions arise:
1. What is an acceptable failure rate for eliminating the weak windows?
2. What is an acceptable test overpressure?

The first question depends on how critical it is to achieve optimum light transmission. In our case we will be applying a wavelength shifting coating (TPB) to the outside of the window in order to shift the 172nm light from Xe excimer decay to a longer wavelength that will transmit through the sapphire which cuts off below 200 nm. Thus maximization of optical transmission is not critical, and so this author proposes to size the window thickness such that 10% of the purchased windows may fail upon application of the test overpressure. This should assure a final thickness not too far from optimum. The answer to the second question is found by comparing slow growth crack rates with fracture toughness, the idea being that a suitable overpressure test will find any flaws that are above a threshold value for slow crack growth rate.

Stress-thickness function:

for thickness t, radius a, pressure q, Poisson's ratio ν, and assume simple edge support condition (rotation allowed, no extra plate material past support), maximum stress is in the radial direction, and is found at center.

Center Moment:

$$M_{rc} := \frac{3+\nu}{16} q\, a^2$$

and maximum stress is, at center of plate

$$\sigma := 6\frac{M}{t^2}$$

ref. 1: Roark's Formulas for Stress and Strain, 6th ed. table 24 case 10b, fixed supports, plate thickness <1/4 least transverse dimension (=2a)

or:

$$\frac{t^2}{a^2} := 6 \cdot \frac{3+\nu}{16} \cdot \frac{q}{\sigma} \qquad t := \sqrt{\frac{3}{8} \cdot (3+\nu) \cdot \frac{q}{S_{max}} \cdot a^2}$$

Maximum allowable stress :

Although one can find strength numbers for sapphire such as these:

Sapphire Yield Strength, from http://www.roditi.com/SingleCrystal/Sapphire/Properties.htm:

$S_{y\_Al2O3} := .275GPa$

Flexural Strength, min., same ref, is more appropriate for our stress distribution:

$S_{f\_Al2O3} := .48GPa$ $\quad S_{f\_Al2O3} = 6.962 \times 10^4 psi$ or, from Kyocera literature $S_{f\_kyo} := 690MPa$

Sapphire and other brittle materials are not well characterized by a single number for ultimate, yield or flexural strength. Unlike metals, there is much more scatter in the data and failure is a strong function of total stressed area or volume and surface condition, as well as other variables. For this reason large safety factors are often used:

LBNL safety manual (PUB-3000) required factors of safety on maximum stress:

FS>=8 required by PUB-3000 for brittle high hazard, for no personnel barrier, We will have a barrier, so FS>=4

These large safety factors are somewhat arbitrary and not satisfactory. It is not very clear what the true factor of safety really is. This is important for us in that we have 60 windows which will need to be very reliable over many pressure cycles. A better method is to use a probabilistic strength determination, such as the Weibull distribution, which relates a probability of failure(or survival) to a stress and area ratio (between actual area and stress relative to a nominal "test" or "characteristic" area and stress respectively. The basis for this distribution is the assumption that, for brittle materials, actual strength is determined not by the material intrinsic strength, but by the presence of volume or surface flaws; the larger the stessed area or volume the more likely there will be a flaw of minimum size to cause a failure at the given stress. The simplest form of the Weibull distribution is the two parameter type, wherein it is assumed that therre is no applied stress that does not have some finite probability of failure. The two parameters are the "characteristic strength" and the Weibull modulus; the characteristic strength is typically defined as the strength at which (1/e) of the total number of standard specimens survive (uniformly stressed area or volume of a unit area or volume , typically 1 cm^2 or $1cm^3$) . the Weibull modulus is a measure of how quickly the probability changes as stress level and area change. A modulus, m = 1 indicates random failure, m<1 indicates infant mortality, and m>1 indicates defect driven strength, as we have. Metals have a very high modulus m>10 which indicates very little sensitivity to defects.

There exists a significant amount of data on sapphire strength, and it has been shown to have a moderate Weibull modulus (failure strength moderately correlated with total stressed area) so we can attempt to choose a maximum strength which will give a low probability of failure.

From "<u>Flexural strength of sapphire: Weibull statistical analysis of stressed area, surface coating, and polishing procedure effects",</u> C. Klein:

for:	characteristic strength	Weibull modulus	characteristic area (uniform biaxial stress)
	$\sigma_c := 975MPa$	$m := 3.4$	$s := 1cm^2$

copy here:
http://www-eng.lbl.gov/~shuman/NEXT/MATERIALS&COMPONENTS/Quartz/sapphire_weibull_Klein.pdf

For pressure loading the tensile stress is nonuniform and the probability function (dp/dA) must be integrated over the area. From "<u>Materials for Infrared windows and Domes</u>, Daniel C Harris, SPIE Optical Engineering Press 1999, Appendix F

Probability of survival:

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$$P_s := e^{-\int_0^{\infty}\left(\frac{\sigma}{\sigma_0}\right)^m dA}$$ (F-9), A substituted for V)

copy here:
http://www-eng.lbl.gov/~shuman/NEXT/MATERIALS&COMPONENTS/Quartz/Harris_book/Weibull_harris.pdf)

The integral above can be expressed in terms of an effective area kA where:

let

$$k := \frac{1}{A}\cdot\int\left(\frac{\sigma}{\sigma_{max}}\right)^m dA \qquad dA := dr\cdot(r\cdot d\theta)$$

However, a more exact formula for effective area under pressure loading is given in: Slow Crack Growth and Fracture Toughness of Sapphire for the International Space Station, Fluids and Combustion Facility, J. Salem:
http://www-eng.lbl.gov/~shuman/NEXT/MATERIALS&COMPONENTS/Quartz/sapphire_window_NASA.pdf

given :

support radius	window outer radius	Poisson's ratio
$R_s := 38mm$	$R_d := 42mm$	$\nu := .29$

$$A_e := \frac{4\cdot\pi\cdot(1-\nu)}{1+m}\cdot\left(\frac{R_s}{R_d}\right)^2\cdot\frac{2\cdot R_d{}^2\cdot(1+\nu)+R_s{}^2\cdot(1-\nu)}{(3+\nu)\cdot(1+3\nu)} \qquad A_e = 15.045\,cm^2$$

where Salem = Harris
$A_e := k\cdot A_w$

k is then:

$$k_2 := \frac{A_e}{\pi\cdot R_s{}^2} \qquad k_2 = 0.332$$

Probability of Survival:

$$P_s := e^{-k\cdot A_w\cdot\left(\frac{\sigma_{max}}{\sigma_0}\right)^m}$$

Let P_s = 90%; i.e. we allow 10% of purchased windows to fail a pressure test. Although this may sound distressing, to significantly better this, we will need to specify a higher polish grade, and/or increased thickness, which may not save anything in the end, and will result in less optical transmission.

$$P_s := .9$$

then, solving for σ_{max}

where: Harris Weibull scaling factor = Klein characteristic strength

$$\sigma_0 := \sigma_c$$

area ratio, actual to characteristic

$$A_w := \frac{\pi\cdot R_s{}^2}{s} \qquad A_w = 45.365$$

effective area (ratio):

$$k_2\cdot A_w = 15.045$$

we find:

$$\sigma_{max} := \sigma_0 \cdot \left(\frac{\ln(P_s)}{-k_2 \cdot A_w} \right)^{\frac{1}{m}}$$

$\sigma_{max} = 226.6\,\text{MPa}$ compare ->> $\sigma_0 = 975\,\text{MPa}$ (uniform) stress at which 50% of 1 cm^2 specimens fail

This answers the first question; to answer the second question, we first ask " What would cause a failure of a previously tested component?". There are several possible answers, such as subsequent damage (perhaps from thermal or mishandling, presence of degrading environments such as stress corrosion inducing substances, one of which, for sapphire, is water, and repeated pressure cycling. Here we only consider repeated pressure cycling, and estimate a maximum of 100 cycles.

To determine how much test pressure to use, we use fracture mechanics (linear elastic). This analysis method relates crack sizes to fracture strength through a "stress intensity factor K, where Y is a geometry factor, usually around unity, σ is the applied stress and a is the $^1/_2$ crack length.

$$K := Y \cdot \sigma \cdot \sqrt{\pi \cdot a}$$

A given material will have a critical stress intensity K_{Ic} , (AKA fracture toughness) where fracture occurs when the following condition is met:

$$K_{Ic} := Y \cdot \sigma \cdot \sqrt{\pi \cdot a}$$

From this we can determine a maximum crack size, a_{cr} associated with the above stress (weeding out all windows having anything greater than this by the pressure test)

for

$Y := 1$ (plane strain condition) $K_{Ic} := 2.5\text{MPa} \cdot \sqrt{m}$ $a_{cr} := \frac{1}{\pi} \cdot \left(\frac{K_{Ic}}{Y \cdot \sigma_{max}} \right)^2$ $a_{cr} = 0.039\,\text{mm}$

Furthermore we can define our test to operating pressure ratio (factor of safety FS) as a ratio of critical crack sizes

$FS := \frac{\sigma_t}{\sigma_o}$ $\frac{\sigma_t}{\sigma_o} := \sqrt{\frac{a_o}{a_t}}$ where : $a_t := a_{cr}$ and test stress: $\sigma_t := \sigma_{max}$

R.O. Ritchie, et. al., in
"Cyclic fatigue-crack propagation in sapphire in air and simulated physiological environments" copy here:
http://www-eng.lbl.gov/~shuman/NEXT/MATERIALS&COMPONENTS/Quartz/sapphire_fatigue_Ritchie.pdf
find that single crystal sapphire, like many other untoughened ceramics and glasses, does not have a true cyclic fatigue behavior over that of static fatigue, that is, crack growth under a monotonic load, also referred to as stress corrosion cracking. They give a value for threshold static fatigue:

$K_{TH} := 1.64\text{MPa} \cdot \sqrt{m}$ for humid air, 85% R.H., where crack velocity is "vanishingly small" i.e.<1 nm/s:

$v_c := 1\text{nm} \cdot s^{-1}$ which would give a time to failure: $t_f := \frac{a_{cr}}{v_c}$ $t_f = 0.448\,\text{day}$ in a humid air environment

This value of threshold may still be appropriate, given that the mechanism for static fatigue is one of Si-O bond dissociation due to chemisorbed mointure at the crack tip; the environment around the crack will be a silicon optical pad under vacuum. Therefore the factor of safety for pressure testing is simply a function of the ratio

$$\frac{\sigma_t}{\sigma_o} := \sqrt{\frac{a_o}{a_t}} \qquad \frac{\sigma_t}{\sigma_o} := \frac{K_{Ic}}{K_{TH}}$$

and resulting design stress is: $\sigma_o := \sigma_t \cdot \frac{K_{TH}}{K_{Ic}}$ $\quad \sigma_o = 148.7\,\text{MPa}$

for maximum applied operating pressure and safety factor:

$q := 16.4\text{bar}$ (MAWPa)

resulting minimum thickness is:

$$t_{min} := \sqrt{\frac{3}{8} \cdot (3 + \nu) \cdot \frac{q}{\sigma_o} \cdot R_s^2} \qquad t_{min} = 4.43\,\text{mm}$$

let actual window thickness be:

$t := 5\text{mm}$

$E_{saph} := 50 \cdot 10^6 psi$

$$D := \frac{E_{saph} \cdot t^3}{12\left(1 - \nu^2\right)}$$

$$y := \frac{-q \cdot a^4}{64D} \qquad y = -0.02\,mm$$

Quartz tensile strength is only: $S_{yt\_quartz} := 5400psi$ $\qquad S_{yt\_quartz} = 37.232\,MPa$

$$S_{max\_qt} := \frac{S_{yt\_quartz}}{FS_b}$$

$$t_{qt} := \sqrt{\frac{3}{8} \cdot (3 + \nu) \cdot \frac{q}{S_{max\_qt}} \cdot a^2} \qquad t_{qt} = \blacksquare\ cm$$

$$700 \frac{kg}{cm^2} \cdot g = 68.647\,MPa$$