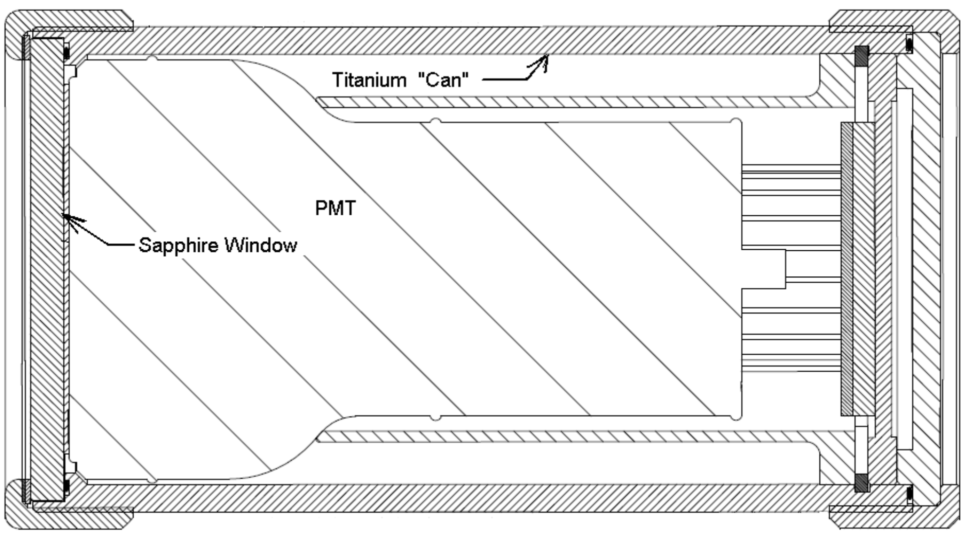
proj: **NEXT-100**
Quartz window
title: **Quartz Window Pressure Safety**

DRAFT

The NEXT100 ANGEL design uses 60 photomultiplier tubes (PMTs) mounted inside pressure resistant "cans" having a high strength window. These "canned PMTs" are mounted inside a pressure vessel filled with Xenon gas at 15 bar(d) operating pressure. Single crystal sapphire is the strongest material available, allowing a thin window. However Suprasil (high purity synthetic fused silica), though substantially weaker, has better optical transmission at 172 nm, and could be considerably cheaper, even at increased thickness. We desire to maximize optical transmission and minimize cost, and so the task at hand is to determine an appropriate window thickness. The figure below shows a longitudinal cross section of the assembly. The inside of each can is kept at vacuum by direct unvalved lines (not shown) leading to an active pumping system; no isolation is possible, and the can cannot become pressurized through slow leakage of Xe through seals. Thus the windows do not present a safety hazard and the usual high safety factors are not appropriate. We do however need a high reliability against failure in operation. To assure this, we will pressure test each window (both sides) in a gas (not hydrostatic) test chamber beforehand to eliminate any weak windows.



Two questions arise:
1. What is an acceptable failure rate for eliminating the weak windows?
2. What is an acceptable test overpressure?

The first question depends on how critical it is to achieve optimum light transmission. In our case, it is not yet clear if will be applying a wavelength shifting coating (TPB) to the outside of the window in order to shift the 172nm light from Xe excimer decay to a longer wavelength that will transmit through the sapphire which cuts off below 200 nm. Thus maximization of optical transmission is highly desired, and so this author proposes to size the window thickness such that 10% of the purchased windows may fail upon application of the test overpressure. This should assure a final thickness not too far from optimum. The answer to the second question is found by comparing slow growth crack rates with fracture toughness, the idea being that a suitable overpressure test will find any flaws that are above a maximum inititial size that would subsequently grow to a fracture critical size over the life of the esxperiment

Stress-thickness function:

for thickness t, radius a, pressure q, Poisson's ratio ν, and assume simple edge support condition (rotation allowed, no extra plate material past support), maximum stress is in the radial direction, and is found at center.

Center Moment:

$$M_{rc} := \frac{3+\nu}{16} q\, a^2$$

and maximum stress is, at center of plate

$$\sigma := 6\frac{M}{t^2}$$

ref. 1: Roark's Formulas for Stress and Strain, 6th ed. table 24 case 10b, fixed supports, plate thickness <1/4 least transverse dimension (=2a)

or:

$$\frac{t^2}{a^2} := 6\cdot\frac{3+\nu}{16}\cdot\frac{q}{\sigma}$$

$$t := \sqrt{\frac{3}{8}\cdot(3+\nu)\cdot\frac{q}{S_{max}}\cdot a^2}$$

$$\sigma := \frac{3}{8}\cdot(3+\nu)\, q\cdot\frac{a^2}{t^2}$$

Maximum allowable stress :

Although one can find strength numbers for quartz such as these:

Quartz Flexural Strength, from hereaeus

$$S_{f\_Suprasil} := 67\mathrm{MPa}$$

Quartz and other brittle materials are not well characterized by a single number for ultimate, yield or flexural strength. Unlike metals, there is much more scatter in the data and failure is a strong function of total stressed area or volume and surface condition, as well as other variables. For this reason large safety factors are often used:

LBNL safety manual (PUB-3000) required factors of safety on maximum stress:

FS>=8 required by PUB-3000 for brittle high hazard, for no personnel barrier, We will have a barrier, so FS>=4

These large safety factors are somewhat arbitrary and not satisfactory. It is not very clear what the true factor of safety really is. This is important for us in that we have 60 windows which will need to be very reliable over many pressure cycles. A better method is to use a probabilistic strength determination, such as the Weibull distribution, which relates a probability of failure (or survival) to a stress and area ratio (between actual area and stress relative to a nominal "test" or "characteristic" area and stress respectively. The basis for this distribution is the assumption that, for brittle materials, actual strength is determined not by the material intrinsic strength, but by the presence of volume or surface flaws; the larger the stressed area or volume, the more likely there will be a flaw of minimum size to cause a failure at some given stress. The simplest form of the Weibull distribution is the two parameter type, wherein it is assumed that there is no applied stress that does not have some finite probability of failure. The two parameters are the "characteristic strength" and the Weibull modulus; the characteristic strength is typically defined as the strength at which (1/e) of the total number of standard specimens survive (uniformly stressed area or volume of a unit area or volume , typically 1 cm^2 or 1cm^3) . The Weibull modulus is a measure of how quickly the probability changes as stress level and/or area change. A modulus, m = 1 indicates random failure, m<1 indicates infant mortality, and m>1 indicates defect driven strength, as we have. Metals have a very high modulus m>10 which indicates very little sensitivity to defects.

There exists a significant amount of data on quartz strength, and it has been shown to have a fairly high Weibull modulus (failure strength weakly correlated with total stressed area and surface finish) so we can attempt to choose a maximum strength which will give a low probability of failure.

From "Characteristic strength, Weibull modulus and failure probability of fused silica glass", C. Klein:

for:	characteristic strength	Weibull modulus	characteristic area (uniform biaxial stress)
	$\sigma_c := 101\mathrm{MPa}$	$m := 10$	$s := 1\mathrm{cm}^2$

$$\sigma_c = 14.6\,\mathrm{ksi}$$

copy here:
http://www-eng.lbl.gov/~shuman/NEXT/MATERIALS&COMPONENTS/Quartz/quartz_weibull.pdf

For pressure loading the tensile stress is nonuniform and the probability function (dp/dA) must be integrated over the area. From "Materials for Infrared windows and Domes, Daniel C Harris, SPIE Optical Engineering Press 1999, Appendix F

Probability of survival:

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$$P_s := e^{-\int_0^{\infty}\left(\frac{\sigma}{\sigma_0}\right)^m dA}$$ (F-9), A substituted for V)

copy here:
http://www-eng.lbl.gov/~shuman/NEXT/MATERIALS&COMPONENTS/Quartz/Harris_book/Weibull_harris.pdf)

The integral above can be expressed in terms of an effective area kA where:

let
$$k := \frac{1}{A}\cdot\int\left(\frac{\sigma}{\sigma_{max}}\right)^m dA \qquad dA := dr\cdot(r\cdot d\theta)$$

However, a more exact formula for effective area under pressure loading is given in: Slow Crack Growth and Fracture Toughness of Sapphire for the International Space Station, Fluids and Combustion Facility, J. Salem:
http://www-eng.lbl.gov/~shuman/NEXT/MATERIALS&COMPONENTS/Quartz/sapphire_window_NASA.pdf

given :

support radius	window outer radius	Poisson's ratio
$R_s := 38mm$	$R_d := 42mm$	$\nu := .17$

$$A_e := \frac{4\cdot\pi\cdot(1-\nu)}{1+m}\cdot\left(\frac{R_s}{R_d}\right)^2\cdot\frac{2\cdot R_d^{\,2}\cdot(1+\nu)+R_s^{\,2}\cdot(1-\nu)}{(3+\nu)\cdot(1+3\nu)} \qquad A_e = 8.637\,cm^2$$

where Salem = Harris
$A_e := k\cdot A_w$

k is then:

$$k_2 := \frac{A_e}{\pi\cdot R_s^{\,2}} \qquad k_2 = 0.19$$

Probability of Survival:

$$P_s := e^{-k\cdot A_w\cdot\left(\frac{\sigma_{max}}{\sigma_0}\right)^m}$$

Let P_s = 99%; i.e. we allow only 1% of purchased windows to fail a pressure test. With the high Weibull modulus of 10, we pay only a small price in increased thickness for this, as opposed to the earlier recomendation of 90% for sapphire, which has a low Weibull modulus of 3.4.

$P_s := .99$

then, solving for σ_{max}

where: Harris' Weibull scaling factor = Klein's characteristic strength

$\sigma_0 := \sigma_c$

area ratio, actual to characteristic

$$A_w := \frac{\pi\cdot R_s^{\,2}}{s} \qquad A_w = 45.365$$

effective area (ratio):

$$k_2\cdot A_w = 8.637$$

we find:

$$\sigma_{max} := \sigma_0 \cdot \left(\frac{\ln(P_s)}{-k_2 \cdot A_w} \right)^{\frac{1}{m}}$$ $\sigma_{max} = 51.4\,\text{MPa}$ compare ->> $\sigma_0 = 101\,\text{MPa}$ (uniform) stress at which 63% of 1 cm^2 specimens fail

This answers the first question; to answer the second question, we first ask " What would cause a failure of a previously tested component?". There are several possible answers, such as subsequent damage (perhaps from thermal or mishandling, presence of degrading environments such as stress corrosion (AKA static fatigue) inducing substances, one of which, for quartz, is water, and repeated pressure cycling. Here we only consider static fatigue as cyclic fatigue in glasses is not a pronounced phenomenon and we have very few applied cycles, probably a hundred at most.

To determine how much test pressure to use, we use fracture mechanics (linear elastic). This analysis method relates crack sizes to fracture strength through a "stress intensity factor K, where Y is a geometry factor, usually around unity, σ is the applied stress and a is the $^1/_2$ crack length.

$$K := Y \cdot \sigma \cdot \sqrt{\pi \cdot a}$$

A given material will have a critical stress intensity K_{Ic} , (AKA fracture toughness) where fracture occurs when the following condition is met:

$$K_{Ic} := Y \cdot \sigma \cdot \sqrt{\pi \cdot a_{cr}}$$ for mode I displacement

From this we can determine a maximum crack size, a_{cr} associated with the above stress (weeding out all windows having anything greater than this by the pressure test)

for

$Y := 1$ (mode I plane strain condition) (ref. 4 in Klein gives 1.12) $K_{Ic} := 0.66\text{MPa} \cdot \sqrt{m}$ $a_{cr} := \frac{1}{\pi} \cdot \left(\frac{K_{Ic}}{Y \cdot \sigma_{max}} \right)^2$ $a_{cr} = 0.052\,\text{mm}$

Furthermore, we can define our test to operating pressure ratio (factor of safety FS) as a ratio of critical crack sizes

$$FS := \frac{\sigma_t}{\sigma_o} \qquad \frac{\sigma_t}{\sigma_o} := \sqrt{\frac{a_o}{a_t}}$$ where : $a_t := a_{cr}$ and test stress: $\sigma_t := \sigma_{max}$

It is generally known that many untoughened ceramics and glasses, do not have a true cyclic fatigue behavior over that of static fatigue, that is, crack growth under a monotonic load. Static fatigue is in actuality, stress corrosion cracking, in humid or wet environments. This is the reason we do not use a hydrostatic pressure test

Without further literature search, it is not clear whether or not static fatigue occurs in the complete absence of water. From "Humidity Dependence on the Fatigue of High Strength Silica Optical Fibers" Armstrong, et al. (copy here:
http://www-eng.lbl.gov/~shuman/NEXT/MATERIALS&COMPONENTS/Quartz/humidity_fatigue_fused_silica.pdf)
crack growth velocity as a function of relative humidity is investigated using various crack growth models.These models all agree well for extremely slow crack growth rates, so we choose model 1, as it is the simplest form.

$$\left(\frac{d}{dt} a \right) := A_1 \cdot \left(\frac{K_I}{K_{Ic}} \right)^{n_1}$$

From figs 3 and 4 of the above ref. we extrapolate (from 20% RH to 0%) to find the values:

$n_1 := 27$

$log\_A_1 := -2$ $A_1 := e^{log\_A_1} \frac{m}{s}$ $A_1 = 0.135 \frac{m}{s}$ (assumed natural log)

We set a desired maximum crack growth rate (velocity) of:

$v_c := 0.165\text{pm}\cdot\text{s}^{-1}$ which would give a time to failure: $t_f := \dfrac{a_{cr}}{v_c}$ $\quad t_f = 10.1\,\text{yr}$

We solve for K_I :

$$K_I := \left(\frac{v_c}{A_1}\right)^{\frac{1}{n_1}} \cdot K_{Ic} \qquad K_I = 2.389\times 10^5 \frac{\text{kg}}{\text{m}^{0.5}\,\text{s}^2} \qquad \frac{K_I}{K_{Ic}} = 0.362$$

Therefore the factor of safety for pressure testing is simply a function of the ratio:

$$\frac{\sigma_t}{\sigma_o} := \sqrt{\frac{a_o}{a_t}} \qquad \frac{\sigma_t}{\sigma_o} := \frac{K_{Ic}}{K_I}$$

and resulting design stress is: $\sigma_o := \sigma_t \cdot \dfrac{K_I}{K_{Ic}}$ $\quad \sigma_o = 18.6\,\text{MPa}$

for maximum applied operating pressure and safety factor:

$q := 16.4\text{bar}$ (MAWPa)

resulting minimum thickness is:

$$t_{min} := \sqrt{\frac{3}{8}\cdot(3+\nu)\cdot\frac{q}{\sigma_o}\cdot R_s^{\,2}} \qquad t_{min} = 12.3\,\text{mm}$$

specify 1/2" thick windows