ANGEL Torispheric Head Design, using (2010 ASME PV Code Section VIII, div. 2, part 4 rules)

DRAFT D. Shuman, LBNL, Apr 1, 2011

These design rules require selecting trial dimensions and iterating to find acceptable stresses. Shown here are final dimensions from repeated iterations.

Torispheric head with multiple ports for sapphire windows. First step determine the following:

I.D.

$D_i := 1.14m$

Crown radius (largest size possible without using part 5, design by analysis)

constraints

$L_{cr} := 1.14m$ $\quad 0.7 \le \frac{L_{cr}}{D_i} \le 1.0 = 1$ (true when conditional = 1)

Knuckle radius, here chosen as smallest size that doesn't start to significantly compromise strength

$r_{kn} := .17m$ $\quad \frac{r_{kn}}{D_i} \ge 0.06 = 1$

(c) Step 3 calculate:

thickness, this is an iterated value after going through part 4.5.10.1 (openings)and 4.5.13 (Spacing of openings) further down in the document

$t_{ts} := 8mm$ $\quad 20 \le \frac{L_{cr}}{t_{ts}} \le 2000 = 1$ initial (no nozzle reinforcement) value: 4.9mm

$\beta_{th} := \mathrm{acos}\left(\frac{0.5D_i - r_{kn}}{L_{cr} - r_{kn}}\right)$ $\quad \beta_{th} = 1.146\,rad$

$\phi_{th} := \frac{\sqrt{L_{cr} \cdot t_{ts}}}{r_{kn}}$ $\quad \phi_{th} = 0.562\,rad$

$R_{th} := \frac{0.5D_i - r_{kn}}{\cos(\beta_{th} - \phi_{th})} + r_{kn}$ $\quad R_{th} = 0.649\,m$ for $\phi_{th} < \beta_{th} = 1$

$R_{th} := 0.5D_i$ <--disabled calculation, because conditional-> $\phi_{th} \ge \beta_{th} = 0$ is false

$R_{th} = 0.649\,m$

(d) Step 4 compute:

$C_{1ts} := 9.31\left(\frac{r_{kn}}{D_i}\right) - 0.086$ for $\frac{r_{kn}}{D_i} \le 0.08 = 0$

$C_{1ts} := 0.692\left(\frac{r_{kn}}{D_i}\right) + 0.605$ $\quad C_{1ts} = 0.708$ for $\frac{r_{kn}}{D_i} \ge 0.08 = 1$

$C_{2ts} := 1.25$ for $\frac{r_{kn}}{D_i} \le 0.08 = 0$

$$C_{2ts} := 1.46 - 2.6\left(\frac{r_{kn}}{D_i}\right) \qquad C_{2ts} = 1.072 \qquad \text{for} \qquad \frac{r_{kn}}{D_i} > 0.08 = 1$$

(e) Step 5, internal pressure expected to cause elastic buckling at knuckle

$$P_{eth} := \frac{C_{1ts} \cdot E \cdot t_{ts}^{\ 2}}{C_{2ts} \cdot R_{th} \cdot (0.5R_{th} - r_{kn})} \qquad P_{eth} = 429\,\text{bar}$$

(f) Step 6, internal pressure expected to result in maximum stress (S_y) at knuckle

$$C_{3ts} := S_{y\_Ti\_g3} \qquad S_{y\_Ti\_g3} := 55000\text{psi} \qquad \text{time independent}$$

$$P_y := \frac{C_{3ts} \cdot t_{ts}}{C_{2ts} \cdot R_{th} \cdot \left(0.5\frac{R_{th}}{r_{kn}} - 1\right)} \qquad P_y = 47\,\text{bar}$$

(g) Step 7

$$G_{th} := \frac{P_{eth}}{P_y} \qquad G_{th} = 9.087$$

$$P_{ck} := \left(\frac{0.77508 \cdot G_{th} - 0.20354 \cdot G_{th}^{\ 2} + 0.019274 \cdot G_{th}^{\ 3}}{1 + 0.19014G_{th} - 0.089534G_{th}^{\ 2} + 0.0093965G_{th}^{\ 3}}\right) \cdot P_y \qquad P_{ck} = 93\,\text{bar}$$

(h) Step 8

$$P_{ak} := \frac{P_{ck}}{1.5} \qquad P_{ak} = 62.01\,\text{bar}$$

(i) Step 9

$$P_{ac} := \frac{2S_{max\_Ti\_g3\_div2} \cdot 1}{\frac{L_{cr}}{t_{ts}} + 0.5} \qquad P_{ac} = 26.3\,\text{bar}$$

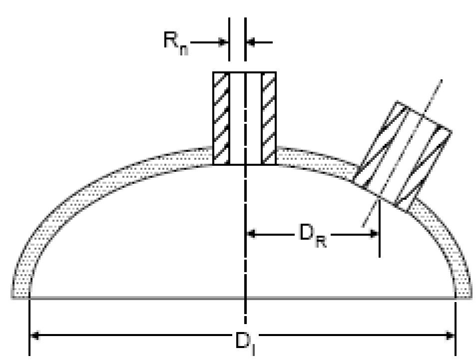
(j) Step 10

$$P_a := \min(P_{ak}, P_{ac}) \qquad P_a = 26.3\,\text{bar}$$ OK, min thickness is 4.9mm (no openings)

4.5.10.1 Radial Nozzle in formed head

We need to accommodate 76mm dia PMT's, so let

$$R_n := 3.8\text{cm}$$

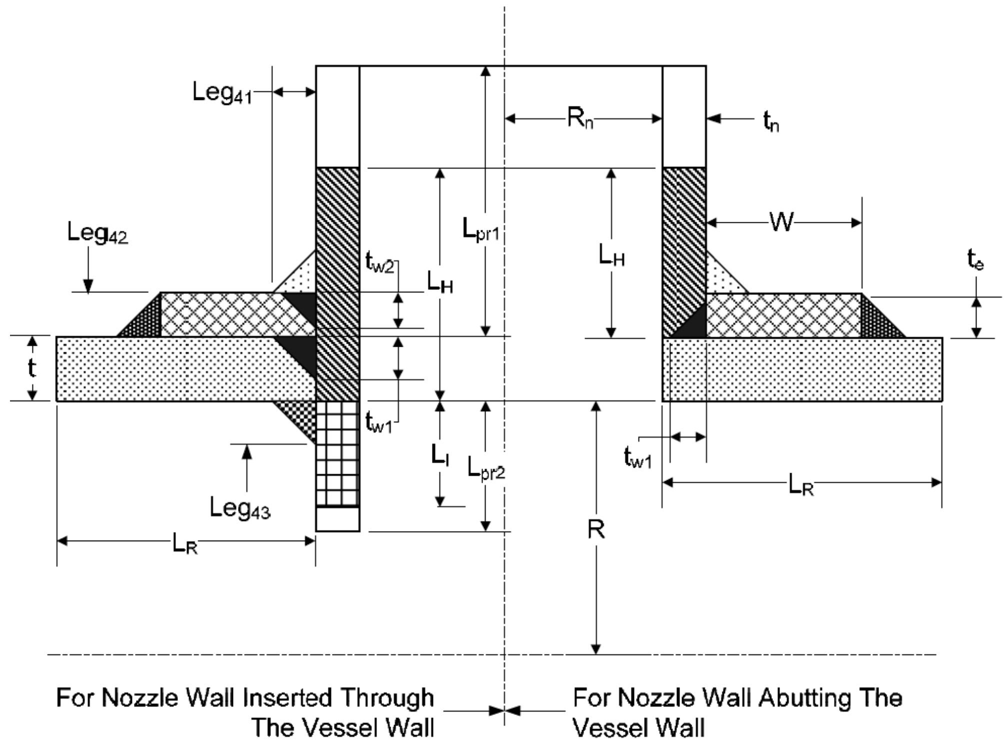


Considering nozzle as separate from head, to obtain a trial thickness

$$t_{n\_int\_min} := \frac{MAWP_{pv} \cdot (R_n)}{S_{max\_Ti\_g3} \cdot E_w - 0.6 \cdot MAWP_{pv}} \qquad t_{n\_int\_min} = 0.466\,mm$$

However, the nozzle not only must withstand internal pressure, but must also help to carry head stresses across the openings made for them.

4.5.10.1 Procedure for Radial Nozzle in a Spherical or Formed head



a) Step 1

$R_{eff} := L_{cr} \qquad R_{eff} = 1.14\,m$ (4.5.64)

b) Step 2- limit of reinforcement along vessel wall, assume integrally reinforced nozzles

We must first determine the various D_i that will be present (see fig x above). We can use "vectors" in mathCAD to perform parallel calculations.

$$L_{R1} := 0.5 \cdot D_i - (D_R + R_n + t_n) \qquad L_{R1} = \begin{pmatrix} 43.9 \\ 34.9 \\ 25.9 \\ 16.9 \end{pmatrix} cm \qquad D_R := \begin{pmatrix} 9 \\ 18 \\ 27 \\ 36 \end{pmatrix} cm$$ (4.5.67)

$\sqrt{R_{eff} \cdot t_{ts}} = 0.095\,m \qquad 2R_n = 0.076\,m$

$L_{R2} := \min\left(\sqrt{R_{eff} \cdot t_{ts}}, 2R_n\right) \qquad L_{R2} = 7.6\,cm$ (4.5.67)

$$L_R := \min(L_{R1}, L, R2) \qquad L_R := L_{R2}$$ (4.5.68)

Nozzle to nozzle minimum spacing, initial, with unmodified limits of reinforcement

$$d_{n2n\_no\_mod} := 2(L_R + R_n) \qquad d_{n2n\_no\_mod} = 22.8\,cm$$

This spacing is greater than desired, part 4.5.13.1 allows for modification of this limit of reinforcement, in order to allow closer nozzle spacing. We momentarily divert to part 4.5.13.1 to recalculate L_R then return to part 4.5.10.1 which is treated immediately below.

4.5.13.1 Spacing Requirements for Nozzles

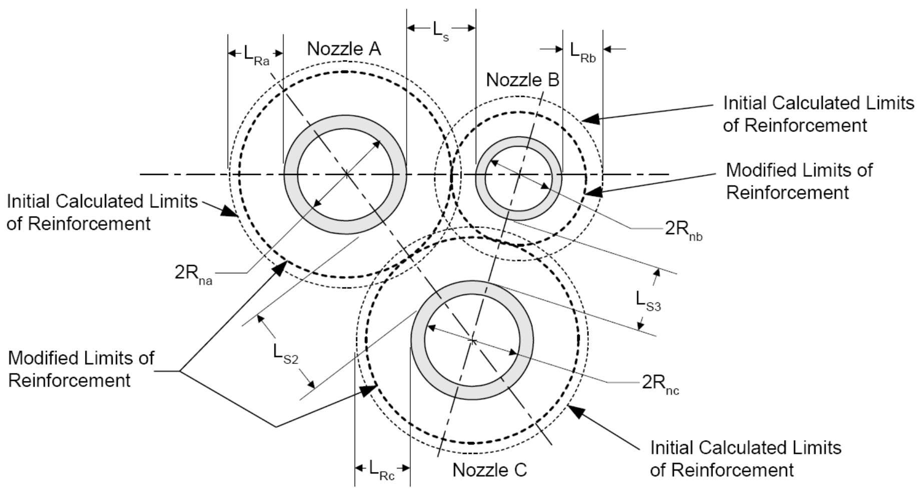


Figure 4.5.12 – Example of Three Adjacent Nozzle Openings

let: $R_{na} := R_n \qquad R_{nb} := R_n \qquad R_{nc} := R_n$

and then let:

$$L_{S1} := L_R \quad L_{S2} := L_R \quad L_{S3} := L_R \quad L_R = 7.6\,cm$$

this seems like the maximum overlap allowable

$$L_R := \min\left[L_{S1}\cdot\left(\frac{R_{na}}{R_{na}+R_{nb}}\right), L_{S2}\cdot\left(\frac{R_{na}}{R_{na}+R_{nC}}\right)\right]$$

for nozzle A, etc. (4.5.129)

Since all nozzles are identical, the modified limit of reinforcement is:

$$L_R := 0.5L_{S1} \qquad L_R = 3.8\,cm$$

Spacing is then:

$$d_{n2n} := 2(L_R + R_n) \qquad d_{n2n} = 15.2\,cm$$

Returning to part 4.5.10.1 with revised L_R:

c) Step 3- limit of reinforcement along nozzle wall projecting outside vessel surface wall.
We have no pad reinforcement, and nozzle projects only outward so:

$t_e := 0mm \quad L_{pr1} := 13cm \quad L_{pr2} := 0cm$

$$L_H := \min\left[\left(t_{ts} + t_e + F_p \cdot \sqrt{R_n \cdot t_n}\right), L_{pr1} + t_{ts}\right] \qquad (4.5.73)$$

where:

$$X_o := D_R + R_n + t_n \qquad X_o = \begin{pmatrix} 0.131 \\ 0.221 \\ 0.311 \\ 0.401 \end{pmatrix} m \qquad (4.5.79)$$

$$C_p := e^{\frac{0.35D_i - X_o}{8t_{ts}}} \qquad C_p = \begin{pmatrix} 65.858 \\ 16.139 \\ 3.955 \\ 0.969 \end{pmatrix} \qquad (4.5.78)$$

$$C_n := \min\left[\left(\frac{t_{ts} + t_e}{t_n}\right)^{0.35}, 1.0\right] \qquad C_n = 1 \qquad (4.5.81)$$

$$F_p := \min(C_n, C_p) \qquad F_p = 0.969 \qquad (4.5.80)$$

$$L_H := \min\left[\left(t_{ts} + t_e + F_p \cdot \sqrt{R_n \cdot t_n}\right), L_{pr1} + t_{ts}\right] \qquad L_H = 1.835\,cm \qquad (4.5.73)$$

d) Step 4- limit of reinforcement along nozzle wall projecting inside vessel surface wall, if applicable

$$L_I := \min\left(F_p \cdot \sqrt{R_n \cdot t_n}, L_{pr2}\right) \qquad L_I = 0\,cm \qquad (4.5.82)$$

e) Step 5- determine total available area near nozzle opening

(material strength ratios)--> $f_{rn} := 1 \qquad f_{rp} := 1$ (4.5.30) (4.5.31)

$$A_T := A_1 + f_{rn}(A_2 + A_3) + A_{41} + A_{42} + A_{43} + f_{rp} \cdot A_5 \qquad (4.5.83)$$

$$A_1 := t_{ts} \cdot L_R \qquad A_1 = 3.04\,cm^2 \qquad (4.5.84)$$

$$A_2 := t_n \cdot L_H \qquad A_2 = 0.55\,cm^2 \qquad (4.5.86)$$

$$A_3 := t_n \cdot L_I \qquad A_3 = 0\,cm^2 \qquad (4.5.83)$$

$$L_{41} := 0cm$$

$$A_{41} := 0.5L_{41}^2 \qquad A_{41} = 0\,cm^2 \qquad (4.5.88)$$

$$L_{42} := 0cm$$

$$A_{42} := 0.5L_{42}^2 \qquad A_{42} = 0\,cm^2 \qquad (4.5.89)$$

$$L_{43} := 0cm$$

$$A_{43} := 0.5L_{43}^2 \qquad A_{43} = 0\,cm^2 \qquad (4.5.90)$$

$$t_e = 0\,cm$$

$$A_5 := 0cm^2 \qquad (4.5.94)$$

$$A_T := A_1 + f_{rn} \cdot (A_2 + A_3) + A_{41} + A_{42} + A_{43} + f_{rp} \cdot A_5 \qquad A_T = 3.59\,cm^2 \qquad (4.5.83)$$

f) Step 6 determine applicable forces

$$t_{eff} := t_{ts} \cdot \left(\frac{t_{ts} \cdot L_R + A_5 \cdot f_{rp}}{t_{ts} \cdot L_R} \right) \qquad t_{eff} = 8\,mm \qquad (4.5.100)$$

$$R_{xn} := \frac{t_n}{\ln\left(\frac{R_n + t_n}{R_n} \right)} \quad R_{xn} = 3.948\,cm \quad R_{xs} := \frac{t_{eff}}{\ln\left(\frac{R_{eff} + t_{eff}}{R_{eff}} \right)} \quad R_{xs} = 1.144\,m \qquad (4.5.98)$$

$$f_N := P \cdot R_{xn} \cdot (L_H - t_{ts}) \qquad f_N = 637.714\,N \qquad (4.5.95)$$

$$f_S := \frac{P \cdot R_{xs} \cdot (L_R + t_n)}{2} \qquad f_S = 3.66 \times 10^4\,N \qquad (4.5.96)$$

$R_{nc} := R_n$ (radius along chord= R_n for radial nozzles)

$$f_T := \frac{P \cdot R_{xs} \cdot R_{nc}}{2} \qquad f_T = 3.393 \times 10^4\,N \qquad (4.5.97)$$

g) Step 7 determine effective thickness for nozzles in spherical, ellipsoidal, or torispherical heads

$t_{eff} = 8 \times 10^{-3}\,m$ same formula as above in step 6 (4.5.100)

h) Step 8 Determine avg. local primary membrane stress and general primary membrane stress at nozzle intersection

$$\sigma_{avg} := \frac{f_N + f_S + f_T}{A_T} \qquad \sigma_{avg} = 198.2\,MPa \qquad (4.5.101)$$

$$\sigma_{circ} := \frac{P \cdot R_{xs}}{2t_{eff}} \qquad \sigma_{circ} = 111.6\,MPa \qquad (4.5.102)$$

I) Step 9 Determine maximum local primary membrane stress

$$P_L := \max\left[(2\sigma_{avg} - \sigma_{circ}), \sigma_{circ} \right] \qquad P_L = 284.8\,MPa \qquad (4.5.103)$$

$S_{max\_Ti\_g3\_div2} = 190.3\,MPa$ $\qquad E_w = 1$

$$S_{allow} := 1.5 S_{max\_Ti\_g3\_div2} \cdot E_w \qquad S_{allow} = 285.4\,MPa \qquad (4.5.43)$$

j) Step 10 Maximum local primary membrane stress must be less than the allowable stress

$$P_L \le S_{allow} = 1 \qquad (4.5.104)$$

k) Determine max allowable working pressure of the nozzle

$$A_p := R_{xn} \cdot (L_H - t_{ts}) + \frac{R_{xs} \cdot (L_R + t_n + R_{nc})}{2} \qquad A_p = 456\,cm^2 \qquad (4.5.108)$$

$$P_{max1} := \frac{S_{allow}}{\left(\frac{2A_p}{A_T} - \frac{R_{xs}}{2t_{eff}} \right)} \qquad P_{max1} = 15.4\,bar \qquad (4.5.105)$$

$$S := S_{max\_Ti\_g3\_div2} \qquad S = 190.3\,MPa$$

$$P_{max2} := 2 \cdot S \cdot \left(\frac{t_{ts}}{R_{xs}}\right) \qquad P_{max2} = 26.3\,bar \tag{4.5.106}$$

$$P_{max} := \min\left(P_{max1}, P_{max2}\right) \qquad P_{max} = 15.4\,bar \tag{4.5.107}$$

Conclusion: A nozzle wall thickness of 3mm is sufficient to both withstand maximum operating pressure and reinforce its own opening in the torispheric head of 8mm thickness, given an e-beam full penetration fusion weld (no added fillet). A diffusion weld could also be used, allowing a pressure safe conical weld and allowing all nozzles to be welded in one oven cycle. The nozzles could be machined with integral fillets, and pads, allowing further reduction in head thickness, possibly down to a minimum of 5mm.