

## *Risk and Reward: Optimizing NEXT*

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### **Introduction**

In this note, we offer a personal perspective for the choices NEXT collaboration should face soon. Our motivations include both urgency and due diligence. The primary driver for this note is the fact that no radio-pure pressure-tolerant PMTs exist. We don't claim to offer an unbiased, well-pruned decision tree, but intend to clarify where some fruitful branches may lie just out of sight. A strongly preferred option emerges. Based on that option, we offer a design study suggesting how a detector would look. We hope that this note and our preferred concept will merit serious attention.

We maintain that any design scenario under consideration should realize a detector with the greatest scientific reach, preserving NEXT's unique combination of excellent energy resolution and event topology by tracking. We explicitly try to avoid any (known) or unnecessary compromise to these capabilities. Our attractiveness for new funding and additional collaborators depends on preserving both of these two capabilities. In addition, allowing needless performance compromises will sap enthusiasm within the collaboration.

Although tracking is equally essential, we concentrate here on ensuring near-optimal energy resolution. The  $^{214}\text{Bi}$   $\gamma$ -ray at 2448 keV is close to the Q-value – within 10 keV – and is clearly very dangerous. Beyond topology tests, can we also manage to achieve some useful separation in energy? We believe this possibility deserves serious consideration.

For energy resolution, an important factor is the statistical contribution “G”, determined by  $n_{\text{pe}}$  the required number of detected photoelectrons per primary electron [1, 2]]. At present, we have set the G factor equal to the Fano factor F. This was a convenient choice, but an arbitrary one. There is no compelling basis for F and G to make equal contributions to the resolution. While F is fixed by atomic physics (unless we exploit a Penning mixture), G is a variable under our control. We may want to make G smaller than F.

For  $F = 0.15 = G$ , the number  $n_{\text{pe}}$  is around 10. We believe that it is prudent to design for a somewhat larger number of  $n_{\text{pe}}$ , perhaps  $n_{\text{pe}} = 30$ . This provides a means to recognize subtle systematic errors with improved precision to study them. But most importantly, a larger  $n_{\text{pe}}$  provides opportunity for improved energy resolution. To approach this goal, we can try to increase the detection efficiency for 175 nm VUV, or increase the total amount of EL produced, or some combination of both. Dynamic range requirements for the PMT + electronics will be larger since we need to see single pe spectra clearly.

The PMTs used in Xenon100 and LUX are both excellent from the standpoint of radioactivity. The limit for the Hamamatsu 1” square Xenon100 PMT is allegedly five bars. We don't have information about the 2” round Hamamatsu PMTs used in LUX,

but almost surely, they will also be quite limited in their capability to withstand pressure much above 1 bar. The Hamamatsu R7378 PMTs used in the 7-PMT TAMU system appear to withstand 20 bars but the actual failure pressure is unknown. These PMTs are perhaps 100 times more radioactive than the Xenon100 PMTs. In the absence of a scheme to place them well behind thick shielding, the R7378A cannot be considered.

### Premises

To begin our discussion, we put forth some premises and supporting comments:

1. NEXT-100 should be capable of supporting a “near-ideal” xenon density  $\rho$ , likely to fall in the range of 10 – 20 bars ( $0.05 < \rho < 0.1 \text{ g/cm}^3$ ). This is because:
  - a. “Near-ideal” density keeps the detector manageably small.
    - i. This minimizes surface area and mass of detector structures, reducing radioactive burden.
    - ii. For a given number of PMTs, the detection efficiency for primary and secondary light is increases if the detector is smaller.
    - iii. Design issues are less complex for ideal density.
    - iv. Any external shielding scenario is less costly at the ideal density.
    - v. A water tank may become difficult to impossible at 5 bars.
    - vi. A vertical orientation may become less feasible at 5 bars.
  - b. Pressure vessel thickness is nearly independent of density, so it helps to keep the system small, as less total vessel material is required.
  - c. Too high a density compromises the tracking function, as the event size becomes too compact relative to diffusion, which scales as  $\rho^{-1/2}$ .
  - d. Simulations probably show a gentle optimum with density for topological background rejection.
2. NEXT-100 should be the final, uncompromised design. This is because:
  - a. That is what we have proposed to do, and been funded to build.
  - b. Even though we have not proven everything we would like, nor have we gained experience in all matters, there is enough knowledge to make a final detector design for NEXT-100.
  - c. Some reasonable risks must be taken to achieve a timely execution of an effective design.
  - d. The expenditure of effort and money to reduce risks to a low level will require time and resources we do not have.
  - e. We may not get everything exactly right, but we must avoid decisions that *a priori* would prevent ultimate success.
  - f. One of the risks we face is that we display inadequate ambition. There is real competition from EXO and elsewhere.
  - g. To anticipate success only in an undefined, subsequent stage II amplifies an avoidable and severe risk: of being too late.
3. NEXT-100 should maintain a primary goal of realizing energy resolution at the Q-value as close as possible to 0.5 % FWHM. This is because:
  - a. The  $^{214}\text{Bi}$   $\gamma$ -ray is very dangerous, known to be only  $\sim 10$  keV away from the Q-value.
  - b. The overlap of two gaussian peaks becomes rapidly smaller when  $\Delta E$  exceeds  $2\sigma$ .

- c. This goal requires more than 10 photoelectrons per primary electron, perhaps as many as 30 – 50, reducing significantly the quantum statistical contribution to energy resolution.
  - d. The option to reduce the Fano factor by Penning effect, requiring a molecular admixture should not be ruled out *a priori*;
  - e. We can learn, and will certainly have to learn, how to make geometric corrections for light collection no matter what design is chosen. With good tracking, these effects may not limit energy resolution.
  - f. If topology criteria show peak separation, one from  $\beta\beta$  0- $\nu$  decay and the other from  $^{214}\text{Bi}$   $\gamma$ -rays, the evidence is much stronger than if there is essentially no energy difference within resolution.
4. NEXT-100 should incorporate a double vessel design. This is because:
- a. Without this feature, the fraction of xenon in the active volume will be much smaller than desired.
  - b. Balancing pressure between the double volumes is a problem that can be managed safely by good engineering design and practice.
  - c. A pressure differential excursion will at most lead to mixing of xenon with a buffer gas (*e.g.*,  $\text{CO}_2$ ) that can be easily separated from xenon.
  - d. If we design the double volume in to begin with, we can test with argon or nitrogen before exposing xenon to the system.
  - e. We should engage experts in this subject, and be prepared to pay for their advice.
  - f. Signal feed-throughs between xenon and buffer gas do not need to support a pressure differential.
5. Digitization of the SiPM signals within the pressurized gas volume should be considered.
- a. The slow signal development permits digitization at 1 MHz or less.
  - b. A 1 MHz or less digitization rate permits a high level of multiplexing, in the range of 64.
  - c. The ADC/multiplexer cards can be small.
  - d. The radioactivity of electronics can be shielded in the same way that the PMTs can, as proposed below.
  - e. Readout by  $\sim 100$  optical fiber penetrators is easily feasible.
  - f. Modern electronics can be made sufficiently reliable that maintenance will not be a major concern.<sup>1</sup>

We turn now to issues of PMTs and operation under pressure:

6. We have to use either the 1" square or the 2" round radio-pure Hamamatsu PMTs. This is because:
- a. Within the collaboration, we have experience with both PMTs (Santorelli – Xenon100, White – LUX).

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<sup>1</sup> IceCube, by paying close attention to quality controls for part selection, handling, manufacturing, and test procedures, realized >99% operational success for vastly more complex electronic systems, permanently deployed in deep south pole ice. NEXT should be able to achieve  $\sim 100\%$  reliability for simple ADC/multiplexers.

- b. No other good options with such low radioactivity appear to exist.
  - c. These PMTs have very good performance.
  - d. The only limitation of the 1” square PMT is the five bar pressure limit.
  - e. The pressure limit for the 2” PMT is probably lower than 5 bars.
  - f. “Hardening” the 2” PMT against pressure is probably more difficult due to its larger size, but might still be a better choice; this needs study.
  - g. But we don’t have any guarantee from Hamamatsu about pressure.
7. We must establish what safety factor is needed against PMT failure in operation.
- a. This is an essential and immediate **engineering requirement**.
  - b. We want to avoid a Super-K style chain-reaction disaster.
  - c. It is not easy to establish what the safety factor really is.
  - d. PMTs may not stand repeated five bar pressure cycling.
  - e. A single individual PMT failure strongly affects energy calibrations.
8. We should avoid a path that requires us to buy, evaluate, modify, and test a large number of PMTs. This is because:
- a. The failure distributions may be non-gaussian.
  - b. It will be hard to establish long-term behaviors with pressure cycles.
9. We will really have to make a choice soon among the potential PMT solutions.
- a. We assert that a performance compromise is avoidable.
  - b. We assert, and argue below, that reasonable choices can be made, without embarking on a crushing R&D program that would exhaust energy, time, enthusiasm, and resources.

### **PMTs and pressure**

Hamamatsu alleges that the Xenon100 style PMTs tolerate a pressure of five bars but not more. To our knowledge, we have no clear declaration or guarantee from Hamamatsu what pressure an ensemble of these PMTs will tolerate, including through repeated pressure cycles. How should we respond to this situation? With this PMT, there are two basic pathways:

1. Do something to make the PMT itself mechanically robust.
2. Protect the PMT from the high pressure by some enclosure.

For either path, it is essential for us to ask and find good engineering answers to questions such as: “What safety factor should be required against failure under pressure?” “How do we determine that the safety factor exists?” “Will this design prevent a Super-K type of chain-reaction catastrophe?” “If we choose either a mechanical reinforcement of individual PMTs, or protection by enclosure, does the solution offer credible long-term safety against pressure cycling failures?”

### **PMT hardening**

Path 1 involves evaluation of weaknesses, reinforcement scenarios and subsequent breaking some number  $N$  of PMTs. The distribution of buckling/fracture/seal failure pressures might show a clear answer, but the number  $N$  may have to unpleasantly large to determine the distribution width and tails. Instead of obvious metal buckling or glass/quartz cracking, a failure of vacuum seal may appear first as the pressure is increased, so tests would have to be made with the PMT in operation, or the photocathode made visible to show the nearly instantaneous change in color with even a slight vacuum loss to air. To our knowledge, it is not known whether the initial PMT failure mode is predominantly seal failure, metal buckling, or glass/quartz fracture, or a

mix of these. To find the answers certainly seems possible, but possibly also very expensive and frustrating as a definitive result may require  $N \sim 40 - 100$ .

A program of hardening the PMT itself seems to require testing to failure of some sample of otherwise functional PMTs. In addition, some reverse engineering seems necessary to see if it is even worth doing large sample testing. One could cut some PMT's open (both live and pressure killed), study the seals, etc., make measurements and comparisons (live to dead), model the PMT in FEA, and calculate stresses. If it then appears feasible, one can try some strengthening schemes in Finite Element Analysis (FEA). Finally, the preferred approach can be tested on a few new PMT's (operating the PMT itself while pressure ramping). The degree of initial success here would determine the sample size number  $N$  of PMT's for further qualification. This could possibly be a good project for a talented engineering grad student with good hands-on skills, given sufficient oversight. Although it seems likely that a fairly simple solution might be found eventually – like just gluing stiffeners on the flat sides – this effort could take, in addition to substantial resources, a long time to complete with adequate confidence. We are quite wary of following this path.

A related alternative path 1 “hybrid” solution might be encapsulation. Here, the entire PMT + base is cast into an adequately thick, optically transparent, solid plastic shell. A cured material such as an epoxy or polycarbonate seems likely to be more robust than a thermoplastic cast. The shell casting would encapsulate solid copper wires to admit HV and return signals. Beyond mechanical strength, a concern here is long-term stability against swelling due to absorption of high-pressure xenon (HPXe). Swelling may either increase pressure directly on the PMT or break the PMT-plastic interface, allowing unacceptable and perhaps sudden pressure increases in the interstitial space over time. Some plastics, however, may not swell when exposed to HPXe. So while this approach might be a good solution, expertise is needed to save time. We do not know who or where to locate the needed expertise on encapsulation techniques.

### **Enclosure for PMT alone**

For path 2, a small housing is needed that:

1. Encloses the PMT and base;
2. Admits light very efficiently to the photocathode;
3. Allows electrical signals in/out;
4. Is not radioactive;
5. Provides high reliability against accumulation of an unnoticed high pressure.

Point 5 requires a very reliable seal, or permanent venting with pump-out capability involving plumbing. Dave noted this avenue as a genuine possibility during his presentation at the January 2010 CM in Valencia, but deemed it “ugly”. That it certainly is, and would surely require a fair engineering effort. But it may be a reasonable path.

### **Branch point?**

However, given that PMT hardening, encapsulation, and enclosure approaches all involve a fair amount of work just to solve a nuisance, it seems to us that the situation at this point deserves some out-of-the box thinking. What we offer next is an attempt

to convert the dilemma (more accurately, the crisis) of the mechanically weak PMT into an opportunity for both safety at pressure **and** improved performance.

**Enclosure for PMT + Wavelength Shifting Bar**

We assert, without real proof at this point, that the multiple challenges enumerated above can be met with an electroluminescent (EL) TPC that employs a circumferential array of scintillator or wavelength-shifter bars (WLS-bar) instead of an array of end-cap PMTs. The bars must act as efficient wavelength-shifters converting both primary xenon scintillation (for  $t_0$ ) and the intense secondary scintillation from EL (to provide energy and tracking measurements). The very large area of the bars compensate for efficiency losses in WLS. A substantial overall increase in detected light relative to the simple end-cap array of PMTs can be realized. Figure 1 indicates the general idea. Figure 2 shows a symmetric design in perspective. While both symmetric and asymmetric TPC designs seem possible, it seems much better to implement an asymmetric design for NEXT-100. And as elaborated below, it seems likely to us that the PMTs and WLS bars will be enclosed in synthetic quartz tubing (QT).

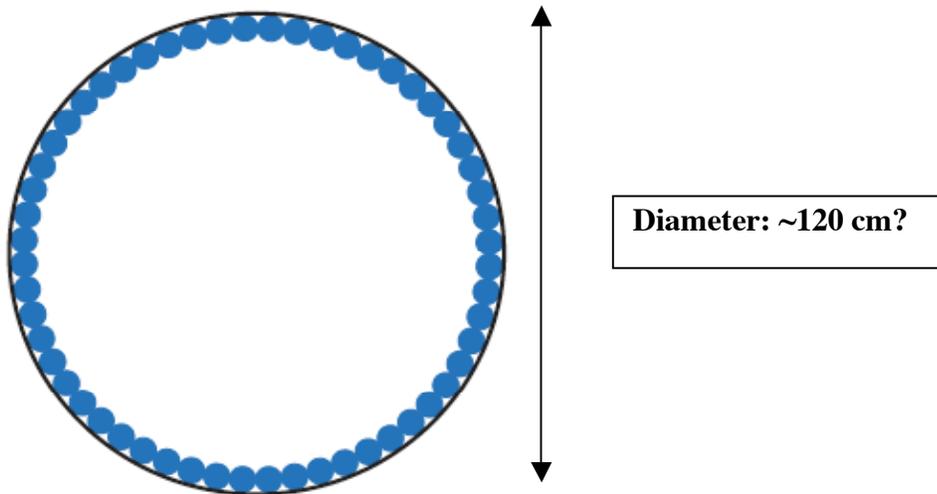


Figure 1. End view of a cylindrical array of circular WLS bars that surround the TPC circumference. These capture both primary and secondary scintillation. PMTs are located at one, or perhaps both, ends of the WLS bars. The primary or coarse TPC field cage is outside the array. Tiny additional electrodes to stabilize electric field at the outer radius of the gas volume are needed, but the optical impact of these can be small. These electrodes could be wire rings wrapped periodically around each QT.

In this scenario, the 175 nm VUV from xenon strikes the WLS bars and is converted from 175 nm VUV to the ~410 nm wavelength optimal range for modern PMTs. The

7.5 eV of xenon VUV is more energetic than optimal for typical plastic scintillator.<sup>2</sup> A rough estimate for the conversion efficiency might be around 60%, but it might be higher or lower, depending on implementation. In any case, WLS conversion occurs anywhere 175 nm strikes the WLS bar. A fairly large fraction of the converted light is trapped internally, perhaps ~60%, and can be reflected at the end back toward the PMT. If scintillator material is used, the bars will also detect ionizing radiation, therefore providing some  $\gamma$ -catcher capability. But the essential function is efficient WLS, rather than  $\gamma$ -catcher.

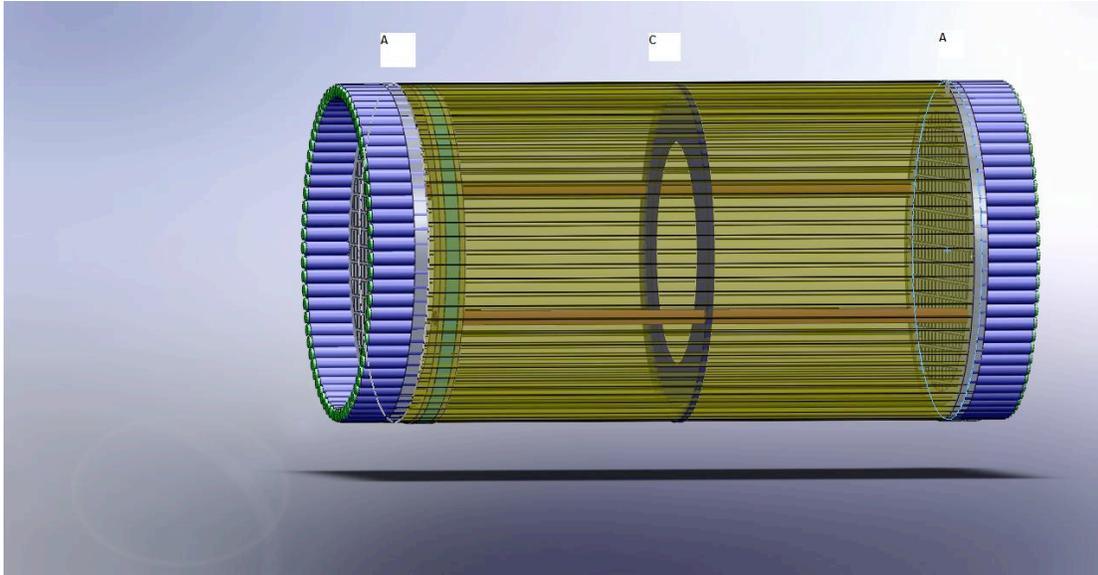


Figure 2. An isometric view of a symmetric TPC is shown with WLS bars and PMTs at both ends (figure by IFIC). An asymmetric version with PMTs only at one end is also feasible, and is preferable for NEXT-100. Not emphasized in this sketch is the possibility to extend the bars well beyond the TPC active volume. That would offer opportunity to implement close-fitting copper shield structures, further reducing the radioactive impact of PMTs.

In the cathode end-cap, there are no PMTs staring into the active volume. Tracking is to be done as presently conceived, with SiPMs at the anode plane. The approximate bar dimensions are diameter ~4.5 cm, length ~150 cm.

A large fraction of the trapped light, ~60%, independent of origin or direction will reach the PMT photocathode. Although the product of these factors is small,  $\sim 0.2 \pm 0.07$ , the overall photon detection efficiency of the WLS bar can be very much larger than a naked PMT. The sensitive area of a WLS bar is roughly  $L \times W$ , which in our case might be 150 cm x 4 cm,  $\sim 600 \text{ cm}^2$ . Adding the conversion loss factor 0.2 for the WLS,

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<sup>2</sup> The WLS process, as suggested below, might also be a two-step scenario with the inner surface of quartz tubing coated with p-terphenyl (TPH), which shifts 175 nm efficiently to  $\sim 330 \text{ nm}$  [3, 4]. This longer wavelength is much more efficiently absorbed and shifted to blue by conventional plastic scintillator.

the effective area is  $\sim 120 \text{ cm}^2$ . This can be compared to the sensitive area of the 1" square PMT:  $\sim 6 \text{ cm}^2$ . The relative sensitivity advantage of a WLS bar is a factor of  $\sim 20$ .

But the cylindrical WLS array has a second related advantage: the number of PMTs needed is an order of magnitude smaller than the "conventional" HPXe EL TPC design concept that employs an array of PMTs in the end-cap plane to detect primary and secondary scintillation, as were evaluated in [1, 2]. If more-or-less uniform coverage over the end-cap is required, then the number of PMTs needed roughly follows an  $r^2$  or  $(M/\rho)^{2/3}$  dependence, where  $M$  is the xenon mass. For the WLS bar array with uniform circumferential covering, the dependence is  $(M/\rho)^{1/3}$ , a much more favorable scaling relationship for large  $M$ . For example, a 120 cm diameter TPC with 1" square PMTs requires about 900 PMTs (assuming a fill factor of 50%). Conversely, the WLS bar solution requires only  $\sim 80$  PMTs. This factor of  $\sim 11$  is a real and huge benefit, reducing system complexity and radioactivity.

But the cylindrical WLS array has a third advantage: the PMTs can be located in a position "behind" the end-cap that introduces significant distance from the active volume and permits partial shielding of the active volume from the PMTs. The reduction in exposure of radioactivity from PMTs to the active TPC volume depends on the specific shielding material, geometry, and shielding thickness. If copper, the attenuation length for a 2500 keV  $\gamma$ -ray is about 3 cm. Shielding thickness of 10 -15 cm seems possible. This corresponds to a factor of maybe  $\sim 50$  over most of the volume. This may be an important advantage, as a major source of radioactivity has been estimated to be the PMT array, even with excellent Hamamatsu Xenon100-style PMTs.

But the cylindrical WLS array has a fourth advantage: both the PMTs and WLS bars can be placed inside cylindrical synthetic quartz tubes (QT). Synthetic quartz has low radioactivity, and is very robust under compression. Large safety factors can be realized with modest wall thickness, as described below. Synthetic quartz offers good transmission at the 175 nm UV wavelength and excellent surface polish. Such tubing is available in various diameters and wall thicknesses of interest from Heraeus (Suprasil), and hopefully other vendors. This offers flexibility, and perhaps a vital separation of the WLS from xenon. Plastic WLS fibers have been observed to display substantial swelling in high-pressure xenon gas [5].<sup>3</sup> The solution in [5] was to seal the 1 mm diameter fibers in tiny mm-scale QT. This solution should work with large  $\sim 40$  mm diameter QT as well.

While it could be argued that the endcap-PMT scenario takes advantage of reflectivity in the barrel, it could also be argued that the WLS-PMT scenario can take advantage of end-cap reflectivity. The WLS appears to offer a substantial advantage for optical efficiency, even if these numbers are optimistic by factor of as much as  $\sim 5$ .

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<sup>3</sup> Conversely, out-gassing from naked WLS bars of electronegative components, including oxygen and water, may conceivably occur. This suggests that xenon should also be protected from direct exposure to the WLS bars.

### **Design advantages for a cylindrical array of WLS-bars**

1. An order of magnitude more light is detected per PMT.
2. Only  $\sim 80$  WLS/PMTs are needed to cover the circumference.
3. PMTs, bases, and WLS are inside sealed QT. This protects the PMTs from pressure and protects WLS and xenon from mutual exposure.
4. The PMTs can be located beyond the active volume in a way that permits good geometric shielding from PMT+base radioactivity.

### **Design issues for a cylindrical array of WLS-bars**

1. Optical performance – is there really such a large improvement?
2. Physical robustness of tube assembly against catastrophic failure
3. High confidence level for system integrity against slow pressure rise
4. Manufacturing, testing, cost, installation, *etc.*...
5. Long-term stability – WLS, chemical, and optical stability, *etc.*...

The main challenge is to find a design that provides a very high level of confidence that the pressure cannot rise unacceptably high in the space holding the PMT. Obtaining confidence that any design protects against pressure (with or without slow leaks) will take some time. Can we choose, with confidence, a good solution in the near future?

### **Optical performance - radial source dependence**

The intensity of light hitting the cylindrical surface will surely display some radial dependence on luminous source position (both primary and secondary scintillation). Bars closer to the luminous source will see more light, and bars farther away will see less. It seems likely that taking the sum (around  $2\pi$  in azimuth) will reduce the radial dependence substantially. The dependence of the total detected light with source radius for a cylindrical WLS array depends on several factors, such as the contribution of EL light that is allowed to emerge radially (between the meshes) and strike the WLS bars directly, and what fraction of detected light comes from reflection at the cathode. Azriel has made an initial calculation of that contribution arriving radially between the meshes. The results show relatively gentle effects up to  $\sim 70\%$  of maximum radius, when the effects become rapidly larger with increasing radius (not a surprise). Combined with the contribution from the rest of the detector, it could turn out that the total is nearly independent, or could be tuned to be by design.

We have already seen from simulations for the conventional scenario that finite field-cage reflectivity leads to a non-negligible radial dependence for light striking the end-cap. Those results depend strongly on assumptions for specular and diffuse reflectivity, which are poorly known. Although the calculation is complex in either case, simulation of a cylindrical WLS bar geometry probably depends less on complex reflectivity. We suspect that in either case, we will learn to make calibrations.

### **Optical performance – continuity at the quartz-WLS-PMT interfaces**

A near-perfect optical coupling between the quartz and the WLS material is desirable, but perhaps not essential. If a gap exists between quartz and WLS, some light will not reach the WLS due to internal reflection at the QT inner surface. Increased absorption of light trapped in QT will reduce any contribution from multiple internal bounces.

Such an interface loss is likely to be around 20%. This loss may be acceptable, given the large apparent gains claimed above. This can be modeled and tested, and could drive the design choices. But some notions either do not impose a gap or exploit this gap to advantage. We return to this issue below.

The optical coupling between PMT and WLS material is also important. The WLS sensitive volume, which is circular, needs to be optically connected to the PMT photocathode, which is square.<sup>4</sup> An optical transition element to couple these two surfaces adiabatically cannot be realized, since the area of the PMT photocathode is much smaller than the WLS cross-section. We suggest below at least one notion to deal with this.

In the consideration of various notions, it seems useful to define two categories, those with only external electrical connections (sealed assemblies), and those with external plumbing plus electrical connections. We start with the first category – sealed assemblies with only electrical connections to the external world.

### **Design concepts with no external plumbing – sealed assemblies**

#### *1. Plastic scintillator casting:*

Plastic scintillator might be cast directly inside QT, enveloping the PMT. It is appealing to have an entirely solid assembly, but plastic scintillator shrinks as it cures. If the plastic does not separate from the inner wall of the QT during cure, persistent stress and temperature cycles may ultimately lead it to do so. A significant difference exists in the thermal expansion coefficients between quartz and plastic. Exposure to large excursions in ambient temperature might lead to separation at the optical interface. Complex stresses may be induced under xenon pressure where the transition begins from circular plastic scintillator to square PMT, compromising the safety factor. Finally, evolution of gas may tend to break the quartz/WLS optical joint and increase internal pressure within the assembly. Nevertheless, it may be that casting can be made to work, and experts may have real answers for all these concerns.

#### *2. “Jellied plastic scintillator”:*

For example, we are led to speculate that it might be possible to “partially cure” the scintillator so that it remains very plastic: perhaps even a super-viscous “jelly” that settles very slowly under its own weight. The scintillator would respond gracefully to slowly forming pressure gradients induced by external pressure changes and temperature differences. The jelly would need to have low vapor pressure and a very low rate of gas generation/evolution. If gas generation/evolution were shown to be negligible, a bubble of argon or nitrogen left inside during sealing ensures that no large pressure differences would arise as quartz compresses under pressure. Seems like an interesting possible solution, but remains *terra incognita*. Again, consultation with experts is essential.

#### *3. Plastic scintillator bars:*

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<sup>4</sup> Putting a square peg in a round hole works sometimes.

The above discussion suggests the desirability of using externally cast, solid plastic scintillator bars. These bars would have to be slightly smaller than the inner diameter of QT to allow for dimensional shrinkage under external pressure. Several options for the thin annular space exist: it could be evacuated, filled with, *e.g.*, argon gas at nominal pressure, or filled with a viscous liquid, either optically transparent to 175 nm or perhaps acting as a WLS. Again, gas generation/evolution over the long-term must be adequately small, so that pressures do not rise to a dangerous level.

The presence of a thin, discrete, annular space may allow better optimization of WLS from VUV 175 nm by specialized doping of the viscous WLS fluid, whereas the thick solid scintillator bar is optimized separately for good transmission and normal scintillator performance. Again, the filler liquid viscosity and a proper bubble of inert gas provide sufficient elasticity to tolerate gracefully dimensional compression of the QT under pressure and protect the PMT.

#### *4. Inorganic scintillator bars:*

The use of scintillator bars made of, *e.g.*, NaI, CeF<sub>3</sub>, or BaF<sub>2</sub> also come to mind. These offer some gamma-catcher performance (but not great unless present everywhere), but these materials are heavy, much more expensive, and quite radioactive in some cases. The crystalline structure of these materials introduces broad energy levels that impose prominent variations in photoluminescence with excitation energy [6]. It is not really known what WLS performance exists at 175 nm, for any of these materials. However, it remains possible that at least one of these materials, perhaps BaF<sub>2</sub>, can provide efficient WLS at 175 nm.

#### *5. Liquid scintillator ?*

We choose to not consider any sealed liquid scintillator (LS) scenario. Sensing of a leak would be externally unavailable unless some electronic pressure sensor is embedded, which would add unwanted complexity. A leak would gradually increase the pressure inside the QT until the PMT fails. No catastrophic implosion seems likely at that point, as LS would gradually fill the PMT; a bubble of pressurized xenon gas might exist, and considerable xenon gas might be absorbed in the LS. However, since the large pressure internal to the QT would persist during a subsequent depressurization of the TPC system, a catastrophic rupture failure is likely. The QT, now under tension, would crack, and LS would spray into the TPC as a sequel to depressurization. Unacceptable.

#### *Long-term stability concerns*

In any of these sealed scenarios – solid-, jelly-, liquid-filled, or combinations thereof – it will be necessary to establish with high confidence that any pressure rise due to gas evolution is negligible. Otherwise, a fluidic connection to the external world would be necessary to detect and bleed off xenon ingress or gas evolution. It would be valuable to have a scintillator expert review these notions and provide consultation.

### **Design concepts with external plumbing**

#### *a. Liquid scintillator - again ! ?*

If a near-perfect optical interface between QT and WLS were made a requirement (not clearly necessary), an obvious approach would be to use liquid scintillator (LS). Filling the QT with LS solves the optical interface problem at a stroke. A double O-ring seal

design, with pump-out, would allow leak detection for either xenon or liquid scintillator, at the cost of more penetrations. The end-plugs should be designed with sufficient attention to elasticity to avoid QT stress build-up under external pressure. A sketch of this notion is provided in figure 3.

If any concept involving fluid components is shown to work acceptably, then it seems clear that the WLS bars, and hence the detector, are best oriented vertically to allow bubbles to flow out at the upper end of the WLS tube during filling. Immersing the PMT also immerses the base and components, improving dielectric strength against any discharges, but may require some testing for component compatibility. If an optical circular-to-square transition piece were used, it would have to be mirrored, as internal reflection would not be present under full LS immersion.

There would need to be two plumbing connections, of course, to permit filling and emptying. Both connections could be at the bottom, with an internal transparent tube extending to the “top” of the liquid volume to allow gas/liquid to be removed/circulated. We suspect that nitrogen or argon would be good to flush water/oxygen out of the LS as well as to fill any bubbles. For an alternative single O-ring design, without pump-out, a leak would introduce a pressure rise that would be externally evident via the filling/emptying tubing.

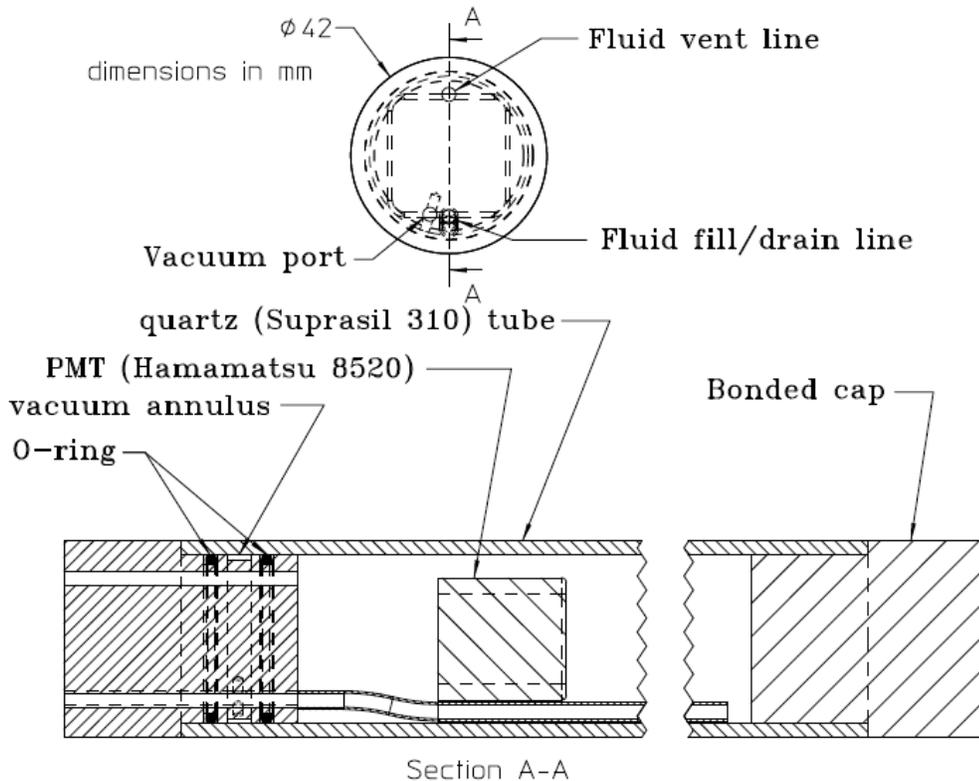


Figure 3. A sketch showing how the 1" PMT might fit in QT filled with liquid scintillator. The orientation would be non-horizontal to permit complete filling and emptying. No optical transition piece is included in this sketch.

The QT + LS scenario offers these benefits:

1. QT has good buckling resistance, offering a large safety factor.
2. The tubes can be pumped out at the same time that the detector is pumped.
3. The tubes are filled at one bar, and PMTs never see pressures beyond 1 bar.
4. The scintillator could be recirculated through a purifier/leak detector.
5. The plumbing allows the WLS tubes to be emptied at will, if necessary.
6. The internal pressure of each WLS tube is measured externally.
7. Even for a fairly large seal leak, the internal pressure rises minimally.
8. Xenon gas gets into the scintillator, rather than scintillator into the TPC.<sup>5</sup>
9. Xenon gas will rise as bubbles and pass to an external detection point.
10. A leak could be detected early and, in principle, the xenon recovered.
11. The system can be shut down gracefully for maintenance.

However, the QT + LS scenario also introduces major issues and severe risks:

1. Two or three external plumbing lines per QT are needed.
2. LS maximizes catastrophic effects of a rupture failure – dumping ~3 liters of LS into the detector from a single tube.
3. If one tube fails catastrophically, would they all fail in a Super-K style catastrophic cascade? The answer is likely, No, since there would be no appreciable gas volume within to provide implosive energy, and the 1" PMT volume is very small. The total liquid inventory in all tubes would be ~300 liters. However, individual plumbing for each tube would still have to be implemented with complete surge isolation from all others, a complication.
4. Even an attractive liquid scintillator such as that based on LAB<sup>6</sup>, with a flash point of 130° C, is still flammable, with non-zero toxicity.

Given the experience at LNGS with the spill from Borexino, a LS scenario would seem to be an uphill battle even if all technical issues were convincingly solved. While the all-liquid scintillator scenario has several attractive aspects, it appears at this point to be too complex and way too scary. We do not recommend this option.

#### *b. Plastic scintillator bars*

The above discussion strongly suggests, again, the use of solid plastic scintillator bars instead of any LS. These bars would have to be slightly smaller than the QT inner diameter to permit assembly and allow for radial compression of the QT under external pressure. The solid plastic scintillator approach admits a particularly interesting set of possibilities. Several options exist for the spaces not occupied by solid plastic - the thin annular space and the two end regions. The QT empty space could be perhaps filled with a viscous liquid, either optically transparent to 175 nm or perhaps serving also as

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<sup>5</sup> In the original PEP-4 TPC, operated at 8.5 bars, a leak occurred in the cooling loop for sector preamps within the pressurized volume. Bubbles of argon-methane appeared at the heat exchanger, running at NTP. But no water passed into the TPC.

<sup>6</sup> Linear Alkyl Benzene, planned for use in SNO+.

an optimized WLS. Alternatively, the QT could be filled with, *e.g.*, argon gas at some nominal pressure.

Finally, the QT could be continuously pumped with rough vacuum. Absorbed oxygen and water would be removed (albeit slowly), possibly improving optical performance with time. An important benefit is continuous, sensitive monitoring by residual gas analyzer (RGA), providing real-time semi-quantitative leak detection. During handling and storage prior to installation, the QT assemblies could be filled with argon at NTP.

One option for the gas-fill or rough vacuum scenario is to route the electrical lines inside the single tubing that connects a QT assembly to the external vacuum systems. Then there is only one feed-through between the xenon containment vessel and the buffer gas vessel. Similarly, there might be only one purely mechanical feed-through between pressurized buffer gas vessel and the external world. The HV and PMT signals would emerge somewhere at a convenient interface at atmospheric pressure. This interesting approach appears to simplify system design significantly. In this concept, the end-caps can be glued in permanently, since a continuous pump-out is foreseen during operation.

We strongly prefer the rough vacuum scenario to a gas-fill. If a leak occurs in some QT assembly, xenon would enter the QT and be soon found by RGA at the pump. An RGA manifold with valves would permit isolation and identification of the source. Appropriate steps could be taken to recapture xenon or initiate shutdown procedures (or possibly doing nothing if the leak is small). The PMT base would be encapsulated with a suitable potting compound to prevent electrical discharges, a good idea even in the absence of vacuum. We presume that the PMTs will tolerate a rough vacuum!

### **Wavelength shifting**

It could be that plastic scintillator is an excellent WLS for 175 nm, but a concern is that the matrix is too strongly absorbent at 175 nm, hampering energy transfer to fluors. PVT and polystyrene absorb 175 nm very strongly [3]. But, increasing the fluor concentration to compensate for this can reduce secondary transparency, leading to an undesirably short attenuation length for secondary fluorescence. However, a solid coating of p-terphenyl (TPH) is essentially 100% efficient in conversion of 175 nm UV [3]. TPH shifts to about 330 nm, a wavelength that plastic or liquid scintillators convert much more efficiently to visible blue light. However, a layer of TPH is translucent, and strongly scatters visible light. A coating of TPH on plastic scintillator would degrade the optical transmission of the blue light and reduce the efficiency of detection.

Instead, the interior surface of the QT could be coated with TPH at an optimum thickness. With TPH, half of the 330 nm light will strike the plastic scintillator and be converted to blue light around 400 nm, an optimum for modern PMTs. While the other half of the 330 nm light goes outward, some other QT assemblies may detect most of this light. Quartz is very transparent at 330 nm, but cuts off part of the xenon 175 nm VUV spectrum. Coating the interior QT surface with TPH avoids loss due to multiple reflections, since conversion would occur at the first encounter with an interior interface. The solid plastic bar would need to be supported within the QT in a way that prevents contact with the TPH. Rubbing between the QT interior surface and plastic

during handling, transport, storage, and installation must be avoided. A vertical orientation of the detector would prevent the danger of long-term sagging of plastic.

It could turn out that there is little difference in total detected light using either a circular WLS bar that nearly fills the volume, or a square WLS that is matched to the PMT sensitive area. The square bar could be oriented to present a maximal area to the 330 nm light, which will emerge most strongly from the surface facing inward to the TPC active volume. A square bar allows more unfilled volume and better pumping, but the implosive energy is higher. A catastrophic QT failure would be worse for the square WLS bar. If the QT safety factor is shown to be large, this may not be a concern.

Of all these notions, we regard the concept of QT + solid plastic scintillator (with TPH coating on QT interior) + single tubing connection (with electrical wiring inside) + rough vacuum + RGA as the most promising option.

### **Physical robustness of QT**

An essential goal is to ensure that a high level of confidence is achieved against catastrophic breakage after the detector is fully assembled and under pressurized operation, whether rising, sustained, or falling. The seal and end-cap design requires good engineering to avoid stress concentration as external xenon pressure is increased. In addition, clear procedures for handling and installation must be in place to avoid damage such as undetected cracks and scratches prior to final closure of the detector. Although any actual fractures will likely require detector shutdown and repair, any WLS bar solutions using fluid are obviously much more serious.

A large calculated safety factor is prudent and necessary. Calculations for 38 mm diameter QT by Derek Shuman using standard formulae indicate that, for a wall thickness of ~2 mm and a length of interest to us, ~150 cm, the long-tube buckling threshold is 212 bars.<sup>7</sup> This is very good news, but does not include the additional impact of axial compression. We should consider 3 mm thickness to gain additional real-life safety, as the buckling threshold increases as the third power of wall thickness.<sup>8</sup> The plugs for the ends of the tubing require careful engineering to eliminate or minimize end-effect stresses that may compromise robustness if not attended to properly. The QT may be out-of-round as supplied, so actual safety factors may be correspondingly smaller.

In any case, QT of ~38 mm inner diameter and 3 – 4 mm wall thickness appears to provide a large ideal “raw” safety factor against catastrophic mechanical failure of this component under pressure. More sophisticated analytical studies and tests are necessary.

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<sup>7</sup> See appendix 1 for Derek’s calculations. Paradoxically, a tube constrained to be circular periodically has a lower buckling threshold; the critical length is ~30 cm.

<sup>8</sup> In this case, still approximate, the buckling threshold becomes more than 700 bars; for 4 mm wall thickness, still not leading to excessive optical absorption, the formula indicates ~1700 bars.

### Double-vessel

In any design, a significant amount of xenon will be found in the purification loop plumbing. Beyond that, in any single pressure vessel design a large fraction of xenon would wind up in numerous large spaces and interstices. A single vessel design must use either a solid HV insulator, or devote an extravagant fraction of xenon (which is a poor HV insulator) to that purpose. A double vessel design concentrates the xenon into the active volume to the greatest extent practical, thereby maximizing sensitivity, and can also exploit the buffer gas as a HV insulation. In the absence of complete engineering designs, with/without a double vessel, we can only guess that the increase in sensitivity will be very attractive in favor of a double vessel.

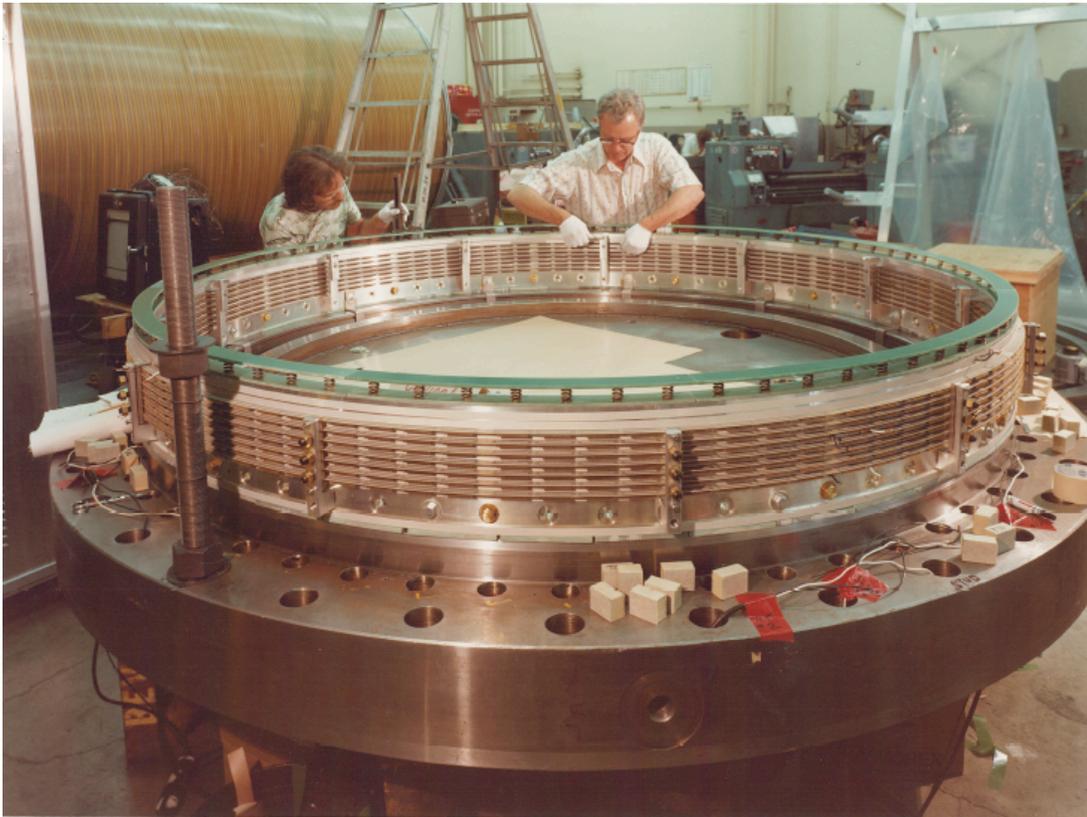


Figure 5. One of the PEP-4 TPC feed-through rings is shown under construction at LBNL, with the lay-up in progress. After the lay-up was complete, a heating stage cured the B-stage epoxy between layers.

In a double vessel design, all signals now need two gas-tight penetrations rather than one, adding more complexity. The  $\sim 80$  penetrations for PMT signals seem manageable, especially if the wiring for signal + HV is inside the pump-out tubing. But, without multiplexing, the SiPMs need several thousand penetrations, which imposes an alarming challenge for either a single vessel or double vessel designs. Cross-talk problems, noise, and other well-known issues regarding transmission of multiple analog signals over considerable distances are persistent challenges.

For comparison, figure 5 shows one of two feed-through rings for the PEP-4 symmetric TPC. Per ring, more than 10,000 signals passed from 8.5 bars argon-

methane to atmospheric pressure. These feed-through rings were constructed with PC-board edge connectors stacked up and fused with B-stage epoxy. This worked very well for that experiment, where there was no significant worry if a leak existed. As figure 5 suggests, these rings were a major effort to construct!

We do not propose nor would we support this approach for NEXT. We show this here, instead, to bolster the case for implementation of a high level of signal digitization and multiplexing within the xenon volume. While optical feed-throughs appear to offer the highest level of compactification, we do not presuppose that the multi-channel electrical feed-throughs developed at SACLAY might not also be a good choice.

A buffer gas such as N<sub>2</sub> or CO<sub>2</sub> provides pressure equalization and HV insulation. Clearly, a double vessel adds complexity and risk. The presence of a double vessel requires the buffer gas to follow, very closely, the xenon pressure under all circumstances to avoid mechanical overstress of the internal vessel. Our perception is that purely mechanical regulators can reliably provide this function under normal circumstances, although redundant electronic monitoring of the pressure differential is essential. A rapid recovery system for xenon is necessary in either design for abnormal circumstances, perhaps involving a large inflatable bladder for quick recovery, followed by a slower cryogenic capture into permanent storage.

### **A double vessel design employing QT with plastic WLS**

We have attempted to integrate all these issues, concerns, risks, benefits, *etc.*, resulting in a double vessel design study. The general idea is shown in figure 6, with some details given in figure 7 – 8. Figure 9 shows a cutaway view. High-resolution drawings and various renderings are available at: <http://www-eng.lbl.gov/~shuman/NEXT/>. The design is intended to suggest how this detector concept might take shape, but does not yet include actual engineering, nor are services for SiPM, power, vacuum, gas recirculation, *etc.*, addressed. As this note is already quite long, we limit our comments, hoping to add more clarity than length.

From figure 6, the xenon containment vessel is supported by three HV standoffs. The xenon containment vessel also supports the weight of the copper ring. The copper ring extends around the PMTs, providing a lip that transfers the entire copper weight to the xenon vessel, as well as more shielding. To support the electric drift field, the xenon containment vessel is made of non-conducting plastic, possibly PMMA or a polycarbonate.

HV dielectric strength is obtained primarily with the buffer gas, with some benefit likely obtained from the plastic xenon containment vessel. The resistor string that grades the field cage voltage is not shown in any of the figures. The field cage rings should, of course, be as reflective as possible and are shown here as circular tubes; a better design may arise with more thought. The QT WLS assemblies are also shown with their own field rings and a resistive stripe to grade the voltage from the cathode to ground. The stripe connects sequentially with all rings of a QT WLS assembly. This seems better than trying to connect all of the QT rings to the main field cage to establish potentials.

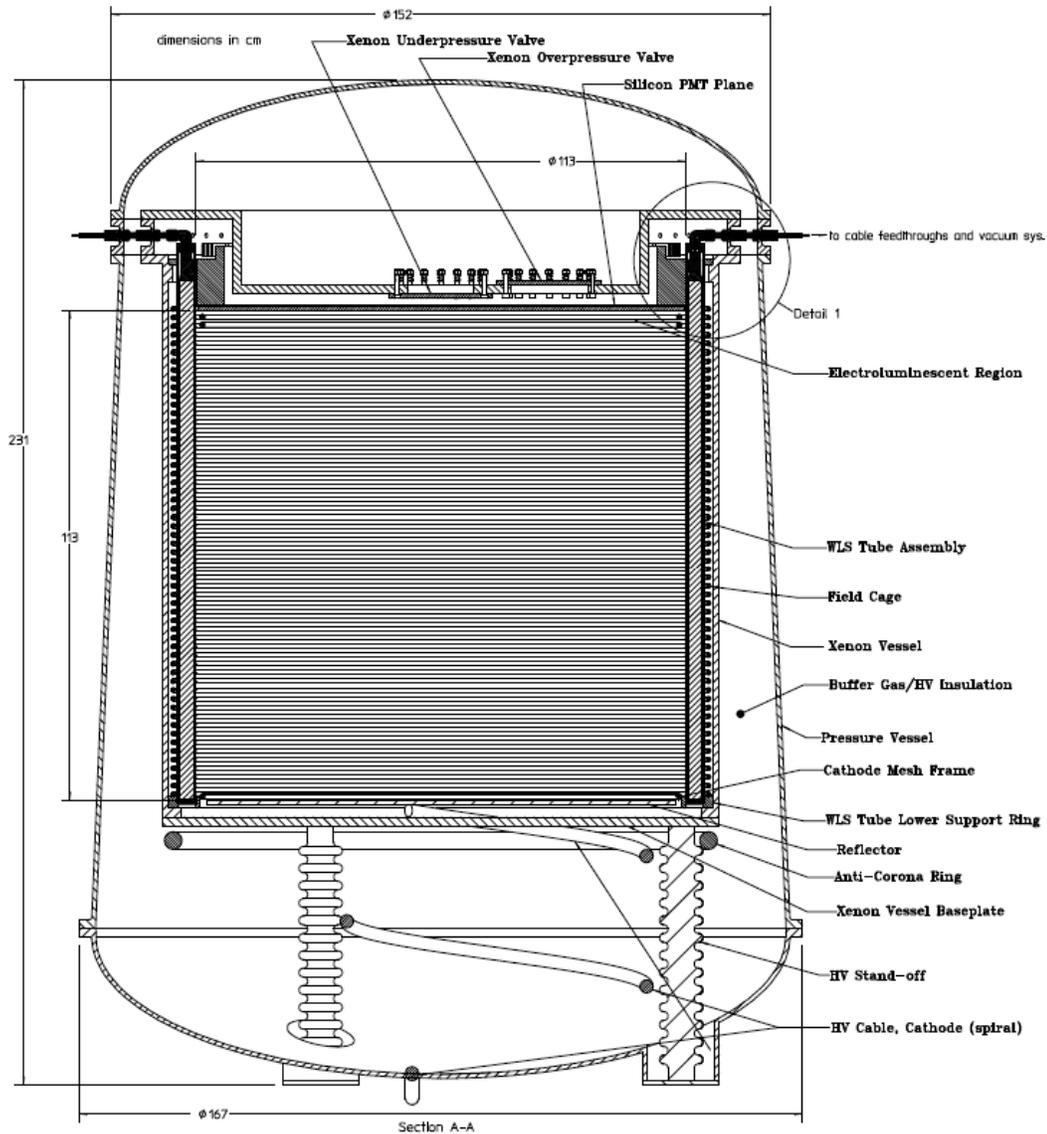


Figure 6. Section view of detector design concept. Dimensions are intended to be suggestive, not definitive. Flanges will be meatier in practice. See text for explanations.

The pressure vessel is shown here as a tapered structure, which affords a larger buffer gas thickness where the HV is highest; more tapering may be advised, or perhaps the volume included at the bottom may be excessive. The tapering adds little to the cost, and we acknowledge that a conventional cylindrical shape may also be fine. All penetrations for signals and voltages (except the really high HV) pass through two concentric annuli. Not shown are gas system ports and flow paths, which have not yet been thoroughly considered. In any event, no gas, vacuum, or electrical connections are foreseen to pass through the top cap.

The really high HV enters at the bottom, shown as a cable with compression seals to avoid leaks of buffer gas and extrusion under pressure. The cable winds around, passing through another compression seal to enter the xenon volume, finally connecting to the

cathode plane. The cathode plane is a simple surface, possibly a mesh with a backing made as reflective as possible for both 175 and 330 nm.

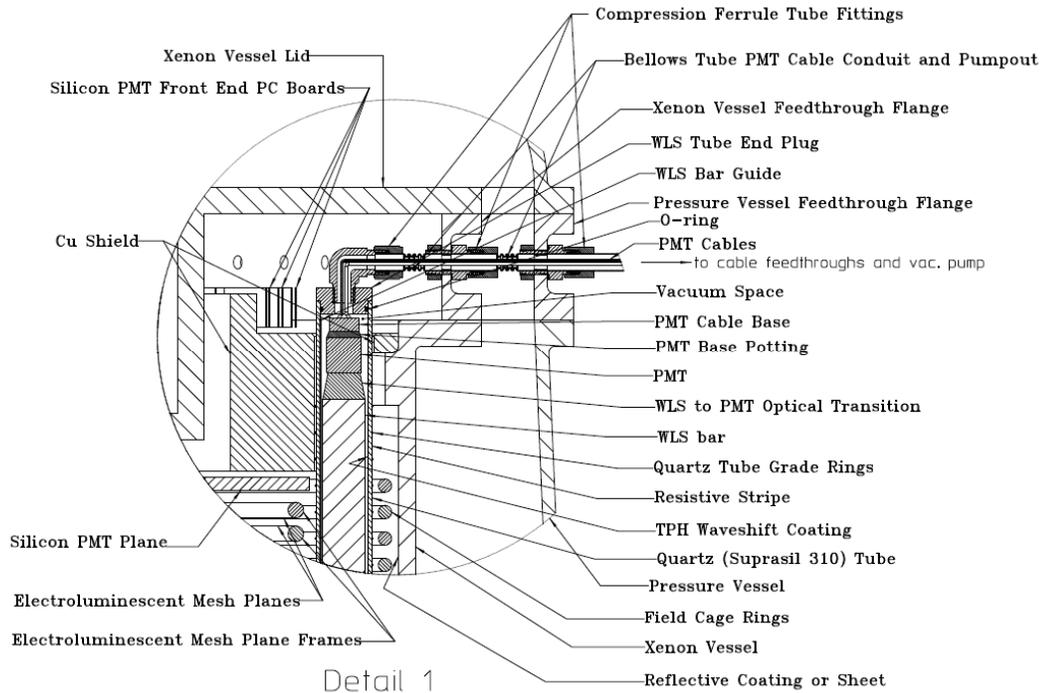


Figure 7. Detail 1, of the corner where most services and signal generation/digitization occur. Please see high-resolution drawings at Derek's website for clearer images of all these important design elements.

Figure 6 suggests flapper valves that permit pressure equalization if a major excursion from equilibrium occurs, but which prevent gas mixing under normal circumstances and minor pressure differentials. Should the gases mix, the primary goal is rapid and complete recovery of xenon, with separation possible offline. Figures 7 – 9 also suggest how important it is to design all services in considerable detail from the beginning.

Because the PMT signals carry precise energy information, it seems better to bring out the PMT analog signals, to be shaped and digitized with high quality high dynamic range external circuitry, rather than digitize internally, inside the xenon vessel. That would put these critical analog signals out of reach for assessment. In figures 7 – 9, a feed-through concept for the QT WLS assemblies is shown, with signal/HV cabling carried within the rough vacuum plumbing. In addition to mechanical simplification, excellent shielding is also realized. Crosstalk and noise are reduced to negligible levels. This could be very important for energy resolution.

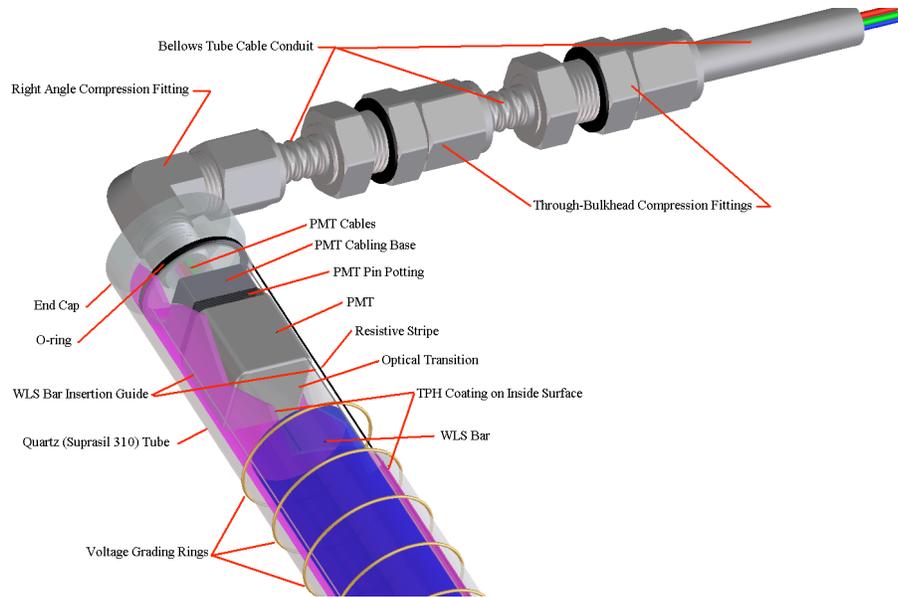


Figure 8. Cutaway view of the PMT end of the QT WLS assembly, showing details of the proposed design concept.

We foresee these penetrators as single structures that pass first from the exterior of the pressure vessel, through the xenon containment vessel, and connecting to the right-angle gas fitting at the base of the QT WLS assembly. To accommodate inevitable small misalignments, two bellows structures are incorporated in each penetrator. These, however, must be nearly straight to avoid instabilities under pressure. Small brackets, not shown, are needed to constrain the QT WLS assemblies, since the bellows provide (by intent) both axial and lateral elasticity, but do not provide force balancing. These brackets need to be positioned near the right angle gas fitting to avoid torques. We imagine that the signal and HV cables are pre-connected to the QT WLS assemblies, passing out the right angle fitting. These cables, perhaps 3 m or so long, must be threaded through the penetrators as they are installed. The cabling must then be fished up to the plenum where the vacuum system and electrical elements are easily separated. Sounds a bit awkward, but needs to be done only once. Still way better than any other scenario we can think of!

However, feed-throughs for the SiPM signals are not shown. As argued above, we believe that the only sensible approach for these signals is local digitization/multiplexing within the xenon volume. The ADC resolution requirement for these signals is modest relative to the QT signals. With optical transmission of digitized waveforms, even with no zero-suppression, the number of penetrations can be reduced by a large factor, probably around 64. This level of compression seems essential. Adding these to the design more or less doubles the pressure vessel penetrators. Better to leak nitrogen than xenon!

The radioactivity of typical components such as capacitors and resistors has made this option appear unattractive, but we are not sure how radioactive the electronics for simple digitization/multiplexing might actually be. Our proposed solution is that the

copper ring shielding – proposed for the PMTs – can also be used to shield the active volume from radioactivity contained in the SiPM electronics. Once the activities are known, it is straightforward to calculate the required copper shielding thickness.

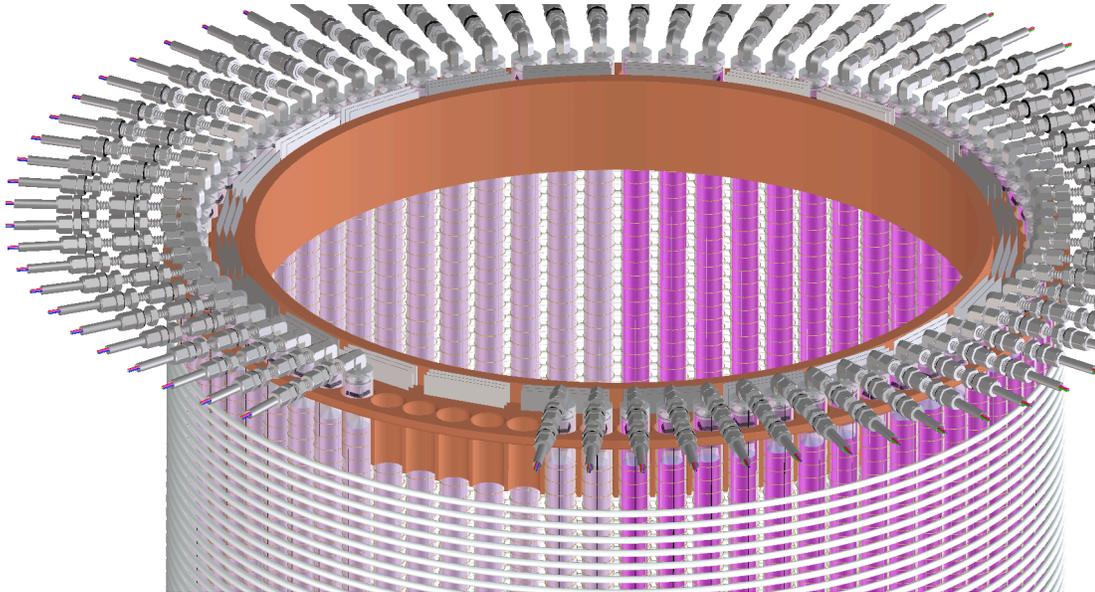


Figure 9. Cutaway view is shown, with pressure vessel and xenon vessel removed, and details of QT-WLS assemblies, field cage, SiPM cards, and the copper shield.

If the SiPMs need only 8-bit digitization at 1 MHz, then it appears that a single layer of cards might be sufficient to convert electrical signals for, *e.g.* 6400 SiPMs to  $\sim 100$  optical fiber paths operating at 500 Mbit/s. In figures 6 – 9, we show not one, but three layers, nestled behind the copper ring to achieve the needed shielding to illustrate possibilities. It does not seem daunting to pass 100 optical signals through two interfaces, only one of which must support a 10 – 20 bar pressure differential. Of course, the digitization/multiplexing approach can also be used if a single vessel design is chosen. The copper ring can also serve as a heat sink for the power dissipated by the electronics. It would be easy to remove the heat by a single cooling loop of copper tubing pressed into a groove in the ring.

The EL plane is shown as allowing some light to emerge radially. This light would impinge mainly on the closest QT WLS bars. This adds a component of detected light that increases strongly with radius. Whether this is a good thing or not needs to be evaluated by simulation. We don't show a separate HV penetrator for controlling the potential across the EL plane, but to provide independent control of the drift and EL fields, such a penetrator is likely to be needed.

### **Summary**

A design concept for NEXT exploiting a quartz tube structure to enclose both PMT and a wavelength-shifter + plastic scintillator bar appears feasible, safe, and attractive. Significant performance advantages may be achieved. At present, the design is not an engineered approach. The R&D phase to demonstrate feasibility, safety, and

performance may be straightforward. The performance advantages appear substantial, and the approach simultaneously solves the problem of pressure-intolerant PMTs. A double vessel appears feasible and preferable. SiPM array signal acquisition may benefit greatly from early digitization and optical multiplexing within the xenon volume.

### Perspective

The weakness of the radio-pure Hamamatsu PMTs against pressure plus the absence of real alternatives drives the design, but also offers opportunity. Although the WLS-bar scenario represents a radical departure from the LOI, PMT pressure weakness allows us to capitalize on a situation that otherwise may lead to severe compromises in scientific reach. It could turn out that the WLS-bar scenario represents the only known pathway to a detector concept that we want to have, and will be able to build, and can build expeditiously with acceptable risk. Unless a more conservative approach can be shown in the immediate future to provide comparable or better performance, the WLS-bar scenario should be considered by the NEXT collaboration as likely the most attractive; more scrutiny, engineering, and quantitative performance evaluations are needed.

A major risk we face is being too conservative, building a safe but scientifically weak detector, and thereby working hard just to fail in a rather boring way.

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Quartz tube theoretical buckling pressures; quick estimate. Formulas from table 35 Roark's Formulas for Stress and Strain, 6th ed. These values are not conservative, as slight imperfections will reduce buckling pressure, but realistic buckling values can be within +/-20% of theoretical. Required Factor of safety on buckling is not clear, it is probably not 8, as required in PUB3000 on strength for brittle materials used in pressure systems. These tubes are not part of the pressure containment system, so a lower factor of safety, based on failure consequence to experiment (only) is warranted.

from [http://www.insaco.com/MatPages/mat\\_display.asp?M=Quartz](http://www.insaco.com/MatPages/mat_display.asp?M=Quartz)

modulus of elasticity

Poisson's ratio

$$E := 10.5 \cdot 10^6 \text{ psi}$$

$$\nu := 0.17$$

try several thicknesses in parallel calculation

$$t := \begin{pmatrix} 1 \\ 1.5 \\ 2 \end{pmatrix} \text{ mm}$$

tube nominal radius  $r := 0.75 \text{ in}$

very long tube ( $l > l_{cr}$ ):

$$l_{cr} := 4.9r \sqrt{\frac{r}{t}} \quad l_{cr} = \begin{pmatrix} 0.407 \\ 0.333 \\ 0.288 \end{pmatrix} \text{ m}$$

buckling pressure:

$$P_{cr\_lt} := \frac{1}{4} \frac{E}{1 - \nu^2} \frac{t^3}{r^3} \quad P_{cr\_lt} = \begin{pmatrix} 26.6 \\ 89.8 \\ 212.8 \end{pmatrix} \text{ bar}$$

for short tube, length  $l$ , or long tube constrained circular at lengths  $l$ :

$$l := 0.5 \text{ m}$$

$$P_{cr\_st} := \sqrt[4]{0.807 \cdot \frac{E \cdot t^2}{1 \cdot r} \cdot \left( \frac{1}{1 - \nu^2} \right)^3 \cdot \frac{t^2}{r^2}} \quad (\text{approx. formula})$$

$$P_{cr\_st} = \begin{pmatrix} 14.2 \\ 39.1 \\ 80.2 \end{pmatrix} \text{ bar}$$

check compressive strength, (no factor of safety here)

$$S_{comp} := 1150 \text{ MPa}$$

Suprasil CG ( no data for Suprasil 310)

$$P_{comp} := S_{comp} \cdot \frac{t}{r} \quad P_{comp} = \begin{pmatrix} 596 \\ 893 \\ 1 \times 10^3 \end{pmatrix} \text{ bar}$$