Statistics of Brittle Fracture

Outlines

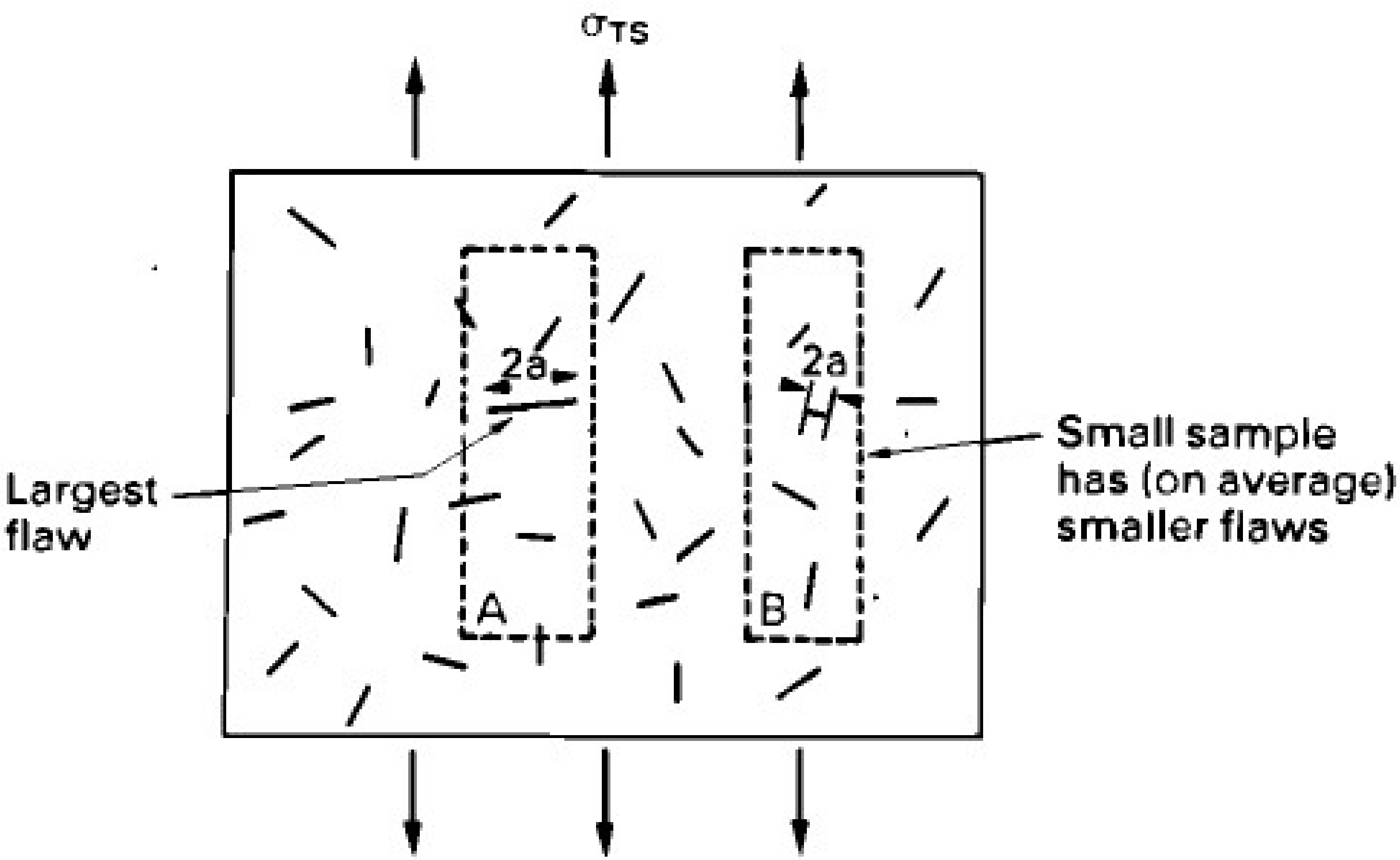
- Statistics of Strength
- Weibull Distribution
- Time-dependence of Ceramic Strength
- Case study – Pressure Windows

Statistics of Strength

<u>Failure Probability, P_f</u>: The probability of the failure of a specimen under certain loading

When using a brittle solid under load, it is not ***possible*** to be certain that a component will not fail. But if an acceptable risk can be assigned to the function filled by the component, then it is ***possible*** to design so that this acceptable risk is met.

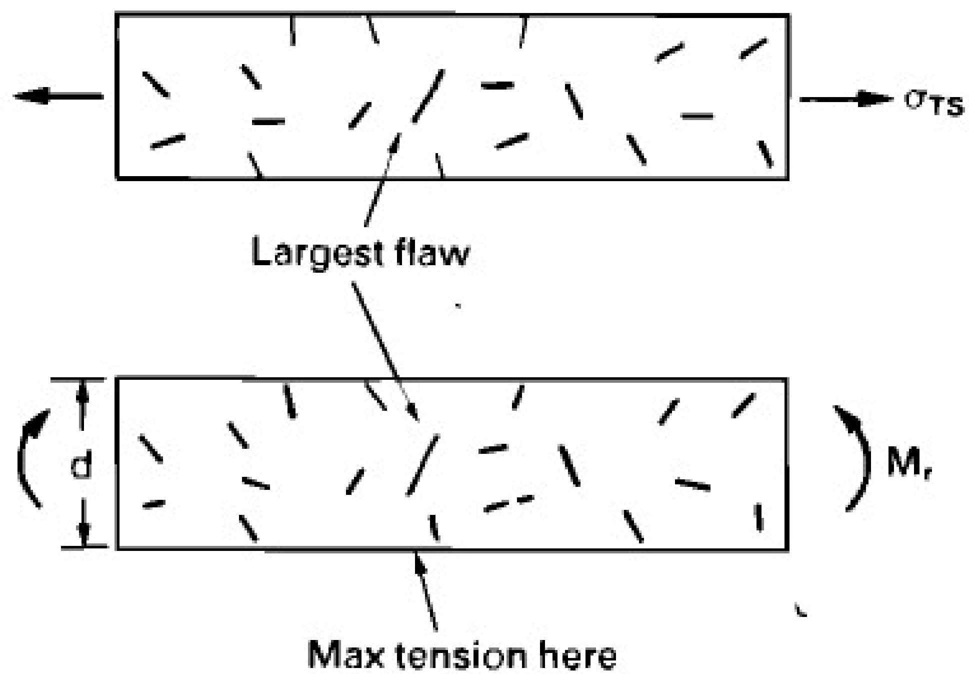
Volume Dependence of Strength



Volume Dependence of Strength

- The average strength of the small samples is greater than that of the large sample, because larger samples are more likely to have larger cracks.
- Ceramic rod is stronger in bending than in simple tension, because in bending only a thin layer close to one surface carries the peak tensile stress.

Volume Dependence of Strength



Ceramics appear to be stronger in bending than in tension because the largest crack may not be near the surface

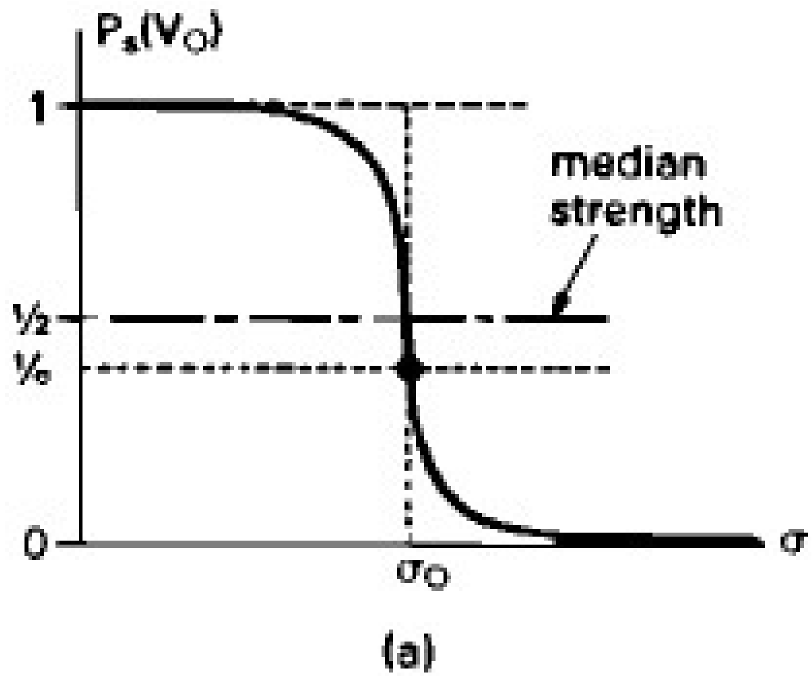
Weibull Survival Probability

Survival Probability ($P_s(V_0)$): The fraction of identical samples, each of volume V_0, which survive loading to a tensile stress σ. $P_s(V_0)$ can be calculated as:

$$P_s(V_0) = \exp\left\{-\left(\frac{\sigma}{\sigma_0}\right)^m\right\} \quad \textbf{(1)}$$

Where σ_0 and m are constants

Weibull Survival Probability



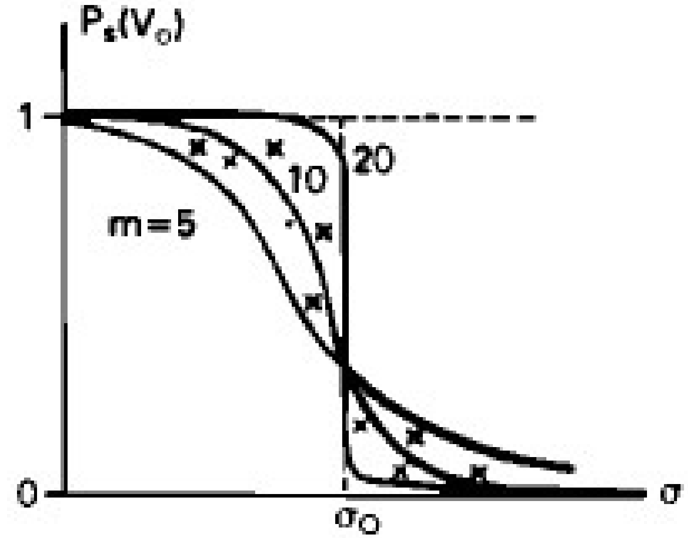
The Weibull Distribution Function

Weibull Survival Probability

Weibull Distribution Function

- **$\sigma = 0$ $\quad\quad P_s(V_0) = 1$**
- **$\sigma \to \infty$ $\quad\quad P_s(V_0) \to 0$**
- **$\sigma = \sigma_0$, $\quad\quad P_s(V_0) = 1/e = 0.37$**
- ***m*** represents how rapidly the strength falls as we approache σ_0 , it is called ***Weibull Modulus***

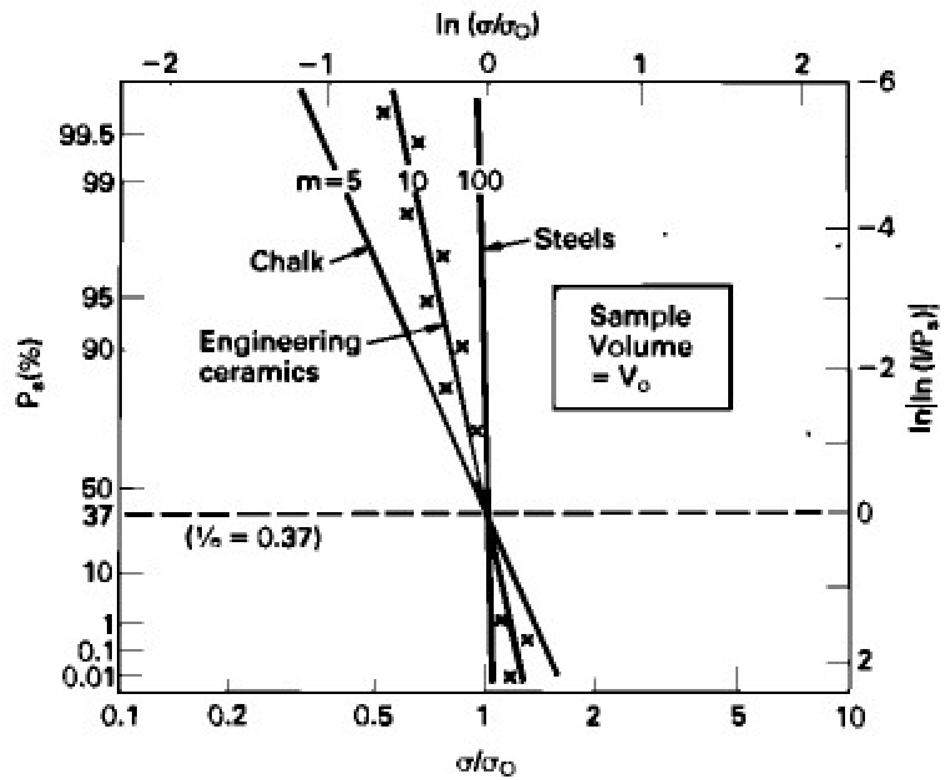
Weibull Survival Probability



(b)

- For brick, cement, m ≈ 5
- For engineering ceramics (Al_2O_3, Si_3N_4), m ≈ 10
- For steel, m ≈ 100

Weibull Survival Probability



Weibull-probability graph

Volume Dependence of Weibull Probability

For one sample with the volume of V_0, WP is $Ps(V_0)$,
For n samples stick together, volume $V = nV_0$, then

$$P_s(V) = \{P_s(V_0)\}^n = \{P_s(V_0)\}^{V/V_0}. \quad (2)$$

This is equivalent to

$$\ln P_s(V) = \frac{V}{V_0} \ln P_s(V_0) \quad (3)$$

or

$$P_s(V) = \exp\left\{\frac{V}{V_0} \ln P_s(V_0)\right\}. \quad (4)$$

Volume Dependence of Weibull Probability

The Weibull distribution can be rewritten as

$$\ln P_s(V_0) = -\left(\frac{\sigma}{\sigma_0}\right)^m. \qquad \textbf{(5)}$$

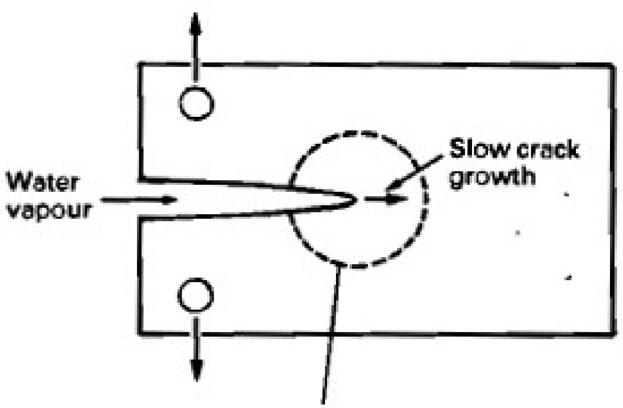
If we insert this result into previous equation, we get

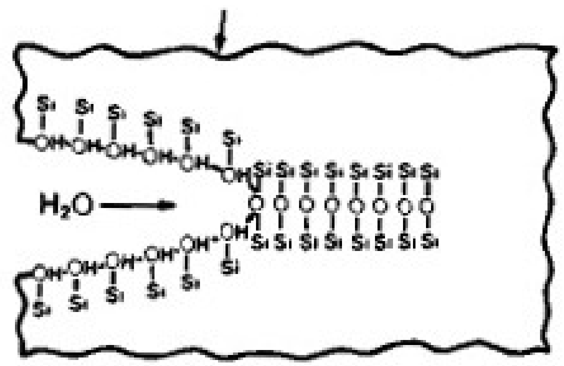
$$P_s(V) = \exp\left\{-\frac{V}{V_0}\left(\frac{\sigma}{\sigma_0}\right)^m\right\}, \qquad \textbf{(6)}$$

or

$$\ln P_s(V) = -\frac{V}{V_0}\left(\frac{\sigma}{\sigma_0}\right)^m. \qquad \textbf{(7)}$$

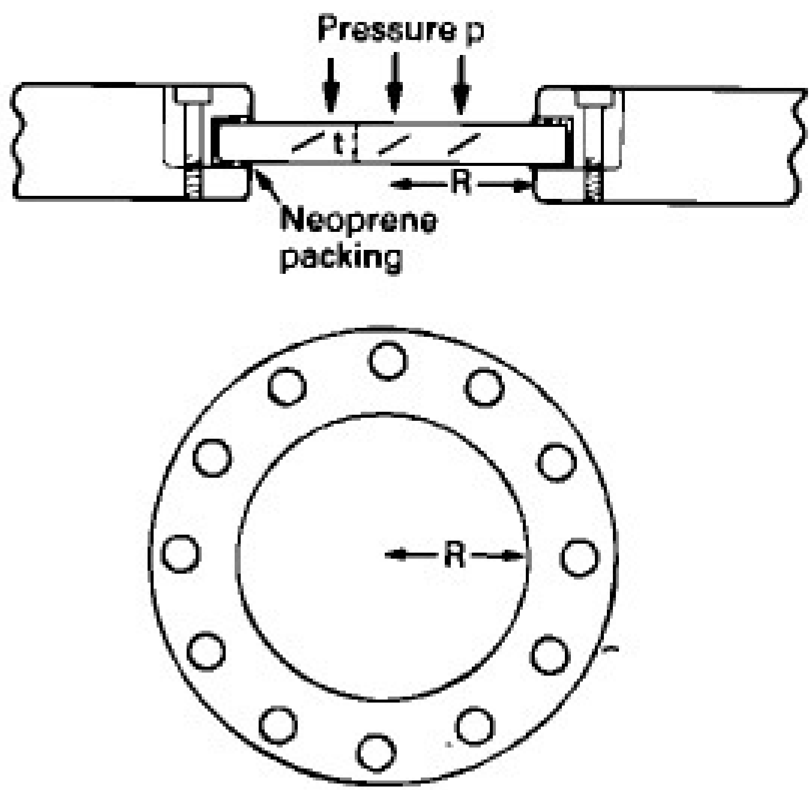
Time-Dependence of Ceramic Strength





$$\left(\frac{\sigma}{\sigma_{\mathrm{TS}}}\right)^n = \frac{t(\mathrm{test})}{t} \qquad (7)$$

Case Study – Pressure Windows



A flat-faced pressure window. The pressure difference generates tensile stresses in the low-pressure face

Case Study – Pressure Windows

The peak tensile stress has magnitude

$$\sigma_{max} = \frac{3(3+v)}{8} \Delta p \frac{R^2}{t^2}. \tag{8}$$

Where ***R*** is the radius and ***t*** is the thickness, ***Δp*** is the pressure difference (0.1MPa), and ν is the Poinsson's ratio, which is about 0.3 for ceramics. Therefore,

$$\sigma_{max} \approx \Delta p \frac{R^2}{t^2}. \tag{9}$$

Case Study – Pressure Windows

Properties of Soda Glass

Modulus E (GPa)	74
Compressive strength σ_c (MPa)	1000
Modulus of rupture σ_r (MPa)	50
Weibull modulus m	10
Time exponent n	10
Fracture toughness K_{IC} (MPa m$^{1/2}$)	0.7
Thermal shock resistance ΔT (K)	84

Case Study – Pressure Windows

(1) Modulus of rupture σ_r = 50MPa, test time is assumed to be 10 min.

(2) Design life is 1000 hours

(3) Failure probability is set to 10^{-6}

Case Study – Pressure Windows

Then

(1) the Weibull equation (eqn. 7) for a failure probability of 10^{-6} requires a strength-reduction factor of 0.25

$$\ln P_s(V) = -(\sigma/\sigma_0)^m$$

Since $\ln P_s = \ln(1-P_f) \approx P_f$

$$10^{-6} = (\sigma/\sigma_0)^{10}$$

$$(\sigma/\sigma_0) = 0.25$$

Case Study – Pressure Windows

(2) The static fatigue equation (eqn. 8) for a design life of 1000 hours requires a reduction factor of 0.4

$$(\sigma/\sigma_{TS})^n = t(test)/t$$

$$(\sigma/\sigma_{TS})^{10} \approx 10^{4}$$

$$(\sigma/\sigma_{TS}) \approx 0.4$$

Case Study – Pressure Windows

For this critical component, a design stress

$$\sigma = \mathbf{50MPa} \times \mathbf{0.25} \times \mathbf{0.4 = 5\ MPa}$$

We apply a further safety factor of S = 1.5 to allow for uncertainties in loading, unforeseen variability and so on.

Then the dimension of the window should be

$$\frac{t}{R} = \left(\frac{S\Delta p}{\sigma}\right)^{1/2} = 0.17.$$

SUMMARY

- **Statistics of Strength**
  - Fracture probability, Volume-dependence of FP
- **Weibull Distribution**
  - Weibull survival probability, Volume-dependence of WSP, Weibull distribution
- **Time-dependence of Ceramic Strength**
- **Case study – Pressure Windows**