1 Introduction

Before the advent of digital methods, a multitude of approaches, theories, and experimental studies had been conducted in order to analyze the behavior of bolted flange joints with metal-to-metal contact beyond the bolt circle. In 1967, Webjörn [1] laid out the first basics of a design method of this type of assembly. Later, Schneider [2] used a beam theory model to solve flanges with metal-to-metal contact. His method was quickly adopted by the American Society of Mechanical Engineers (ASME) Code and forms the basis of Appendix Y [3]. Pinder and Sze [4] focused on experimental methods to determine the influence of the bolts with washers on the flange metal-to-metal contact. The authors have identified the seat preload and rigidity of the bolts as design parameters. They noted that increased values of these parameters reduces the load on the bolts in the operating conditions and increases the contact pressure between the two flanges. Webjörn and Schneider [5] applied their theoretical and experimental findings to another type of metal-to-metal contact flanges known as compact flanges. No sign of separation was observed below a bolt preload of 80% of the elastic limit. The authors noted 6% bolt load change at higher pressure, which lead them to conclude that cyclic pressure below a certain limit does not cause fatigue of the bolts or the deterioration of the joint.

Webjörn [6,7] compared the different metal-to-metal contact flanges with and without the application of external loads, such as the bending moments and misalignment shear loads, taking into account the effect of temperature and corrosion. He concludes that the tension on the bolts increases on average by 5% due to external efforts and offered some practical advice for the design of assemblies. Webjörn [8] stated that the ASME Code flange design is conservative because the elements of the assembly are designed separately. He recommended a comprehensive study that takes into account the interaction between all components. Lewis et al. [9] described a method for determining the initial gap between flat face flanges with initial wedge before applying pressure. They considered various configurations and wide flanges with positive and negative slopes. They observed that, in the case of the flange with a positive slope, the initial rate of leakage increases dramatically with the extension bolts. While in the case of the flange with a slightly negative slope, the leak rate decreases with the initial expansion bolts. Lewis et al. concluded that the leak rate in a slightly negative slope case is less sensitive to the bolt hydrostatic end force and bolt stretch, whereas the leak rate in a slightly positive slope case is very sensitive to the bolt stretch. They added that the quality of the surface flanges plays a crucial role in the seal and has a greater influence than the distortion induced by the crushing of the joint. Fessler et al. [10] conducted a linear elastic finite element deflection study on flanges with wedges. The authors concluded that for flanges with a slightly positive slope, the leak paths are closed, if the bolts are tight enough, and therefore, the leak rate decreases considerably. Hyde et al. [11,12] continued the finite element work of Fessler and added experimental studies to determine the tension on the bolts and the loss of contact pressure between the flanges. They noted that the flanges with a small slope are tighter that those of flat and parallel faces. They mentioned that the effect of the axial load is the most dominant among the efforts applied. This dominance is intensified when the contact surface decreases. Most of the recent work on flanges with metal-to-metal flanges is dedicated toward the study of compact flanges. References [13–16] gave comparative and parametric studies on compact flanges.

A new approach that considers the flange as a plate with a central hole is presented. The one-dimensional aspect of the flange considered by Schneider is to be verified. In addition, referring to Schneider’s work [2], the flange ring could be treated as an annular plate with either clamped ends or simply supported ends, depending on the remaining contact area. This paper compares three models—discontinuous beam model, continuous beam model, and plate model—against the more accurate FE analyses and discusses a few parameters of design importance.
2 Analytical Modeling

The complex structure of the bolted joint may be represented by the structural elements of a known behavior, the free body diagram of which can be drawn with forces and moments applied, as shown in Fig. 1. In the analysis, the gasket located near the bore is soft enough so as its reaction is small compared with the initial bolt load. Applying the force and moment equilibrium together with the continuity of displacement and rotation generates a system of equations that can be used to solve for the unknowns $M, Q, H_B, H_C$, and $M_C$.

First, applying the theory of beams on the elastic foundation [17] to determine the displacement and rotation of the cylinder at the junction with the flange subjected to internal pressure and discontinuity edge loads $Q$ and $M$ yields

$$u_i = \frac{6(1 - v^2)}{E_0\beta^3} Q - \frac{6(1 - v^2)}{E_0\beta^3} M + \frac{(2 - v)B^2}{8E_0} P$$

(1)

$$\theta_i = \frac{12(1 - v^2)}{E_0\beta^3} M - \frac{6(1 - v^2)}{E_0\beta^3} Q$$

(2)

The radial flange displacement considered as a thick cylinder is given by

$$u_f = \frac{B\gamma P}{2E} - \frac{B\gamma Q}{2\eta E} + \frac{r}{2} \theta_i$$

(3)

Establishing compatibility between the cylinder and flange gives a system with two equations and three unknowns $Q, M$, and flange rotation $\theta_i$. The system is statically indeterminate and requires additional equations to solve the problem

$$u_f = u_i$$

(4)

$$\theta_f = \theta_i$$

(5)

When pressure is applied, the hydrostatic end force has the effect of opening the joint. Therefore the bolts are stretched by an additional elongation of $y_B$ after being initially tightened to $y$. The force displacement relationship is given by

$$H_B = \frac{2(y_B + y_B)K_B\eta_B}{\pi C}$$

(6)

The bolt elastic spring $K_B$ is given by

$$K_B = \frac{E_B\eta_B}{L_B}$$

(7)

and the initial bolt stretch is related to the initial bolt load by

$$y_B = \frac{W_B}{A_B E_B}$$

(8)

Using the equations of static equilibrium of force and moment in Fig. 1, the contact force $H_C$ and its location $b$ are expressed as

$$H_C = H_B - H_D$$

(9)

$$h_C = \frac{(H_B K_D - M_C + M + \frac{Q}{2})}{H_B - H_D}$$

(10)

Additional equations are required to determine the separation of the two flanges at any location and especially at the inner edge to facilitate the selection of the diameter of the seal. Schneider [2] applied the discontinuous beam theory to a sector of a flange that has two different widths. In order to validate this assumption for flanges with metal-to-metal contact beyond the bolt circle, it is proposed as a first step to replace the model of the discontinuous beam by a continuous or a discrete beam of a linear varying width, and as a second step, to model the flange as a circular plate with a hole in the center. Obviously, although the equations are slightly more difficult to manipulate, it allows a more precise analytical evaluation of the design parameters.

2.1 Model Based on the Discrete Beam Theory. An expression of the moment of inertia based on the varying width of an element of a unit arc of the flange (Fig. 2) and corrected for the plate effect can be written as

$$E l(x) = \frac{E l^3}{12(1 - v^2)} \left[ \frac{1 - a_0}{L} x + a_0 \right]$$

(11)

with

$$a_0 = 1 + \frac{2L}{B}$$

and

$$L = h_D + h_C$$

(12)

Equation (11) may be expressed as
\[ E I(x) = C_{11} x + C_{12} \]  
with \[ C_{11} = \frac{E t^3}{12(1 - \nu^2)} \left[ 1 - a_0 \right] \] and \[ C_{12} = \frac{E t^3}{12(1 - \nu^2)} a_0 \]

Neglecting the edge moment \( M_c \) at the contact point where the rotation is zero, as in Ref. [2], the moment as a function of the distance \( x \), \( M(x) \) is given by:

\[ M(x) = \frac{H_{p2} h_D + M + Q t}{2} x = C_{13} x \]

with

\[ C_{13} = \frac{H_{p2} h_D + M + Q t}{2} \]

For \( h_c < x < L \):

\[ M(x) = -H_{p2} x + H_{p2} L + M + Q t \]

The rotation is given by integration of the moment over \( E I \). At the inner edge or \( x = L \) this gives:

\[ \theta_{1}(r) = \int_{0}^{r_{c1}} \frac{C_{13} x}{C_{11} x + C_{12}} dx + \int_{r_{c1}}^{L} \frac{C_{13} x}{C_{11} x + C_{12}} dx \]

\[ = \frac{C_{13}(C_{11} h_{c} - C_{12} \ln(C_{11} h_{c} + C_{12}) + C_{12} \ln(C_{12}))}{C_{11}^2} + \frac{C_{14} L C_{13} + \ln(C_{11} L + C_{12}) C_{13} C_{11} - C_{12} C_{14}}{C_{11}^2} \]

\[ + \frac{C_{14} h_{c} C_{11} - \ln(C_{11} h_{c} + C_{12}) C_{13} C_{11} - C_{12} C_{14}}{C_{11}^2} \]

The flange separation or displacement is obtained by double integration of \( M \) over \( E I \) and applying the condition that the rotation is zero at the contact reaction location:

\[ y_{f1} = \frac{C_{13}}{2 C_{11}^2} [h_{c}^2 C_{12}^2 - 2(C_{11} C_{12} h_{c} - C_{12}) \ln(C_{11} h_{c} + C_{12})] \]

\[ + \frac{C_{14}}{2 C_{11}^2} (2 C_{11} C_{12} h_{c} + 1) + \ln(C_{12}) + 2 C_{12}^2 \ln(C_{12}) \]

A system of equations in Eqs. (1)–(6), (9), (10), (19), and (20) is then solved for the unknowns \( M, Q, H_B, H_C, h_C, y_{f1}, u_{f}, \theta_{1}, u_{f}, \) and \( \theta_{1} \).

### 2.2 Model Based on the Plate Theory

Young [18] expressed the deformation \( y_{f}(r) \) and rotation \( \theta_{1}(r) \) of a circular plate with a central hole under different boundary conditions and subjected to different loads (Fig. 3). For a circular plate clamped at the outer edge and subjected to a ring load \( W \) applied at a radius \( r_o \):

\[ y_{f}(r) = y_{f0} + \theta_{f} r F_{1} + M_{r} r^2 D_{1} F_{2} + Q_{f} r^3 D_{1} F_{3} - W r^3 D_{1} G_{3} \]


3 Finite Element Model Used for Validation

ANSYS 3D 20-node solid elements SOLID169 with three degrees of freedom per node [20] were used to model the flange and bolts, as shown in Fig. 4. Because of symmetry of the geometry and loading, only one portion of the flange, delimited by the plane between two bolts and the plane passing through the bolt axis including half of the bolt, was modeled. The axial displacement of the whole flange face contact area was constrained to move down. The initial seating or bolt-up load was applied by imposing an equivalent axial displacement to all bolt nodes that belong to the section lying on the symmetrical plane.

In studying the contact between two bodies, the surface of one body is taken conventionally as a contact surface and the surface of the other body as a target surface. For rigid-flexible contact, the contact surface is associated with the deformable body, and the target surface must be the rigid surface. For flexible-flexible contact, both contact and target surfaces are associated with deformable bodies. The contact and target surfaces constitute a “contact pair.” The CONTA174 contact element is associated with the 3D target segment elements using a shared real constant set number. CONTA174 is an eight-node element that is intended for general rigid-flexible and flexible-flexible contact analyses. CONTA174 is applicable to 3D geometries and is applied for contact between solid bodies or shells.

Comparisons between the two developed analytical models, i.e., the Schneider model and Finite Element Model (FEM) results, were conducted on two flange geometries of 10-in. and 24-in. inside diameter. These are carbon steel integral hubless flanges with dimensions shown in Table 3. A displacement equivalent to 24.7 ksi bolt-up stress was applied to the 10-in. flange before applying an internal pressure.
psi). In the case of the 24-in. flange, a 23 ksi bolt-up stress was applied before applying the internal pressure (50–450 psi). These flanges were used in pairs.

4 Comparison and Discussion of the Results

The parameters for the analysis are the contact force between the flanges, its position relative to the bolt circle, the rotation of each flange, flange separation, and the bolt stress. These parameters are important design indicators that can describe the behavior of the assembly. They are also used to compare the different methods.

Figures 5 and 6 show the contact forces and their locations as a function of pressure for both cases. These two parameters affect the calculation of moments and flange rotation. It is to be noted that for the two flanges the contact force is located between the outer edge and the bolt circle. Therefore, the rotation at the outer edge is zero and the clamped plate case is applicable. The rotation at the bore is shown in Fig. 7 for all three methods. Although the rotations are the same for the cases treated, the analytical model developed can handle assemblies of nonidentical flanges, having different rigidities (thicknesses and materials) for which the rotations will be different. It is worth noting that if the internal pressure of the fluid increases, the contact between the flanges moves to the outer edge of flange and separation will be higher. In the case where the fluid pressure is relatively large, the contact is at the outer edge, and the flange rotation at this location is not null.

Table 3 Flange dimensions in inches

<table>
<thead>
<tr>
<th>B</th>
<th>A</th>
<th>C</th>
<th>t</th>
<th>g₀</th>
<th>g₁</th>
<th>d₂</th>
<th>nₜ</th>
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<tr>
<td>10</td>
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<td>14</td>
<td>1.25</td>
<td>3/8</td>
<td>3/8</td>
<td>1(1/8)</td>
<td>16</td>
</tr>
<tr>
<td>24</td>
<td>32</td>
<td>29.5</td>
<td>2</td>
<td>1(1/16)</td>
<td>1(1/16)</td>
<td>1(1/4)</td>
<td>24</td>
</tr>
</tbody>
</table>

Table 4 shows the comparison of the bore rotation of the two flanges with FEM for internal pressures of 200 psi and 400 psi applied to flanges of 10-in. and 24-in., respectively. Even though the rotations are slightly higher with FEM due to the presence of the holes, the beam and plate theories give good predictions.

On the sealing point of view, separation between the two flanges is the main parameter to determine if the compression on the seal in the operating conditions is adequate. Figure 8 shows flange separation at the bore as a function of pressure for the models compared with FEM and summarized in Table 5. The values obtained by the numerical finite element models are a benchmark to evaluate the analytical models. The models based on plate and continuous beam theories compare well with finite element analysis relative to the continuous beam model. In addition, at the higher pressures, the discontinuous beam model predicts much lower separation than the other two models. This in-
indicates that there will be cases where the modeling of circular flanges by beam theory may not be conservative.

Figures 9 and 10 show the radial distribution of the flange contact stress and separation obtained from the FE study. The separation from the plate model is added for comparison. A liftoff is shown at the bore vicinity during boltup and slightly bigger during operation. The metal-to-metal contact flanges have the advantage of not bending the flanges comparatively to raised face flanges. The contact pressure between the two flanges is much higher at the bolt position than between bolts. Nevertheless, the separation at these two locations is very similar and very close to that predicted by the analytical model based on the plate theory. This suggests that the variation of separation in the circumferential direction is not significant. Obviously, in general, separation is not constant and depends on bolt spacing. Notwithstanding, such information is useful to select the position of the seal based on its ability to spring back. The elastic recovery of the gasket is necessary to fill the space produced by flange separation as a result of the pressure and other external loads. Furthermore, in order to maintain a sufficient force on the seal, it is recommended to locate the latter as close as possible to the bolt hole, where flange separation is relatively smaller compared with that at the bore. However, as a trade off, this may result in a higher hydrostatic end force.

Another parameter worth investigating is the bolt load variation with pressure. It is clear that the bolt load always increases since

<table>
<thead>
<tr>
<th>Flange rotation at bore (deg)</th>
<th>Schneider</th>
<th>Continuous beam theory</th>
<th>Plate theory</th>
<th>FEM</th>
</tr>
</thead>
<tbody>
<tr>
<td>B10</td>
<td>0.0187</td>
<td>0.0182</td>
<td>0.0179</td>
<td>0.0204</td>
</tr>
<tr>
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<td>0.0101</td>
<td>0.0100</td>
<td>0.0097</td>
<td>0.0117</td>
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</table>

<table>
<thead>
<tr>
<th>Flange separation comparison with FEM</th>
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<tr>
<td>Flange (pressure)</td>
</tr>
<tr>
<td>-------------------</td>
</tr>
<tr>
<td>B10 (400 psi)</td>
</tr>
<tr>
<td>B24 (200 psi)</td>
</tr>
</tbody>
</table>
separation causes the bolt to elongate more. It is therefore important to evaluate the amount of increase especially if the bolts are tightened near yield or subject to fatigue. Figure 11 shows the comparison between the three models as the pressure is increased in the two cases. While the increase is somewhat exponential, it remains that there is a good agreement between the three methods. In addition, Table 6 includes the FEM results for validation.

5 Conclusion

An analytical method has been developed to analyze flanged joints with metal-to-metal contact beyond the bolt circle. This method provides additional considerations comparatively to the current method when designing such flanges. The developed model based on the plate theory has the advantage of showing rationality and consistency in the approach of treating metal-to-metal flanges since the same structural theoretical analysis as raised face flanges is used. Besides being more comprehensive, the model based on the plate theory gives more confidence on its ability to predict flange separation and the bolt load increase during operation.

Nomenclature

\[ \beta = \left[ 6(1 - \nu^2) / B g_0 \right]^{1/4} \]

\[ \theta = \text{rotation (rad)} \]
\[ v = \text{flange Poisson ratio} \]
\[ \gamma = \nu + (K^2 + 1) / (K^2 - 1) \]
\[ A = \text{flange outside diameter (in.)} \]
\[ A_B = \text{total bolt stress area (in.}^2) \]
\[ A_p = \text{pressurized area encircled by flange ID (in.}^2) \]
\[ B = \text{flange inside diameter (in.)} \]
\[ C = \text{bolt circle diameter (in.)} \]
\[ d_B = \text{bolt nominal diameter (in.)} \]
\[ D_0 = \text{flange centroid diameter (in.)} \]
\[ E = \text{modulus of elasticity (psi)} \]
\[ g_0 = \text{shell thickness (in.)} \]
\[ g_1 = \text{hub thickness (in.)} \]
\[ H_B = \text{bolt force (lb)} \]
\[ H_C = \text{contact force (lb)} \]
\[ H_D = \text{hydrostatic end force (lb)} \]
\[ h_C = \text{radial distance from the bolt circle to } H_C \text{ (in.)} \]
\[ h_D = \text{radial distance from the bolt circle to } H_D \text{ (in.)} \]
\[ K = \text{ratio of outside diameter to inside diameter of flange } (A/B) \]
\[ K_B = \text{bolt uniaxial stiffness (lb/in.)} \]
Fig. 11  Bolt load increase with pressure

\[ \ell_B = \text{initial effective bolt length (in.)} \]

\[ L = \text{equal to } H_C + H_D \text{ (in.)} \]

\[ M_C = \text{moment per unit circumference at the flange contact (in.lb/in.)} \]

<table>
<thead>
<tr>
<th>Flange</th>
<th>Bolt stress (psi)</th>
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<tbody>
<tr>
<td></td>
<td>Schneider (in.)</td>
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<tr>
<td></td>
<td>Continuous</td>
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<tr>
<td></td>
<td>beam theory</td>
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<tr>
<td></td>
<td>Plate theory</td>
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<tr>
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</tr>
<tr>
<td>B10</td>
<td>24,715</td>
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References

[3] 2007 “Flat Face Flanges With Metal-to-Metal Contact Outside the Bolt Circle,” ASME Boiler and Pressure Vessel Code, Sec. VIII, Div. 1, Appendix Y.