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OPTIMAL DESIGN OF FILAMENT-WOUND COMPOSITE PRESSURE VESSELS

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In this paper, the effect of the width of winding band on the stability of winding pattern is studied. Some key design variables are calculated to optimize the dome geometry. The influence of winding process parameters on the slippage tendency is also considered.

1. Introduction

Composite pressure vessels are being widely used in commercial and aerospace industries, for example in rocket motor cases, fuel tanks, portable oxygen storage bottles, and so on. They offer a high stiffness and strength combined with a low weight and an excellent corrosion resistance. Composite pressure vessels are commonly constructed with a filament overwrap of fiberglass, carbon fiber, or Kevlar in customized resin systems. Various properties can be achieved through an appropriate selection of fiber type, fiber orientation, and resin matrix of the composite structure required for the applications in question. The strong and stiff fibers carry the load imposed on the composite, while the resin matrix distributes the load across the fibers. The process for producing a composite pressure vessel is termed filament winding. The techniques for filament-wound composite pressure vessel can be classified into two main types: geodesic winding and in-plane winding, which consist of three basic steps. First, the fibers are impregnated with a resin. To obtain good results, the impregnation must be done in a carefully controlled manner, by pulling the fibers through a basin filled with the resin. Then, by the use of a winding machine, the fibers are positioned onto a mandrel, which has the shape of the pressure vessel required. Finally, the wet fibers are removed from the winding machine and placed in an oven, where the resin is cured under well-defined conditions of temperature and time. Typical pressure vessels are generally designed with a central cylindrical section and two spherical end caps with optional polar openings. The relative dimensions of different sections of the vessel are designed according to the corresponding space and weight requirements and the pressure levels that the vessel is expected to withstand. Along with thickness and length dimensions, the shape of the end caps also plays a vital role in the design. This is due to the fact that the dome regions undergo the highest stress levels and are the most critical locations from the viewpoint of structure failure. The design concept requires that the composite pressure vessels provide extremely high efficiencies in meeting the overall yielding and buckling failure criteria. Moreover, the slippage tendency of the band at its edges must be taken into account, especially when utilizing the in-plane winding technique.

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Various methods, utilizing analytical and experimental approaches, have been presented for designing the dome shapes of pressure vessels. During the past two decades, several authors have performed detailed analyses of dome shapes by using the theory of orthotropic plates. For example, Young and Lloyd [1] presented an overview of motor-case trade studies and discussed design methods for improving the performance of motor cases. Hofeditz [2] discussed the use of the netting theory and orthotropic analysis methods to solve design problems involving dome shapes. Hojjati et al. [3] developed a technique for designing dome contours based on the theory of orthotropic plates. Lin et al. [4] established design methods for dome shapes based on failure criteria for composite materials and on simple formulas for determining dome contours based on the theory of orthotropic plates. However, the structural efficiencies have recently become the main consideration in designing pressure vessels. Fukunaga et al. [5] described an analytical approach to the optimal design of dome shapes based on the performance factor. But the performance factor is rather complex and unpredictable because of the lack of a maximum burst pressure for the dome structure. Tackett et al. [6] evaluated the effects of shallow dome profiles of carbon pressure vessels on performance efficiencies by using a combined analytical and experimental method. To ensure the stability of winding pattern, Denost [7] presented a new design method for wound vessels subjected to an internal pressure. This method provided a simple solution to the problem of producing wound vessels of different configurations, including the possibility of various openings in the end domes. Lossie and Brussel [8] dealt with the filament winding problem and described some compromises that can be accepted to exploit the advantages of composites despite the winding limitations. Howard and Widera [9] presented a design tool for simple contours and included the effects caused by the use of a wide winding band on the pattern stability and membrane stresses. Di Vita et al. [10] presented a mathematical model for determining non-slip winding paths for composite structures of general shape.

Since filament-wound composite pressure vessels tend to fail in their dome parts, the design of these parts is the most important issue for such vessels. A number of factors must be taken into account in designing dome contours, including the strength of the materials selected, the effect of end openings, the winding stability, geometric variables, and so on. The winding stability and dome shapes must be chosen carefully to obtain an optimal design. Several previous studies have examined the optimal design of composite pressure vessels, but the effects caused by the width of the winding band on the stability of winding pattern in the dome have not been studied. Therefore, the aim of this study is to optimize the design of composite vessels operating under an internal pressure. A practical design example is presented and investigated by using an optimal design procedure. The design variables used in the optimization problem include the ratio a/b of major and minor axes of the dome part, the winding angle, and the band width. The effects of winding process parameters on the slippage tendency at the edges of the band are also considered. The results presented may prove to be helpful for designers of composite pressure vessels. In addition, the procedure employed in this study can also be utilized during the primary design stage.

2. Mathematical Model

Figure 1 illustrates a composite pressure vessel. A cylinder and two domes, both filament-wound, are the principal components of the vessel. This study focuses mainly on the dome shell, which is suitable for the end closure of a typical pressure vessel. Our aim is to establish an optimal design procedure for the dome shape of composite pressure vessels. Figure 2 shows a design flowchart. The geometry of the dome shell is shown in Fig. 3. Since the design configuration depends on the winding technique used, we will consider the in-plane winding. The pre-assigned design parameters are the internal radius of the cylindrical section r_{cyl}, the internal pressure P, the radius of the polar opening of end caps r_0, the height of the metal adaptor h_c, and the material properties of the wound filament, which are all considered constant. Based on these assumptions, we calculate the optimal geometry, winding angle, and band width, with account of a failure criterion, limitations on the slippage tendency, and the necessary strength of the dome. Mathematically, the optimization problem consists in maximizing an objective functions $F(x)=F(x_1,x_2,\ldots,x_n)$ with constraints $G_k(x_1,x_2,\ldots,x_n)\le 0$, $k = 1, 2, \ldots, m$, where x_i, $1\le i\le n$, are the design variables.

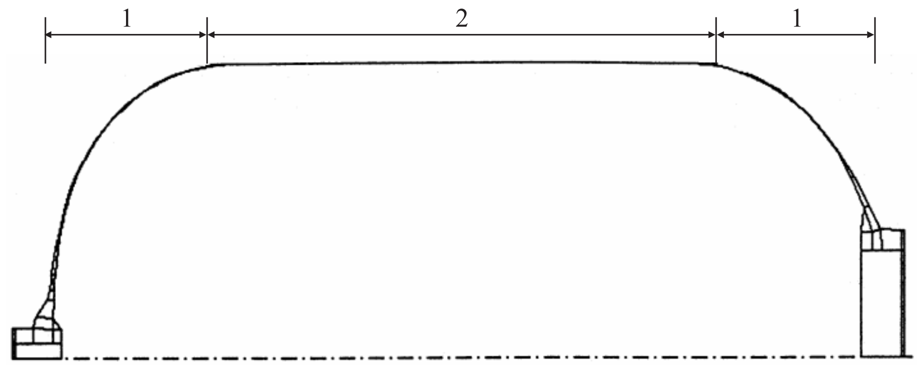

Fig. 1. Geometry of a composite pressure vessel: 1 — dome; 2 — cylinder.

2.1. Objective function. Composite pressure vessels are expected to withstand a maximum internal pressure at a maximum internal volume and a minimum weight. Therefore, the evaluation criterion is PV/W, where P, V, and W are the internal pressure and the volume and weight of the shell, respectively. Thus in this study, we assume that $F(x) = PV/W$.

The internal pressure P is assumed to be constant in this study. The internal volume V can be determined by the formula

$$V = \sum_{i=1}^{n} \pi \left[\frac{(r_{i+1} + r_i)}{2} \right]^2 \Delta x,$$

and the weight of the fiber overwrap at the dome regions is

$$W = \sum_{i=1}^{n} 2\pi \left[\frac{(r_{i+1} + r_i)}{2} \right]^2 \left[\frac{(t_{d,i+1} + t_{d,i})}{2} \right]^2 \Delta x \rho,$$

where r_i is the radius of the structure, $t_{d,i}$ is the fiber thickness, Δx is the length per unit element, and ρ is the specific weight of fibers.

2.2. Design variable. The internal radius of the cylindrical section r_{cyl}, the internal pressure P, the radius of polar openings r_0, the height of the metal adaptor h_c, and the material properties are fixed. As design variables for the optimization problem, we take the ratio a/b between the major and minor axes of the dome part, the winding angle α_c, and the band width BW.

2.3. Design constraints. The design constraints are derived from the requirements associated with the method of manufacture of the vessel, the dome geometry, the internal pressure, the strength of the structure, the slippage tendency, and the failure criterion. Let us consider these factors one by one.

(1) Winding angle.

In this study, the in-plane winding technique with the use of a machine for polar winding is considered. The winding angle α_c at the cylindrical parts is closely related to the winding capability of the machine. An excessive winding angle leads to a considerable fiber slippage, therefore, this angle must be limited.

(2) Dome shape.

When the dome shape is flattened, the internal volume and weight of the pressure vessel increase, thus reducing the bursting pressure and the efficiency of the structure. Moreover, when the overall length/ diameter ratio of the pressure vessel exceeds two, the slippage tendency increases, which explains why the dome shape must be neither too flat nor too peaked. A sphere or a semisphere is the best design choice.

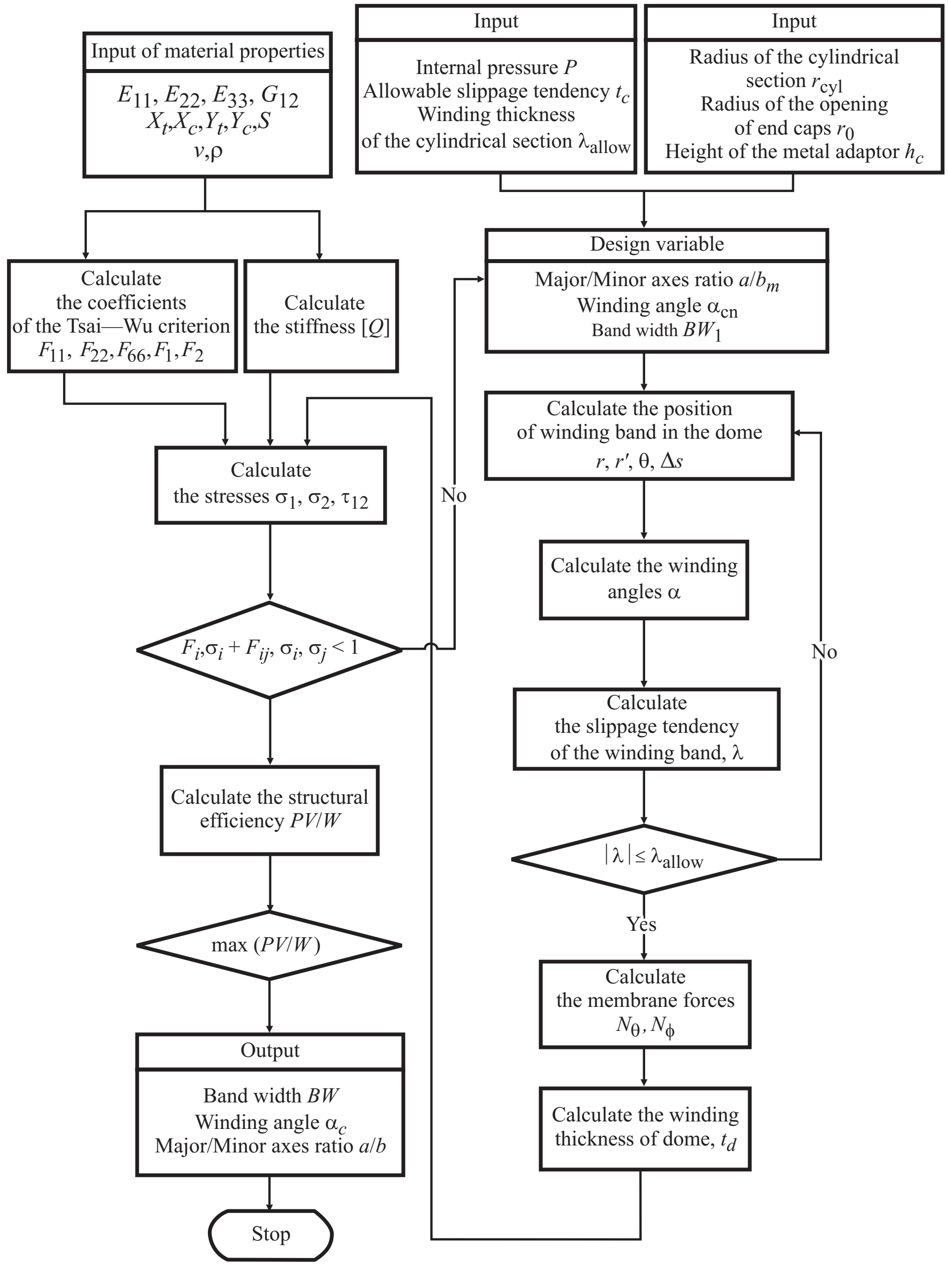

Fig. 2. Procedure for solving the optimization problem considered.

(3) Limitations on the internal pressure.

Generally, the major load imposed on pressure vessels is an internal pressure P, and its maximum tolerable value P_{max} determines the performance needed. The pressure required P_r must not exceed this value, $P_{max} \geq P_r$.

(4) Allowed slippage tendency.

The allowed slippage tendency, which is used as a basis for checking whether the fiber slippage occurs during the winding process, is intimately related to the required smoothness of mandrel surface, the selection of materials, the winding techniques, and the manufacture process. To prevent fibers from slipping during winding, the maximum slippage tendency λ_{max} of the winding band must be below the allowed value λ_{allow}, $\lambda_{max} < \lambda_{allow}$.

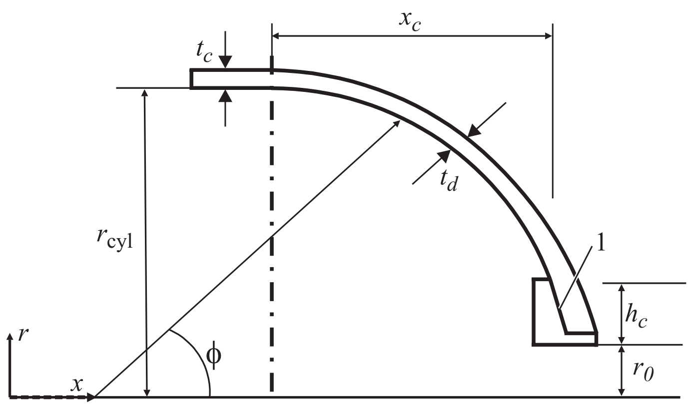

Fig. 3. Geometry of the dome shell: 1 — ellipse–cone junction.

TABLE 1. Properties of the AS4-GP/EPON9405/CA9470 Composite Material

Density ρ			1570 kg/m^3
Young's moduli	E_{11}		147 GPa
	E_{22}		9.66 GPa
	E_{33}		9.66 GPa
Shear modulus G_{12}			4.97 GPa
Poisson ratio ν			0.3
Longitudinal tensile strength X_t			2.3 GPa
Longitudinal compressive strength X_c			1.19 GPa
Transverse tensile strength Y_t			53.9 MPa
Transverse compressive strength Y_c			53.9 MPa
In-surface shear strength S			96.04 MPa
Fiber volume fraction v_f			0.75
Single-layer thickness h_0		Plane winding	0.255 mm
		Hoop winding	0.235 mm

(5) Failure criterion.

The safety of the structure is determined by a failure criterion. For our purposes, the Tsai–Wu criterion was assumed:

$$F_i \sigma_i + F_{ij} \sigma_i \sigma_j < 1,$$

$$F_{11} = \frac{1}{X_t X_c}, \quad F_{22} = \frac{1}{Y_t Y_c}, \quad F_{66} = \frac{1}{S^2}, \quad F_{22} = -\frac{1}{2\sqrt{X_t X_c Y_t Y_c}},$$

$$F_1 = \frac{1}{X_t} - \frac{1}{X_c}, \quad F_2 = \frac{1}{Y_t} - \frac{1}{Y_c}, \quad F_6 = 0,$$

where X_t and X_c are the longitudinal tensile and compressive strengths, Y_t and Y_c are the transverse tensile and compressive strengths, and S is the interfacial shear strength.

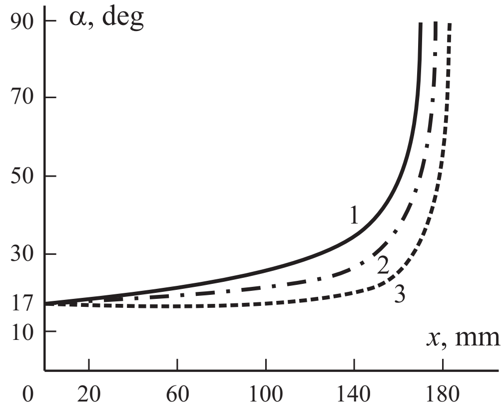

Fig. 4. Winding angles in the dome region of the laminated shell: outside (1); in the middle (2); inside (3).

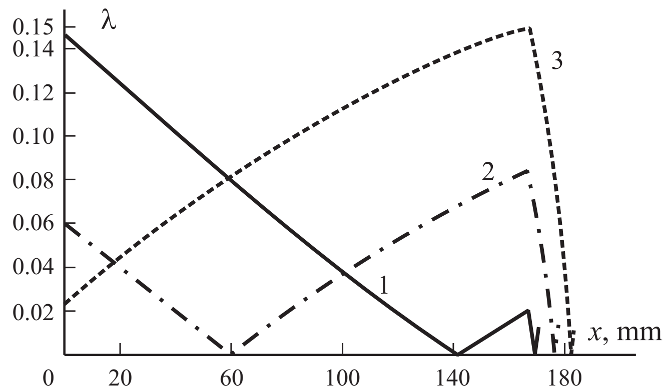

Fig. 5. Slippage tendency of the winding band over the dome shell: outside (1); in the middle (2); inside (3).

3. Optimal Design Example of a Composite Pressure Vessel

The dome shape of composite pressure vessels changes depending on the winding patterns used. In the present study, the in-plane winding technique with the use of an AS4-GP/EPON9405/CA9470 high-strength composite material is considered. Table 1 lists the main properties of the material.

The radius r_c of the cylindrical section of the mandrel is 250 mm, and the radius r_0 of end openings is 50 mm. The metal boss at the polar opening is made of a homogeneous metal, and its height h_c is 50 mm.

The imposed internal pressure P is 11.72 MPa. The winding thickness at the cylindrical section is 1.72 mm.

The design parameters a/b, BW, and α_c have the following ranges: $1.00 \le a/b \le 2.00$, $1\text{ mm} \le BW \le 50\text{ mm}$, and , $0° \le \alpha_c \le 45°$.

In view of the fact that the friction coefficient in the wet winding adopted was 0.2, the allowed slippage tendency was limited to 1.5.

The tensile strength of fibers of the composite is 2.3 GPa, and stresses in the pressure vessel are limited by the Tsai–Wu strength criterion.

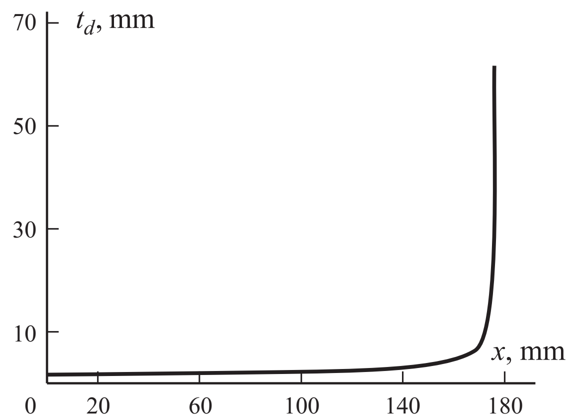

Fig. 6. Variations in the shell thickness t_d over the dome region.

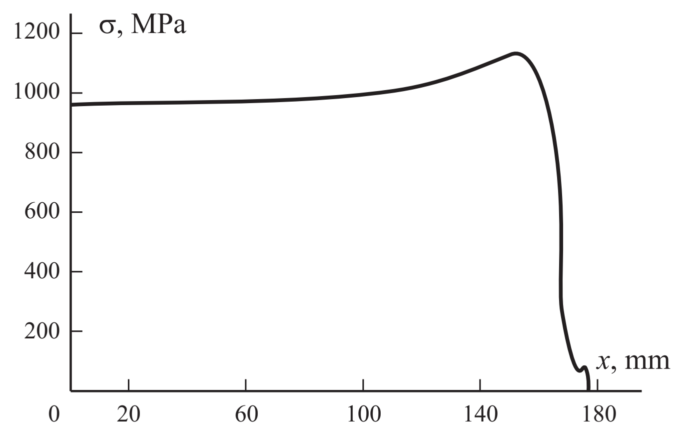

Fig. 7. Longitudinal stresses σ in fibers in the dome region.

4. Results and Discussion

By using the procedure for optimizing the filament-wound composite pressure vessels shown in Fig. 2, it was found that the optimal design point is $(a/b, BW, \alpha_c) = (1.37, 42\text{ mm}, 17°)$, where the structural efficiency PV/W reaches its maximum value.

Figure 4 illustrates the distribution of the winding angle at every circumference point of the dome. This angles was increased from 17° at the dome/cylinder interface to 90° at the polar opening of the dome.

Both inside and outside of the winding band, the entire fiber band completely meets the allowed slippage tendency, which is below 0.15, as shown in Fig. 5. The maximum slippage tendency occurs inside of the winding band close to the openings and is equal to 0.1492.

Figure 6 shows variations in the shell thickness t_d over the dome region. Since the radius of meridional curvature of the pressure vessels and the fiber thickness decrease gradually away from the dome/cylinder interface toward the end opening, the shell thickness also increases in the x-axis direction. Since the winding angle near the polar opening of the dome region approaches 90°, the shell thickness tends to infinity due to fiber accumulation. However, this thickness should be made constant to be connected with other components in practical applications.

Figure 7 shows that the longitudinal stress in fibers increases markedly at the ellipse-cone junction and close to the end opening. The maximum of this stress is 1.1 GPa, which is below the longitudinal tensile strength of fibers 2.3 GPa, and thus the strength criterion if satisfied.

5. Conclusions

Thanks to their high specific strength and stiffness, light weight, and excellent quality, filament-wound composite pressure vessels are attractive for various applications in military, aerospace, and commercial industries. The in-plane winding technique considered in this study provides a simple but effective way of creating pressure vessels with various ends. However, fibers tend to slip during the winding process, which adversely affects the performance of the vessels. Therefore, the problem of assuring the stability of winding pattern and avoiding the fiber slippage is of great concern. The analyses in this study were focused on the application of in-plane winding technique to the dome shape design to prevent the fiber slippage during the winding process and to determine an appropriate band width. The design procedure considered allowed us to minimize the number of input parameters and thus to save considerable financial and labor costs.

REFERENCES

1. M. H. Young and B. A. Lloyd, "Rocket case performance optimization," in: 17th National SAMPE Technical Conference (1985).
2. J. T. Hofeditz, "Structural design considerations for glass pressure vessels," in: 18th Annual Meeting of the Reinforced Plastics Division (1963).
3. M. Hojjati, A. V. Safavi, and S. V. Hoa, "Design of dome for polymeric composite pressure vessels," Compos. Eng., **5**, No. 1, 51-59 (1995).
4. Y. C. Lin and W. C. Hwang, "Design of dome contour for filament-wound rocket motor cases," Trans. Aeronaut. Astronaut. Soc. R. O. C., **27**, No. 1, 61-70 (1995).
5. H. Fukunaga and M. Uemura, "Optimum design of helically wound composite pressure vessels," Compos. Struct., **1**, 31-49 (1983).
6. E. W. Tackett, G. A. Kulkarni, and S. B. Kulkarni, "Carbon pressure vessel performance with changing dome profiles," in: AIAA/SAE/ASME 20th Joint Propulsion Conference (1984).
7. J. P. Denost, "New design concepts for filament-wound pressure vessel with unequal polar openings," in: AIAA/SAE/ASME 18th Joint Propulsion Conference (1982), pp. 1-7.
8. M. Lossie and H. Van Brussel, "Design principles in filament winding," Compos. Manufact., **5**, No. 1, 5-13 (1994).
9. W. E. Howard and G. E. O. Widera, "Design and analysis techniques for filament-wound composite pressure vessel domes," Polymer. Syst. ASME, **88**, 57-65 (1999).
10. G. DiVita, M. Marchetti, P. Moroni, and P. Perugini, "Designing complex-shape filament-wound structures," Compos. Manufact., **3,** No. 1, 53-58 (1992).