

11th US NATIONAL CONGRESS  
ON COMPUTATIONAL MECHANICS

# Tailoring Topology Optimization to Composite Pressure Vessel Design with Simultaneous Consideration of Fiber Angle and Material Distribution

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Acknowledgments:



# Outline

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- Introduction
- Motivation and Objective
- Multilayer Composite Shell FEM
- Topology Optimization Method
- Design problem formulation
- Results
- Conclusions and future work



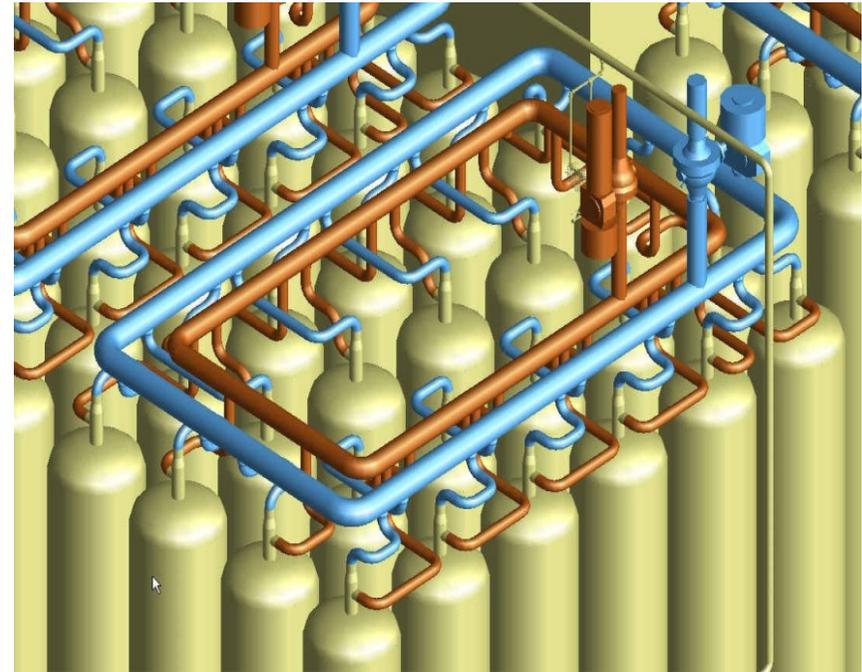
# Introduction – CNG Pressure Vessels

## Compressed Natural Gas (CNG) Pressure Vessels



CNG pressure vessels

CNG Cargo Containment System



CNG inside cars and buses

CNG – More attractive costs for compression, loading and unloading



# Introduction – CNG Pressure Vessels

## Comparison of CNG and LNG (Liquefied Natural Gas)

### Size of investment for a 500MMscf/d plant

	CNG	LNG
Reserves:	Modest	Large
Processing cost:	MM\$30-40	MM\$750-2000*
Transportation costs:	MM\$230/ship	MM\$160/ship
Unloading costs:	MM\$16-20	MM\$500-550
Total investment:	\$1-2 billion**	\$2-3 billion**

\* Depending upon the location of the production site

\*\* Depending upon the number of ships used for the transport of the gas.

MMscf/d - million standard cubic feet per day

MM\$ - million dollars

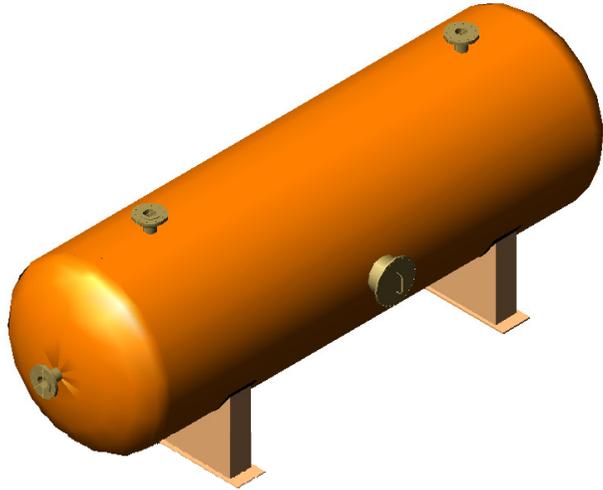
<http://www.hebrewenergy.com/terms/>



# Introduction – CNG Pressure Vessels

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## Types of CNG Pressure Vessels



Homogeneous (stainless steel)

## Composite CNG Vessels



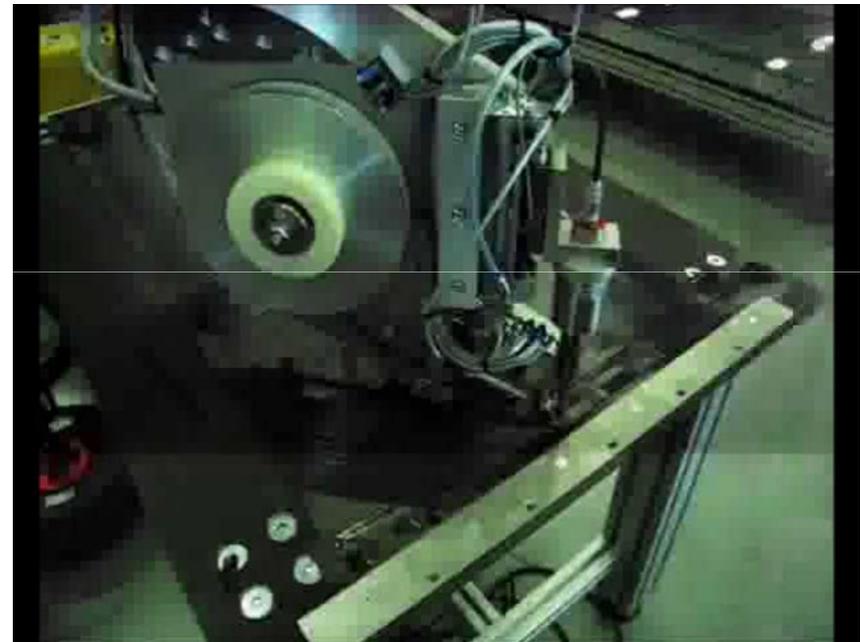
# Introduction - Motivation

## Manufacturing Processes for CNG Pressure Vessels

Winding (traditional) – entire vessel layer has same fiber angle



Fiber Placement – same layer can have different fiber angles



Fiber placement → it allows manufacturing of more complex composite vessel configurations with different topology and fiber angle distribution → topology optimization method



# Introduction - Motivation

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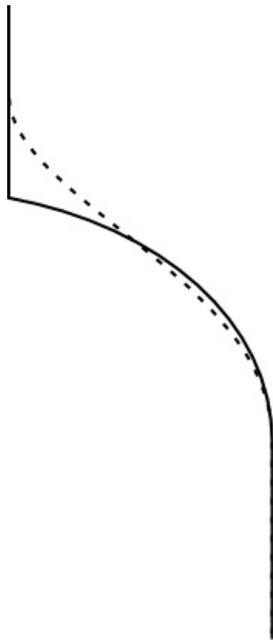
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Costs of CNG vessel transport are too high → mass of gas/mass of vessel ratio is low → Optimization techniques can be applied to design pressure vessels in order to increase the ‘mass of gas/mass of vessel’ ratio. However, usually only parametric or shape optimization have been applied to improve the design of pressure vessels.

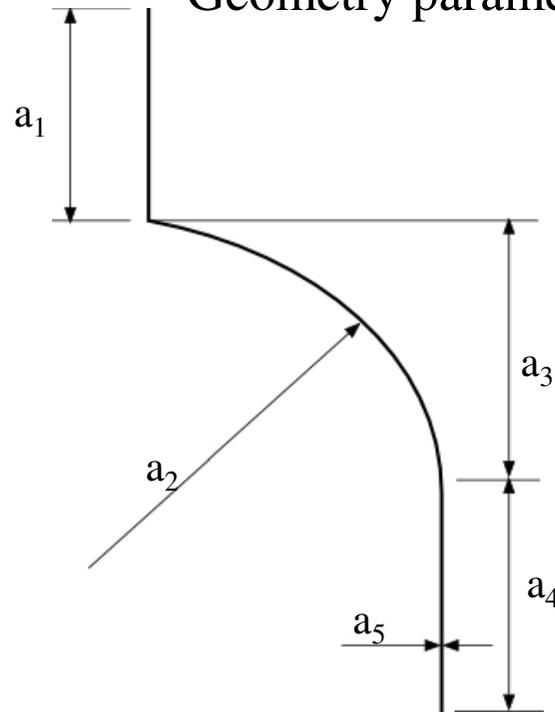


# Introduction - Motivation

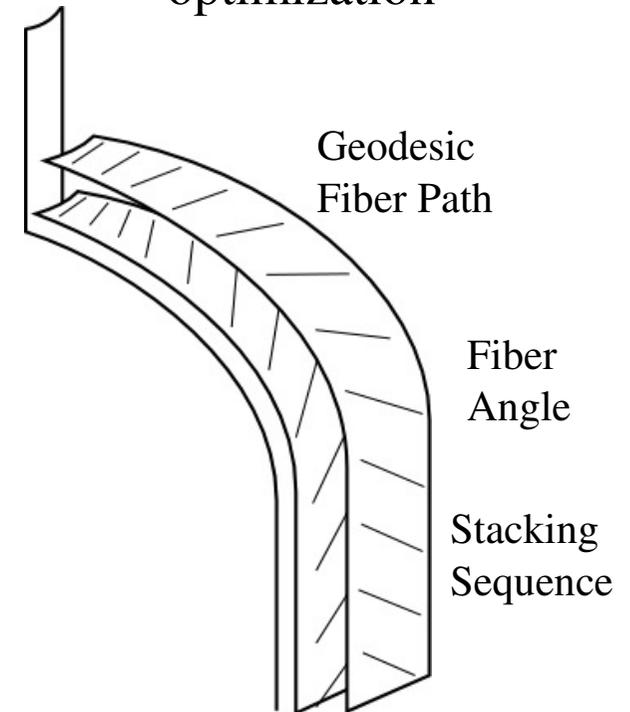
Shape optimization  
by splines



Parametric optimization  
Geometry parameters



Shape + Parametric  
optimization



- Homogeneous tanks: Zhu and Boyle 2000; Carbonari et al. 2011
- Topology nozzle optimization: Liu et al. 2001
- Composite tanks: Fukunaga and Uemura 1983; Vafaesezat 2009; Kim et al. 2005

Winding → Geodesic Contour:  
Shortest path between two  
points on a surface → no  
slippage tendency between the  
fiber and the mandrel during  
manufacturing.



# Objective

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To develop a topology optimization formulation for designing composite pressure vessels considering the optimization of:

- Material Distribution
- Fiber local orientation

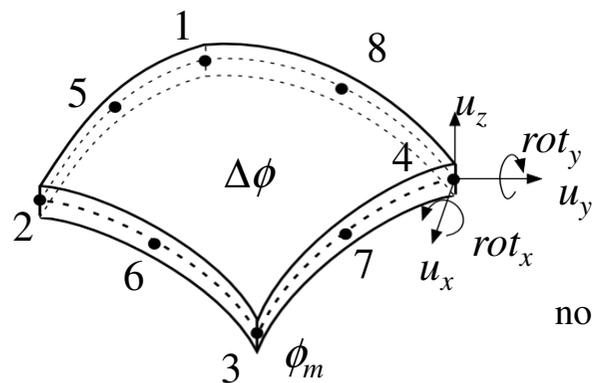
to minimize Tsai-Wu P-norm subjected to composite volume constraint.



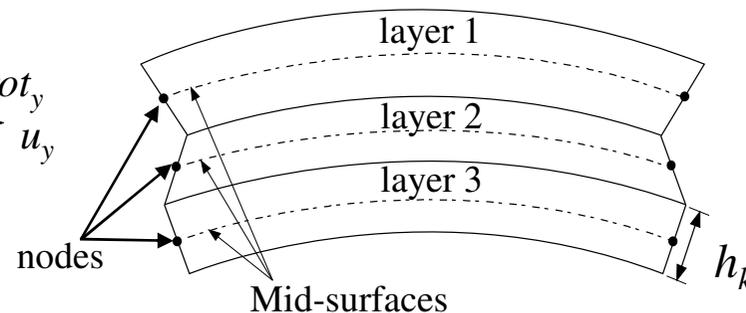
# Finite Element Method

- Shell element – first order shear deformation theory based on the 3D solid degenerated element and the Reissner-Mindlin kinematical assumptions;
- 8-node shell element with 5 nodal dofs – 3 translations and 2 rotations;
- “Layer-wise” theory to model laminated structures ➡ more accurate results.
- Composite layers - fibers are oriented to improve the performance of the structure

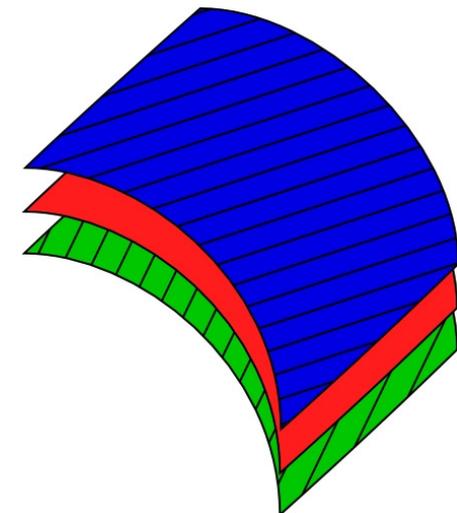
(Reddy, 2002; Ahmad et al, 1970; Kögl & Bucalem, 2005)



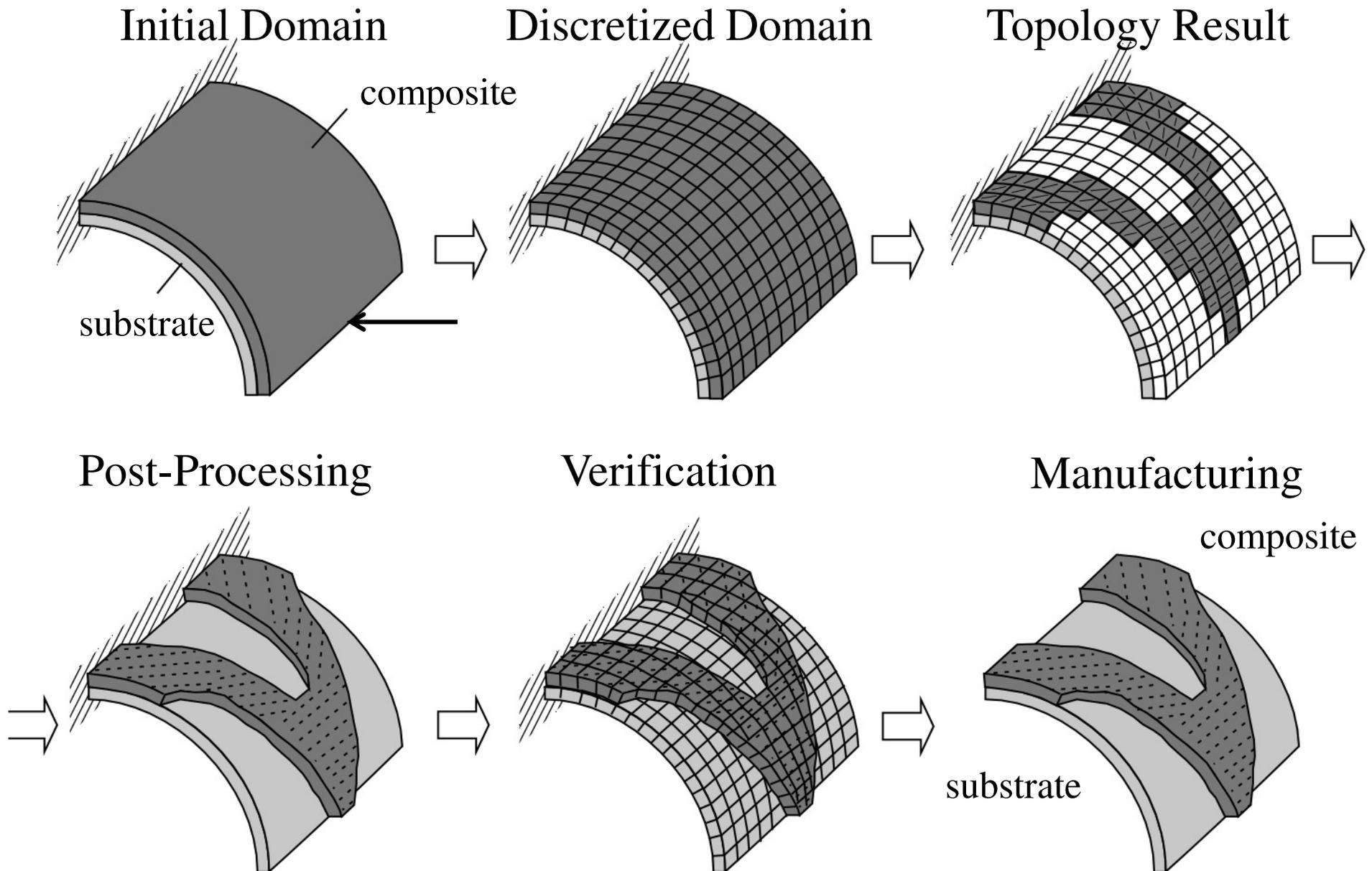
Layer-wise displacement field



Multilayer Composite Shell Structure



# Topology Optimization Method



# Material Model

## Material distribution model

### SIMP

(Simple Isotropic Material  
with Penalization)

$$\mathbf{C}_e = \gamma_e^{p_c} \mathbf{C}_0$$

(Rozvany, 2001)

## Fiber orientation model

### DMO

(Discrete Material Optimization)

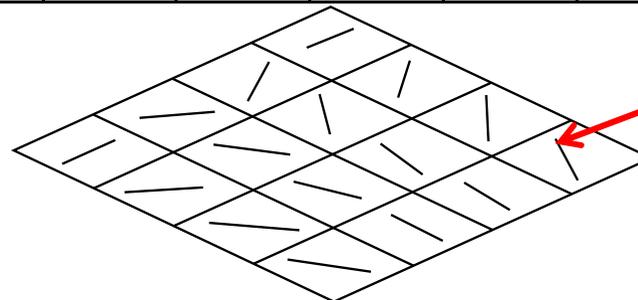
$$\mathbf{C} = \sum_{i_c=1}^{n^c} w_{i_c} \mathbf{C}_{i_c} = w_1 \mathbf{C}_1 + w_2 \mathbf{C}_2 + \dots + w_{n^c} \mathbf{C}_{n^c}$$

$$w_{i_c} = \frac{\hat{w}_{i_c}}{\sum_{j=1}^{n^c} \hat{w}_j}, \quad \text{where } \hat{w}_{i_c} = (\theta_{i_c}^e)^{p_\theta} \prod_{l=1, l \neq i_c}^{n^c} (1 - (\theta_{i_c}^e)^{p_\theta})$$

(Lund, 2009)

## Material Candidates – 13 angles

0°	-85°	-75°	-60°	-45°	-30°	-15°	15°	30°	45°	60°	75°	85°
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13 orientation variables  
 $\theta_i^e$  at each element



# Numerical Implementation

## Nodal Pseudo Density

○ Pseudo Density  $\gamma_k$

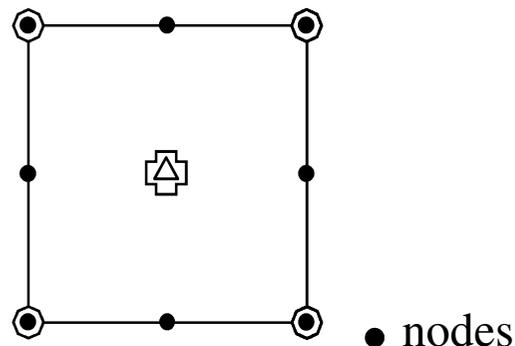
$$\gamma_e = f(\gamma_k)$$

## Nodal Variable

○ Topology Variable  $d_k$

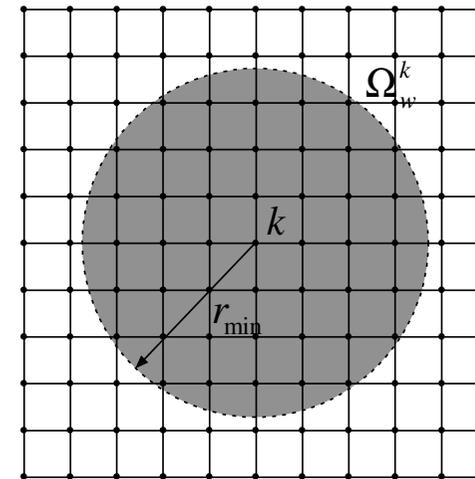
## Element Variables

△ Orientation Variable  $\theta_{i_c}^e$

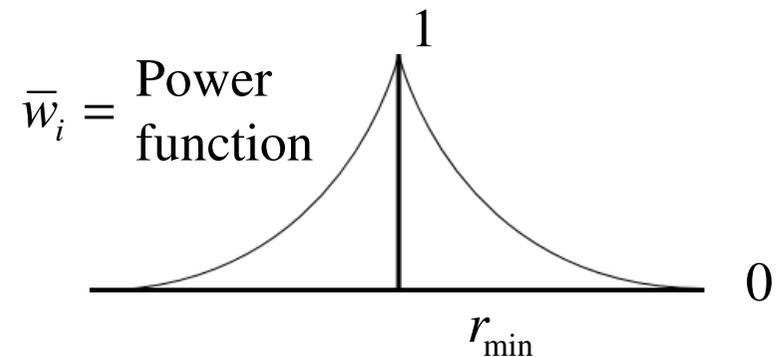


## Projection technique

(Guest et al., 2004)



$$\gamma_k = f(d_i, \bar{w}_i) \quad \tilde{\theta}_{i_c}^e = f(\theta_{i_c}^e, \bar{w}_i)$$

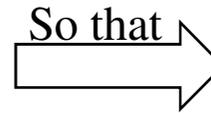


(Almeida et al., 2010)



# Problem Formulation

Minimize  $P$ -norm of Tsai-Wu failure criteria at composite layers



Internal Pressure can be increased

$P$ -norm

$$\mathfrak{S} = \left( \sum TW_i^p \right)^{1/p} \quad i = \text{Gauss points at each element}$$

Tsai - Wu

$$TW_i = Y_1 \sigma_{11_i} + Y_2 \sigma_{22_i} + Y_{11} \sigma_{11_i}^2 + Y_{22} \sigma_{22_i}^2 + 2Y_{12} \sigma_{11_i} \sigma_{22_i} + Y_{66} \tau_{12_i}^2 \leq 1$$

$$Y_1 = \frac{1}{F_{11}^T} - \frac{1}{F_{11}^C} \quad ; \quad Y_2 = \frac{1}{F_{22}^T} - \frac{1}{F_{22}^C} \quad ; \quad Y_{11} = \frac{1}{F_{11}^T F_{11}^C}$$

$$Y_{22} = \frac{1}{F_{22}^T F_{22}^C} \quad ; \quad Y_{66} = \frac{1}{F_{66}^2} \quad ; \quad Y_{12} = -\frac{\sqrt{Y_1 Y_2}}{2}$$

$F_{11}^T, F_{22}^T$  Traction failure stresses along directions 1 and 2

$F_{11}^C, F_{22}^C$  Compression failure stresses along directions 1 and 2

$F_{66}^2$  Shear failure stress

Minimize  $\mathfrak{S}$   
 $d_i \quad \theta_i^e$

Subject to:  $\int_{Volume} \gamma_e dV \leq V_{frac}$

$$0 < d_i \leq 1$$

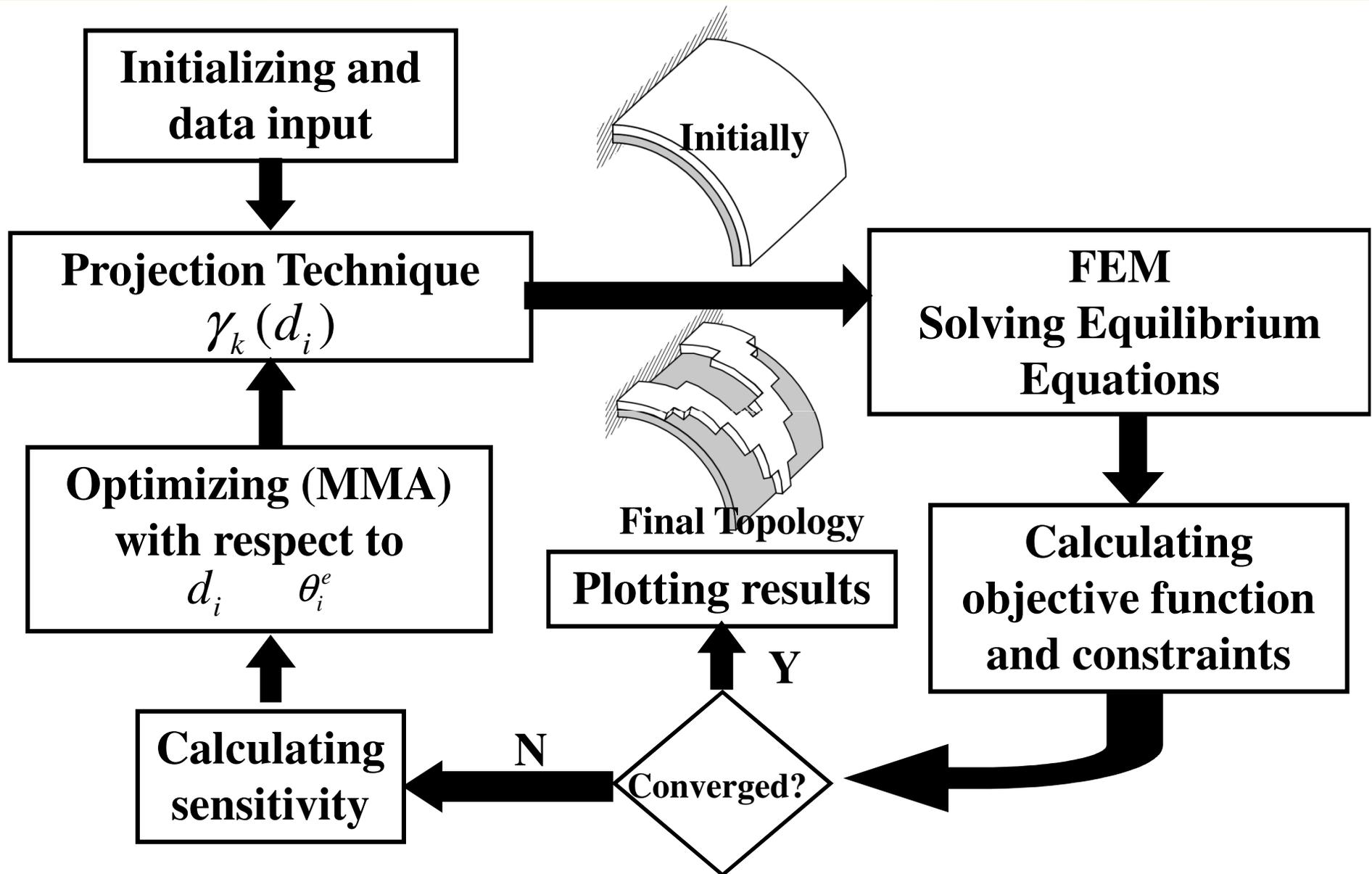
$$0 \leq \theta_i^e \leq 1$$

Solved using MMA  
(Svanberg, 1987)

(Pereira et al. 2003, Le et al. 2010)



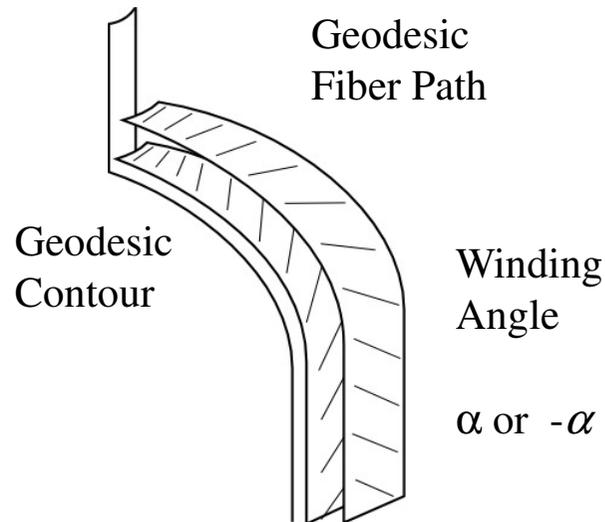
# Topology Optimization Flowchart



# Problem Formulation – Two Approaches

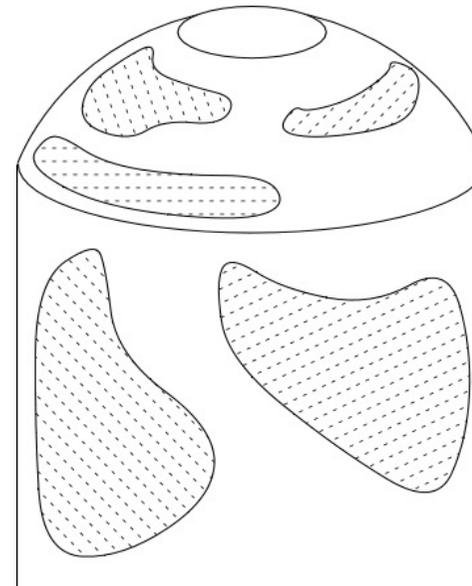
## Case 1 – Winding

- Winding Angle Optimization;
- No material is removed ;
- Geodesic Contour is updated to fit the geodesic fiber path according to winding angle ( $R\sin\alpha = cte.$ );
- Winding angle of alternated layers are equal to  $\alpha$  or  $-\alpha$  ;
- Cyclic Symmetry is considered for FEM model.



## Case 2 – Fiber placement

- Topology and fiber angle optimization (simultaneously);
- Vessel contour does not change;
- Resultant angles can be different for each finite element;
- Entire FEM model considered (more generic).

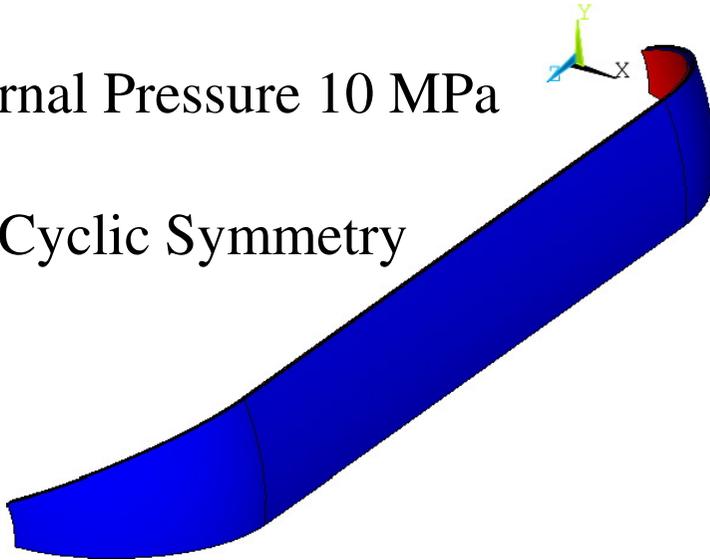


# Problem Design – Case 1

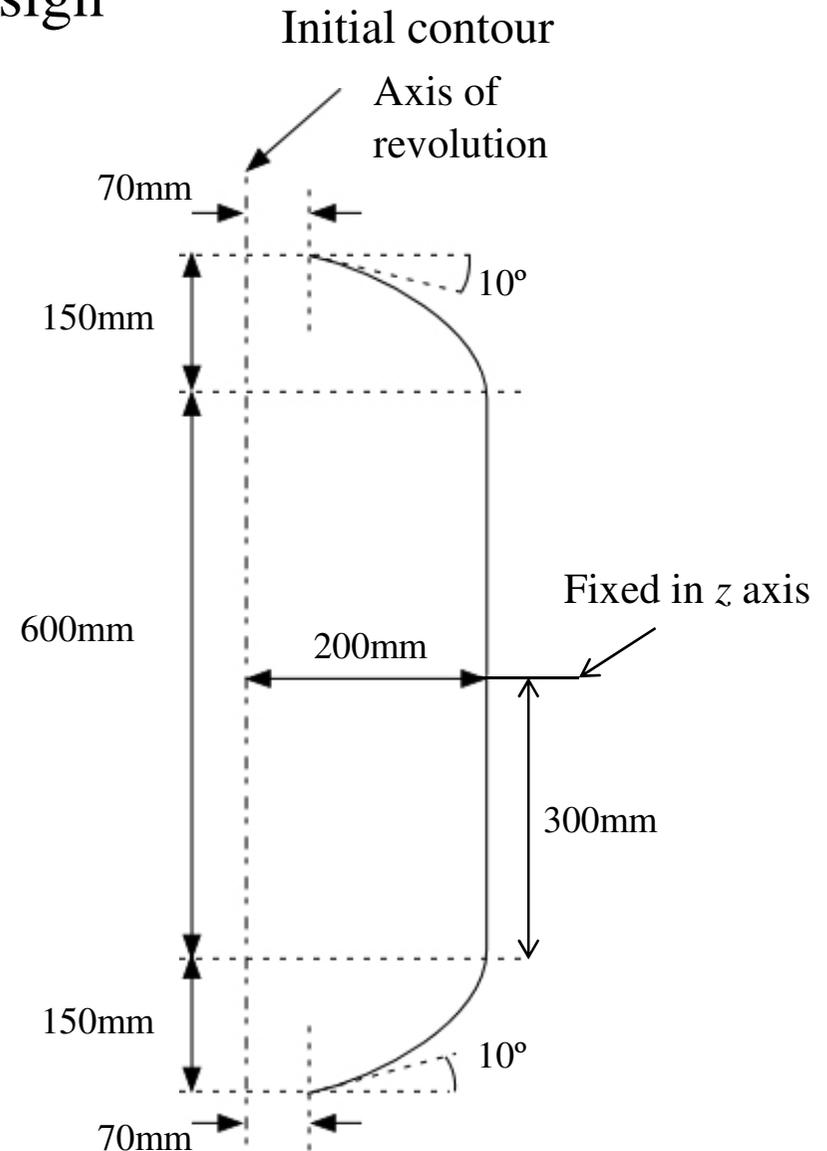
## Winding-type Design

Internal Pressure 10 MPa

Cyclic Symmetry



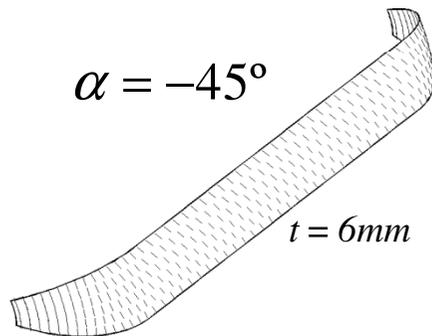
- ‘inner’ layer – Aluminum – red –  $t = 4mm$
- ‘other’ layers – Glass Fiber – blue –  $t = 6mm$   
(sum of all layers thickness is kept constant)
  - 3 examples explored
    - 1 composite layer
    - 2 composite layers
    - 3 composite layers



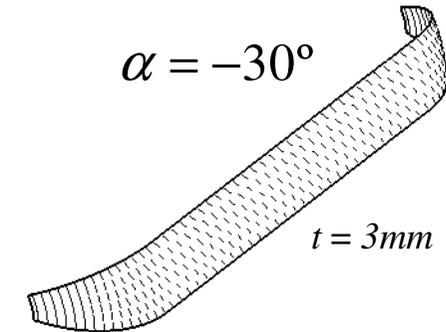
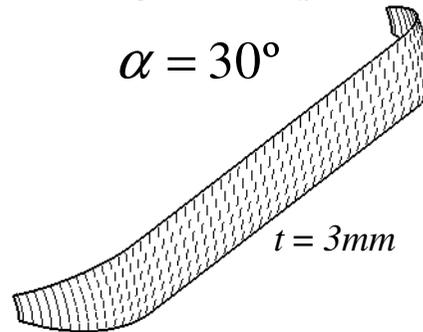
# Results – Case 1

N° composite layers	Optimal winding angle (discrete)	Maximum Tsai Wu value for 10 MPa	Maximum optimized internal pressure
1 (not optimized)	0° (hoop direction)	0.76	11.9 MPa
1	-45°	0.11	22.8 MPa
2	30°	0.21	17.9 MPa
3	30°	0.12	22.5 MPa

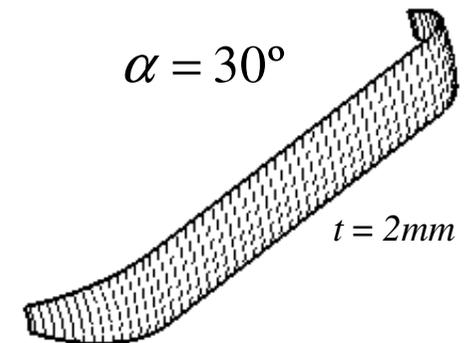
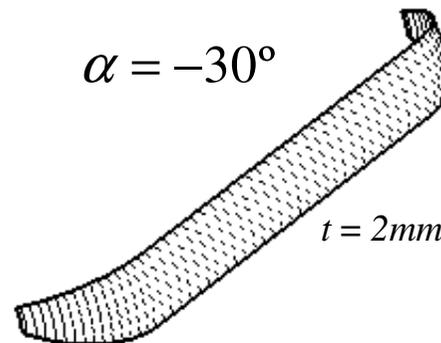
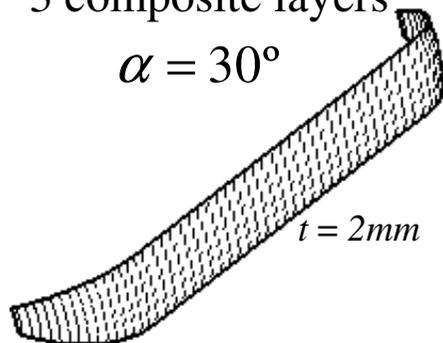
1 composite layer



2 composite layers

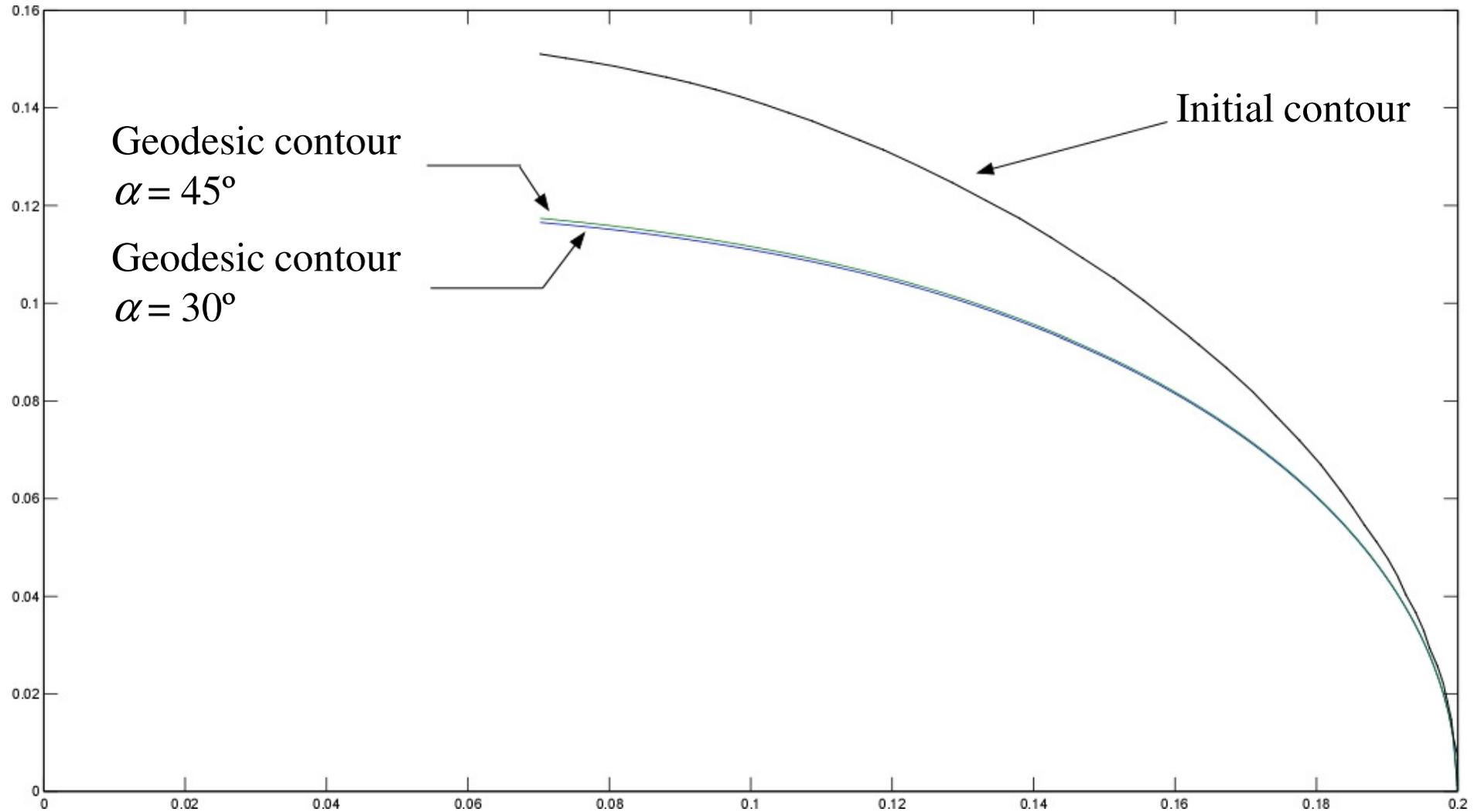


3 composite layers



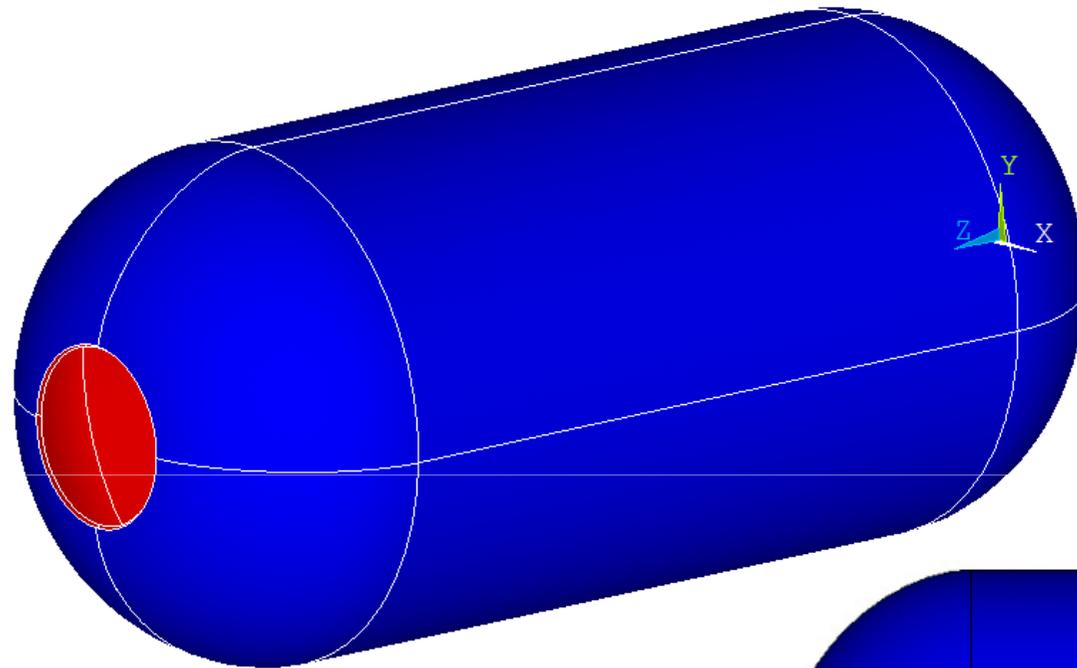
# Results – Case 1

## Dome Contour



# Problem Design – Case 2

## Fiber Placement-type Design

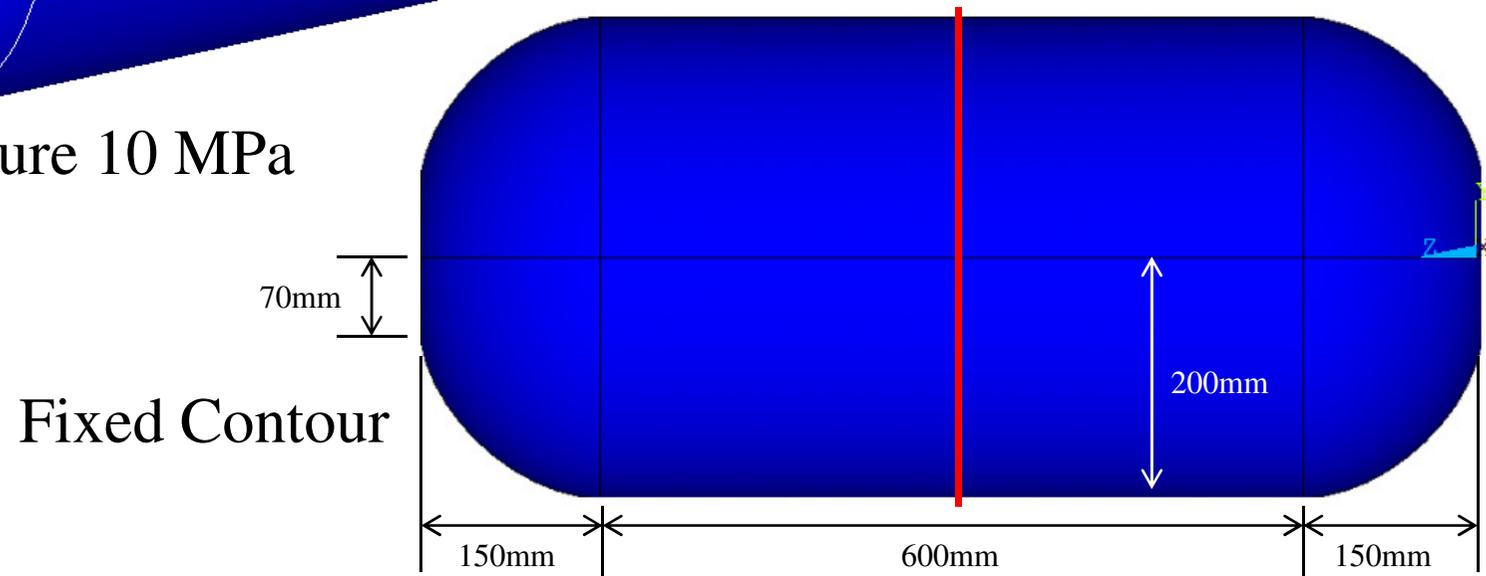


Internal pressure 10 MPa

2 Layers

- ‘inner’ layer – not optimized  
Aluminum – red –  $t = 4mm$
  - ‘outer’ layer – design domain  
Glass Fiber – blue –  $t = 2mm$
- Initial mass: 11Kg  
50% volume constraint

Fixed in  $z$  axis

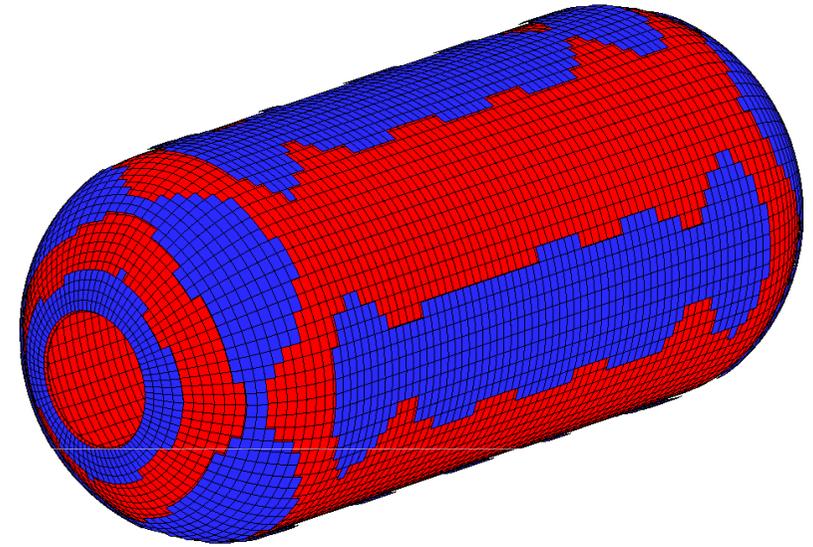
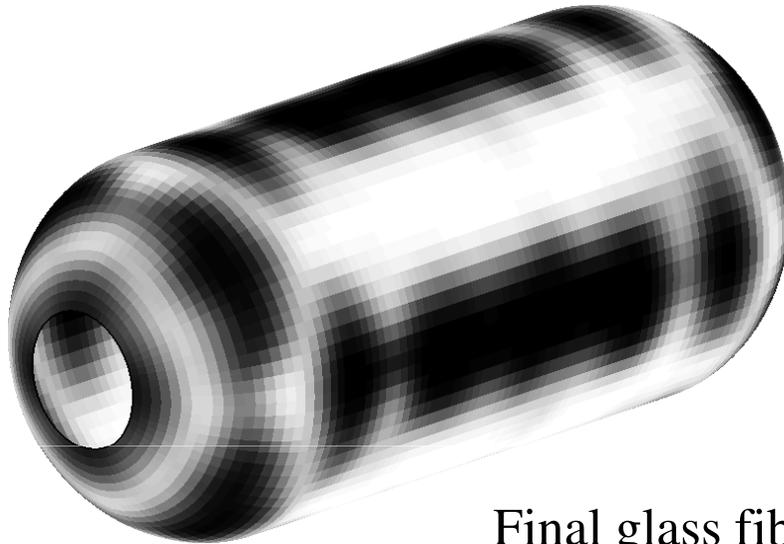


# Topology Result – Case 2

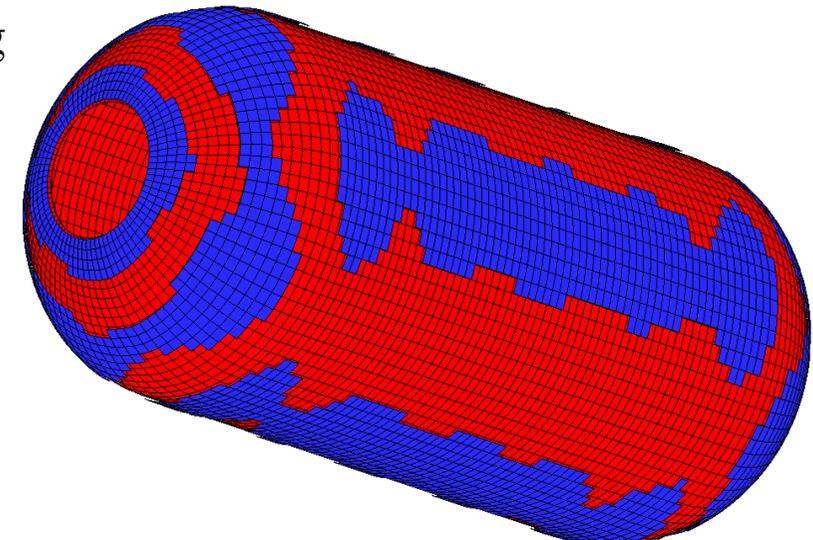
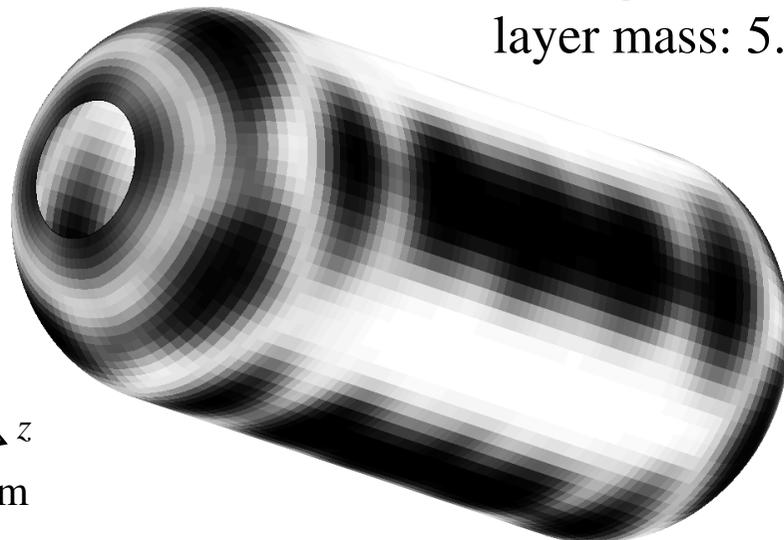
Final mesh topology

Post-Processed Topology

View 1  
Top-front



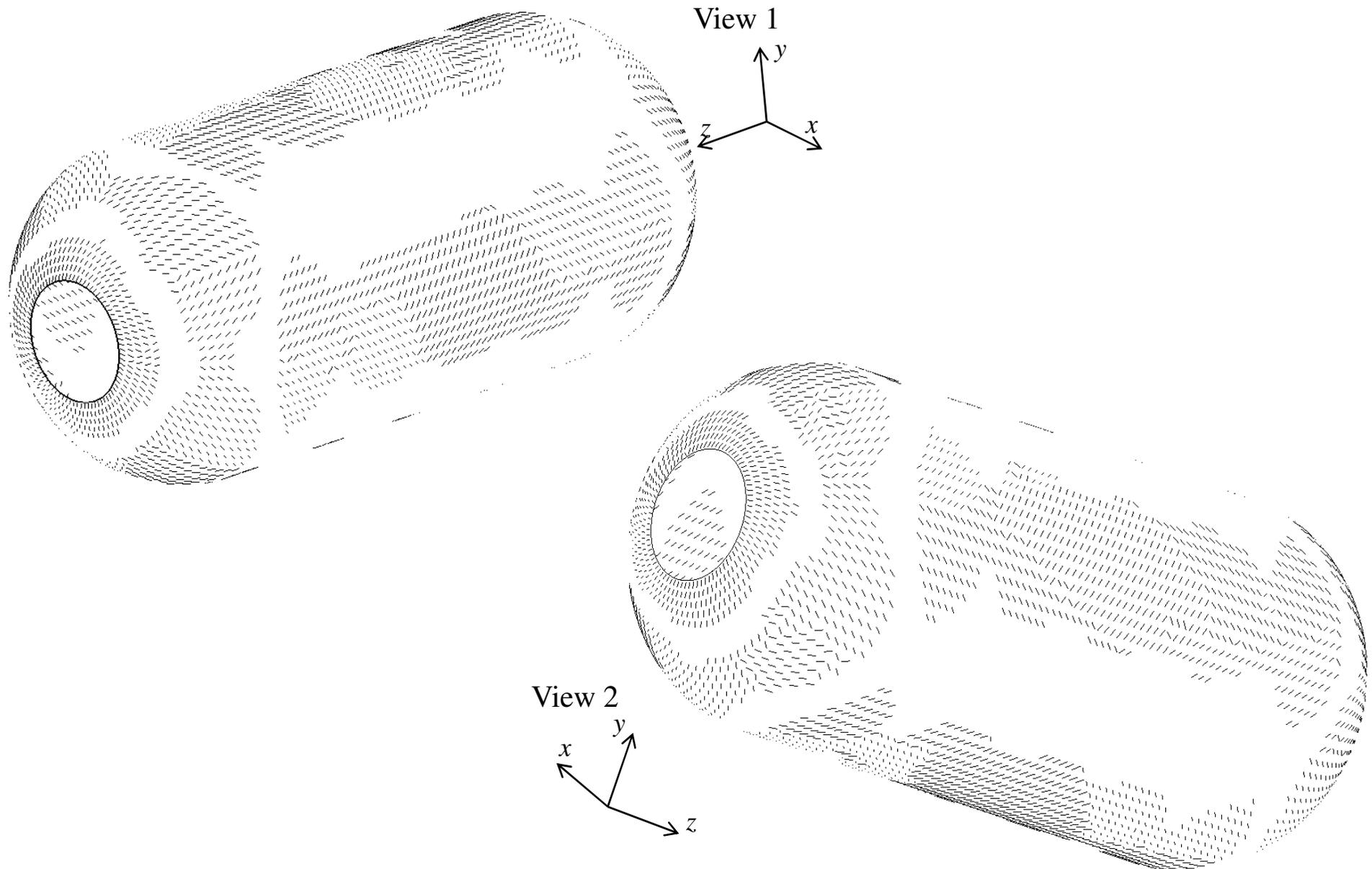
View 2  
Back-bottom



Final glass fiber  
layer mass: 5.5 Kg

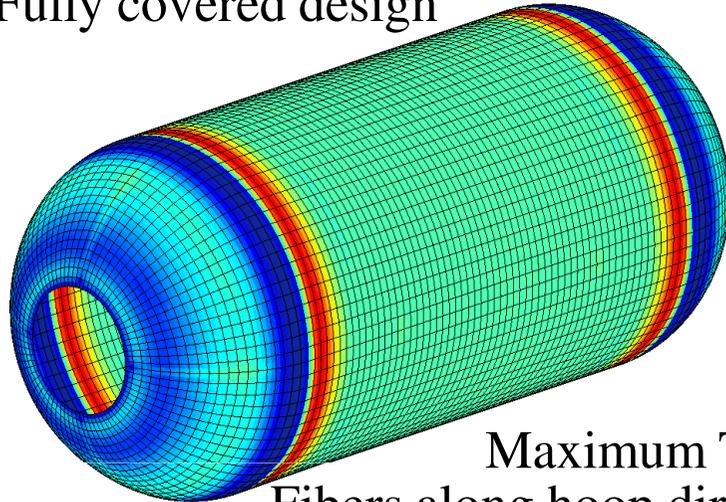


# Fiber Orientation Result – Case 2



# Tsai Wu criteria – Result – Case 2

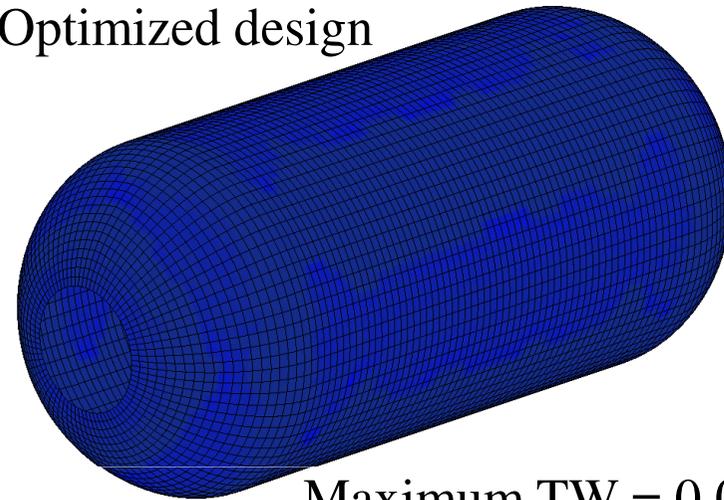
Fully covered design



Maximum TW = 0.98  
Fibers along hoop direction

Internal pressure **9.1 MPa**

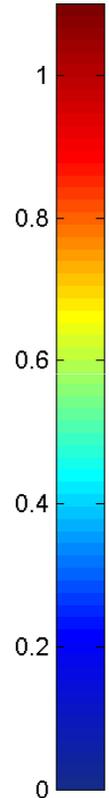
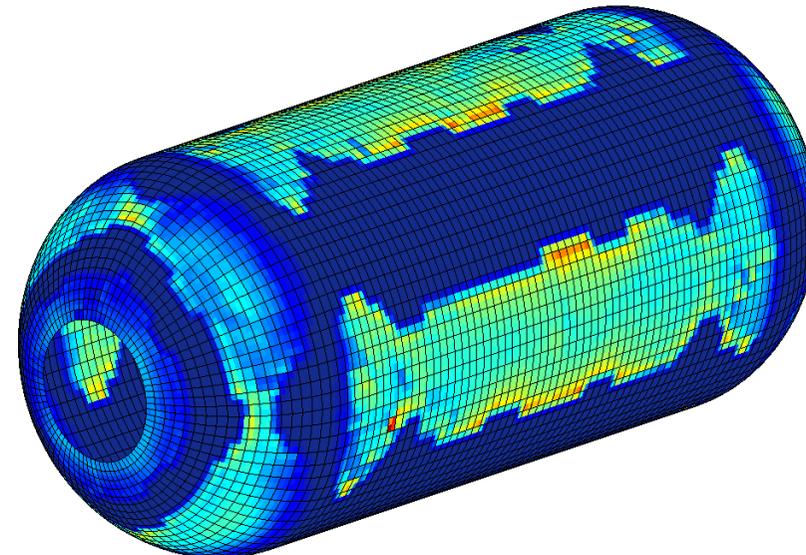
Optimized design



Maximum TW = 0.08

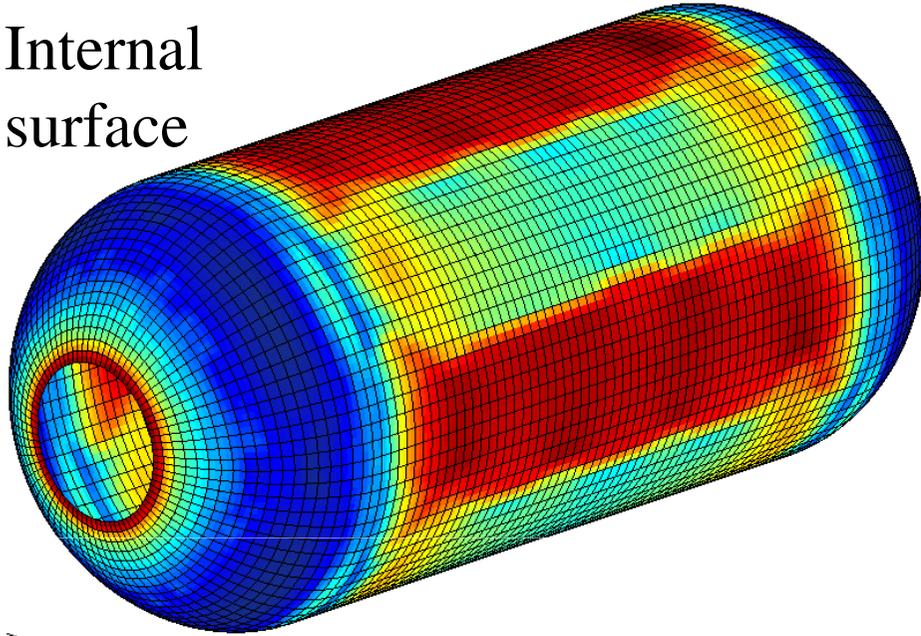
Maximum internal  
pressure of optimized  
vessel = **20 MPa**

Improvement **119,8%**

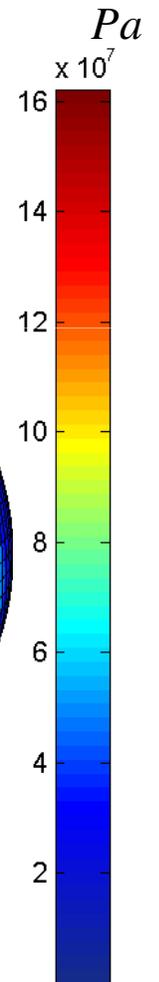


# von Mises – Result – Case 2

Internal  
surface

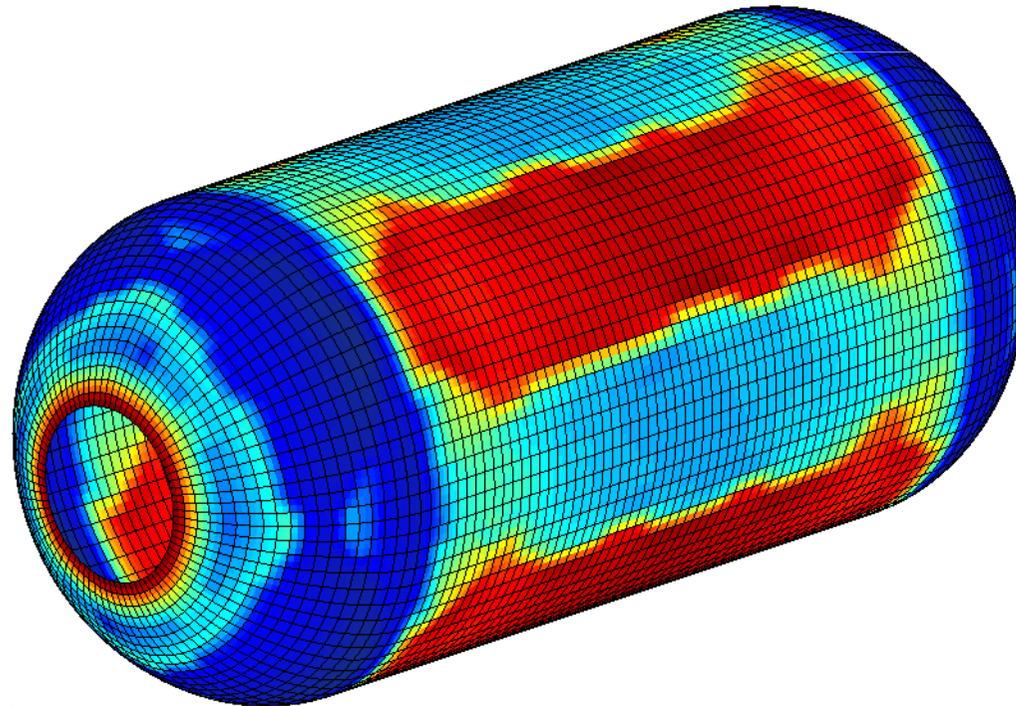


von Mises distribution  
at Aluminum layer



$$\sigma_y = 400MPa$$

External  
surface



# Conclusion

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- ✓ A parametric and topological optimization approaches have been implemented for winding-type and fiber placement-type vessel configurations, respectively;
- ✓ The proposed topology optimization formulation optimizes simultaneously material distribution, and fiber local orientation to minimize Tsai-Wu P-norm for CNG composite pressure vessel subjected to composite volume constraint;
- ✓ These approaches reduce the Tsai-Wu failure criteria value, allowing higher pressures inside the optimized pressure vessels, thus, increasing the quantity of storage gas and the ratio (gas mass)/(tank mass);

## Future Work

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- Thermal loads will be considered;
- The procedure will be extended to other types of vessels, such as, articulated vessels.



# The End

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*Thank you*

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