OAK RIDGE NATIONAL LABORATORY
operated by
UNION CARBIDE CORPORATION
for the
U.S. ATOMIC ENERGY COMMISSION

UNION CARBIDE

ORNL-TM-102

COPY NO. - 5

DATE - January 5, 1962

INSTANTANEOUS AND TIME-DEPENDENT COLLAPSE OF OFHC COPPER VESSELS BY EXTERNAL PRESSURE

C. R. Kennedy

ABSTRACT

Mechanical properties of annealed OFHC copper have been obtained from the literature and supplemented by testing at the Oak Ridge National Laboratory (ORNL). The data include short-time tensile properties, elastic constants, and creep strength values from room temperature to 800°F. Annealed OFHC copper exhibits a very low proportional limit; thus, the yield strength determined by the offset method is ambiguous and considerable scatter results. The remaining measured properties, however, are fairly consistent with small deviations resulting from differences in the final annealing treatment. A master curve for determining the critical pressures for collapse of cylindrical vessels is given and methods of obtaining the critical pressure for collapse of cylindrical vessels are demonstrated for both the instantaneous and time-dependent cases. These methods are applicable only for nearly perfect cylinders; thus, it is important either to maintain close dimensional control or use appropriate safety factors in determining the critical pressure for collapse.

INSTANTANEOUS AND TIME-DEPENDENT COLLAPSE OF OFHC COPPER VESSELS BY EXTERNAL PRESSURE

INTRODUCTION

The experimental apparatus used in the ORNL fusion experiment utilizes copper liners to obtain the necessary low pressures; i.e., 10^{-9} mm Hg. To eliminate the outgassing of the copper, the whole vessel must be baked out prior to each experiment at temperatures to 800°F and for durations to 24 hr. Thus, to incorporate a practical design in these vessels, two major questions must be answered: (1) What external pressure will cause the thin-wall vessel to collapse? (2) What, if any, permanent (plastic) strains will result from the high-temperature bake-out excursions? These design problems cannot be solved without a knowledge of the static, as well as the time-dependent mechanical properties, of OFHC annealed copper.

STRENGTH PROPERTIES OF OFHC COPPER

A literature search for the available mechanical properties was greatly simplified by a recent copper-industry-sponsored literature survey[1] of the properties of OFHC copper. This survey is reasonably inclusive and, although a large amount of mechanical property data is reported, elastic constants for high-temperatures and creep data for temperatures above 573°F were not included. Thus, it was necessary to obtain the elastic constants experimentally and extrapolate the creep data to the temperatures of interest. Short-time tensile tests were also performed on OFHC copper sheet and given an anneal of one hour at 800°F to check the validity of the literature data.

Short-time tensile test data comparing the ORNL results with those taken from the literature are given in Fig. 1. The ultimate tensile strength results, although demonstrating a fair degree of scatter at room temperature, compare very favorably at the higher temperatures. The 0.2% offset yield strength data,

[1] Technical Survey – OFHC Brand Copper, American Metal Company, Ltd., 61 Broadway, New York 6, New York (1957).

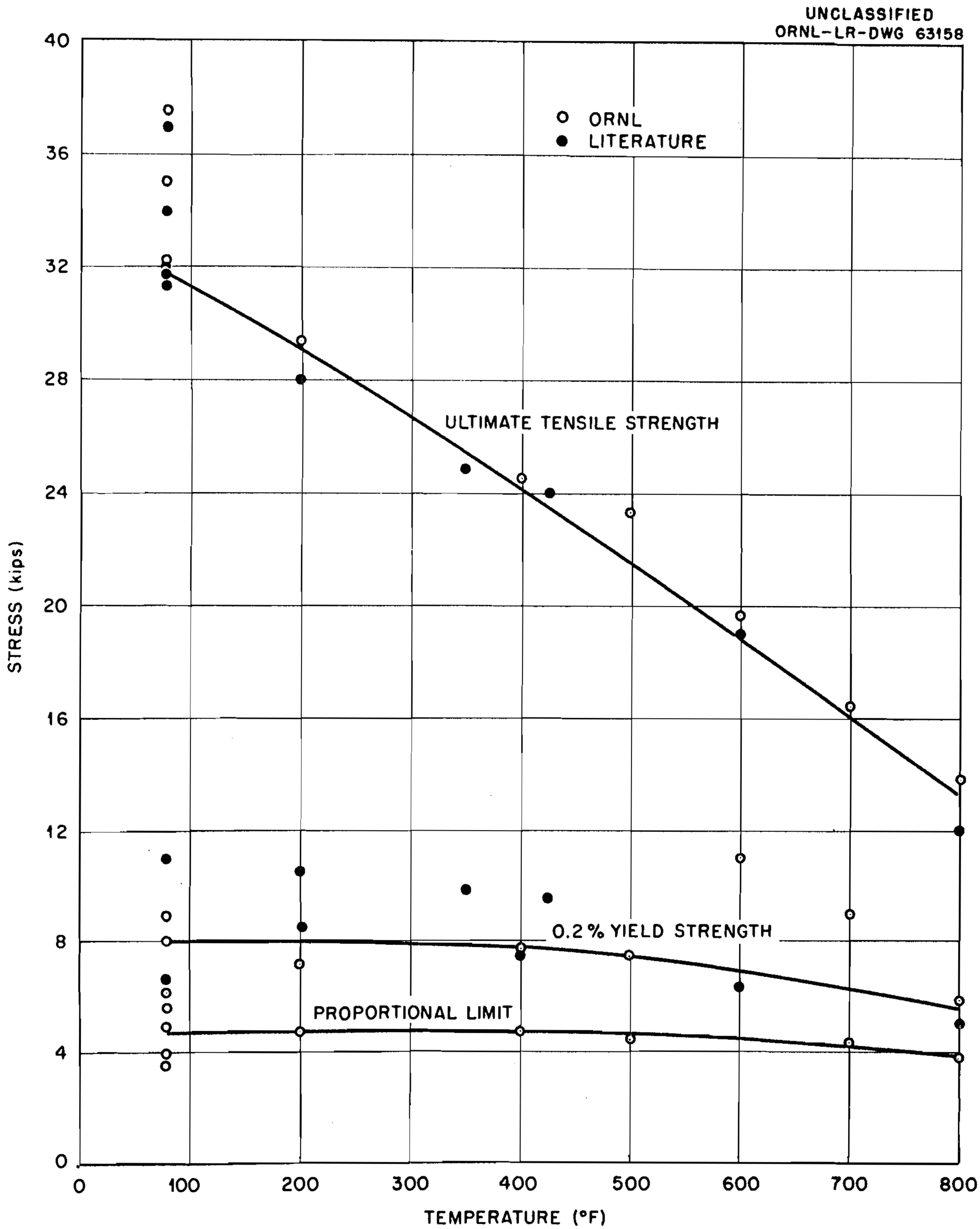

Fig. 1. Tensile Data for OFHC Copper in the Annealed State.

however, show a great deal of scatter. This is, in general, due to the experimental difficulties inherent in measuring the yield strength of materials with low proportional limits. The curve for the yield strength values shown in Fig. 1 represents an arbitrary curve drawn through the conservative region of the points.

The elastic constants were determined experimentally using a sonic method. The equipment used is an elastomat which determines the resonate frequencies in both the axial and torsional directions. These frequencies are converted to Young's modulus and the shear modulus; therefore, Poisson's ratio can be determined by the relationship between the two modulus values. The results obtained are given in Fig. 2.

The available creep data and the extrapolated values are given in Figs. 3 and 4. The method of extrapolation used was simply to extend the curves shown in Fig. 3 to higher temperatures. The values obtained were checked by Dorn's method.[2] Although not considered to be precise in this homologous-temperature range, a fairly close agreement was demonstrated.

The ASME unfired pressure vessel code stresses[3] for annealed, oxygen-free copper are also shown in Fig. 3. This code is based, in general, upon either two thirds of the yield strength or the stress to produce a strain rate of 10^{-5}%/hr. It is obvious that the oxygen-free copper listed by the ASME is stronger than OFHC copper. Therefore, new design stresses for annealed OFHC copper, based upon the ASME rules, are given in Fig. 5.

It is immediately evident that the design stresses determined by the ASME rules are extremely low at the elevated temperatures. These stress values are so chosen for extended lifetimes where, as mentioned previously, the lifetime of one of these vessels is at most 1000 hr at the high bake-out temperatures. Therefore, the allowable stress can be raised to some level which will not produce damaging deformation through creep.

---

[2]R. L. Orr, O. D. Sherby, and J. E. Dorn, "Correlation of Rupture Data for Metals at Elevated Temperatures," Trans. Am. Soc. Metals 46, 113–28 (1954).

[3]"Rules for Construction of Unfired Pressure Vessels," ASME Pressure Vessel Code, Sec. VIII, Am. Soc. Mech. Engrs., New York (1956).

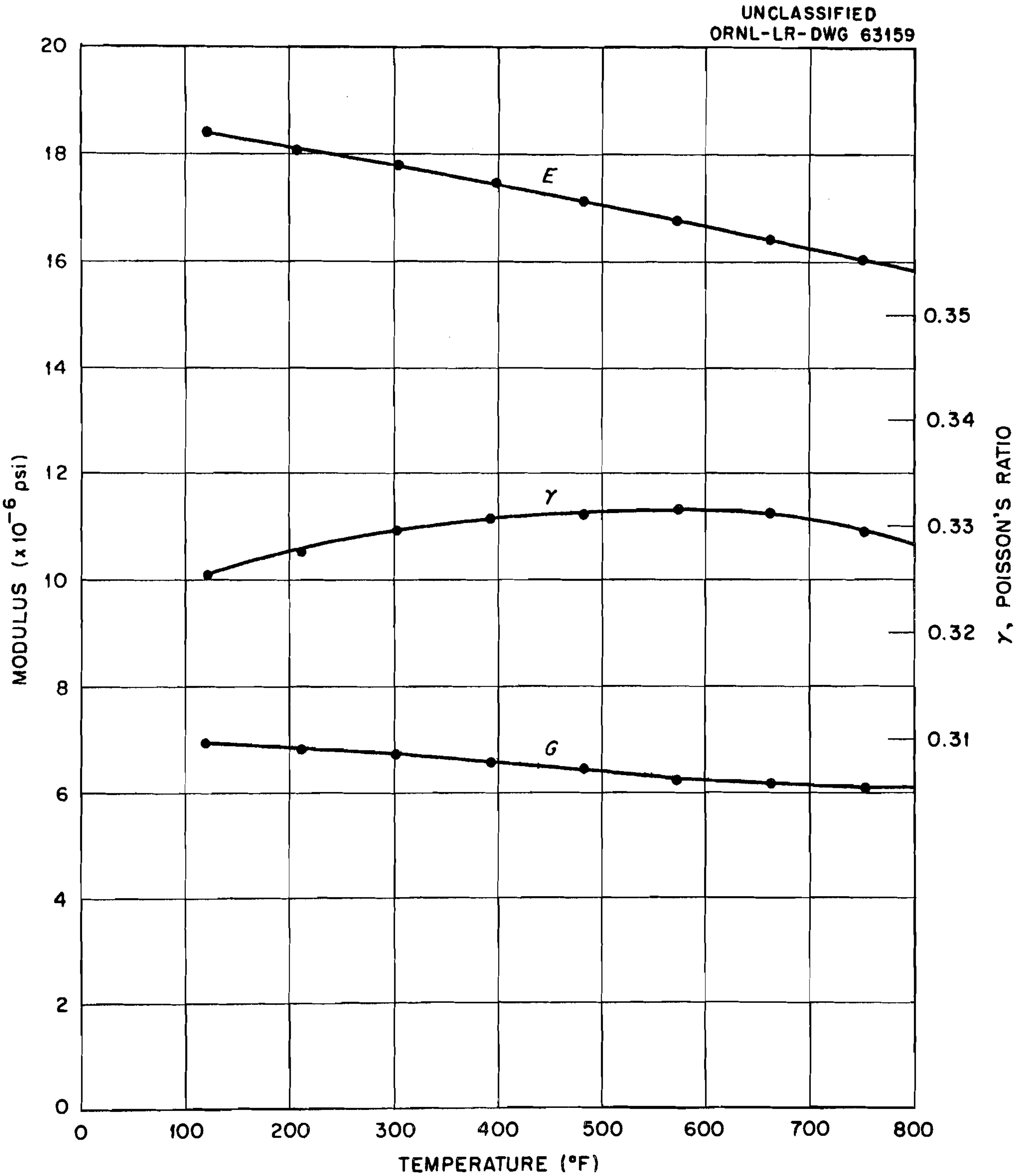

Fig. 2. Young's and Shear Modulus of Elasticity and Poisson's Ratio for OFHC Copper in the Annealed Condition.

UNCLASSIFIED
ORNL-LR-DWG 63160

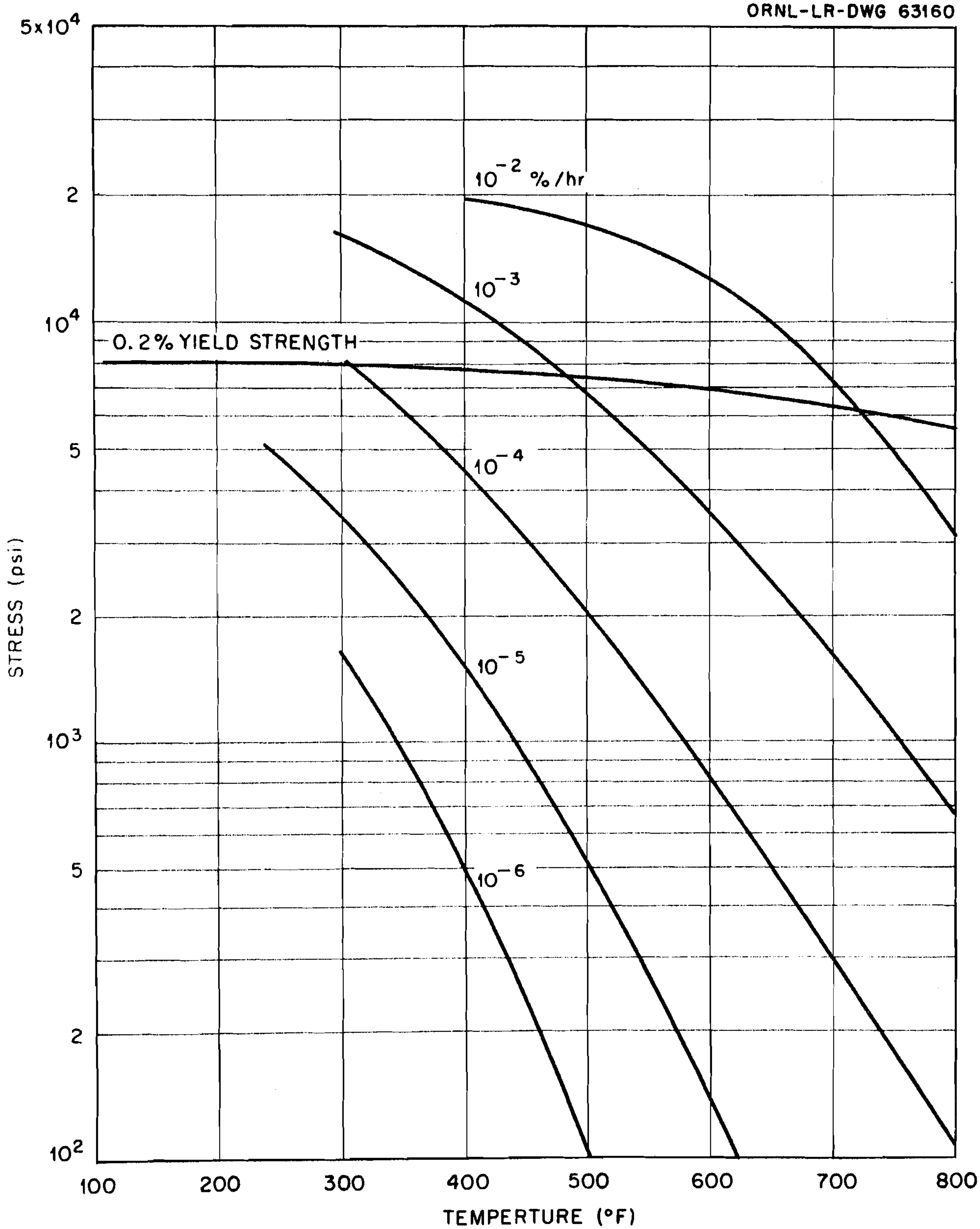

Fig. 3. Stress to Produce Designated Strain-Rates Versus Temperature for OFHC Copper in the Annealed Condition.

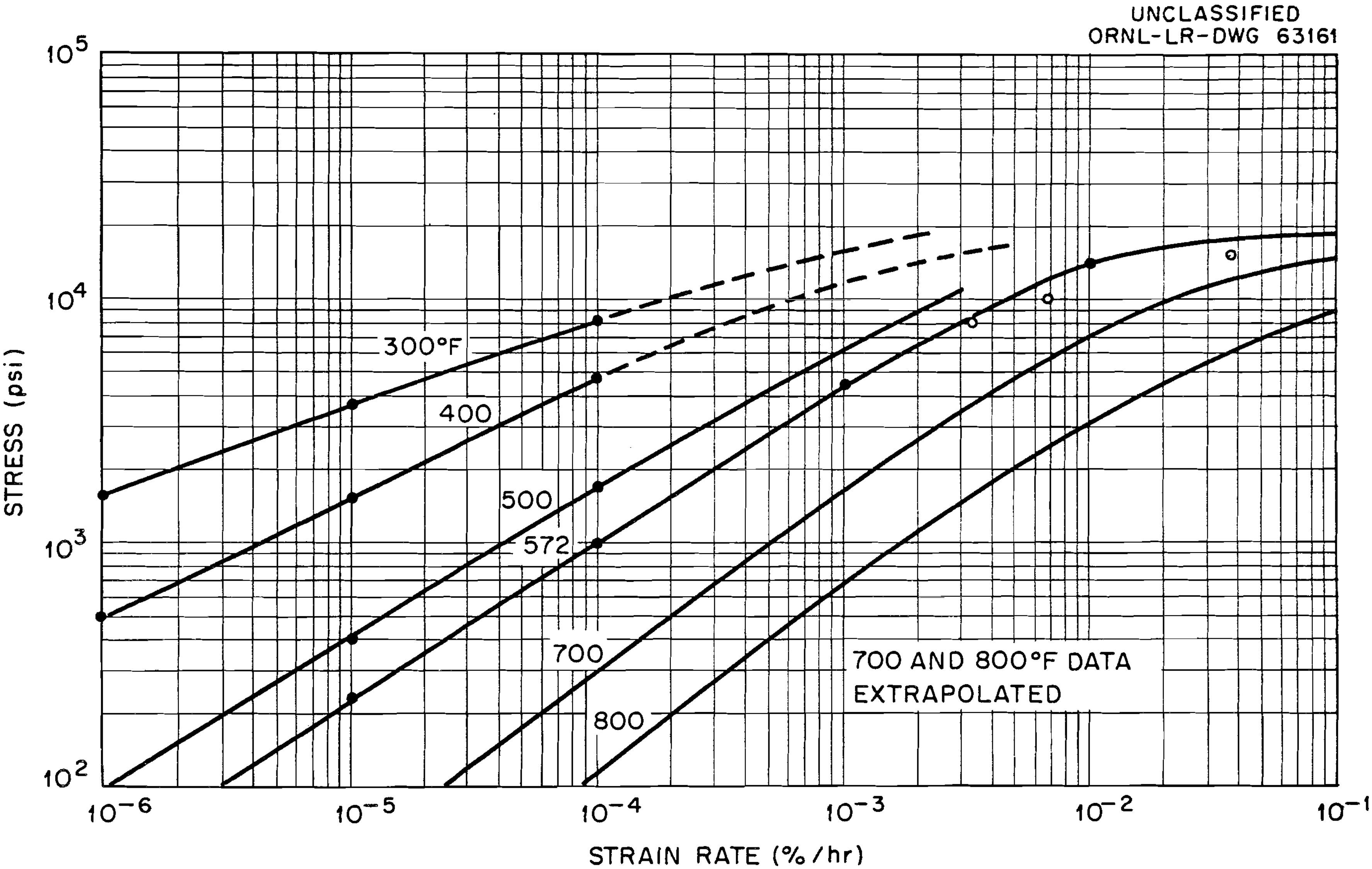

Fig. 4. Strain Rate Versus Stress for OFHC Copper in the Annealed Condition.

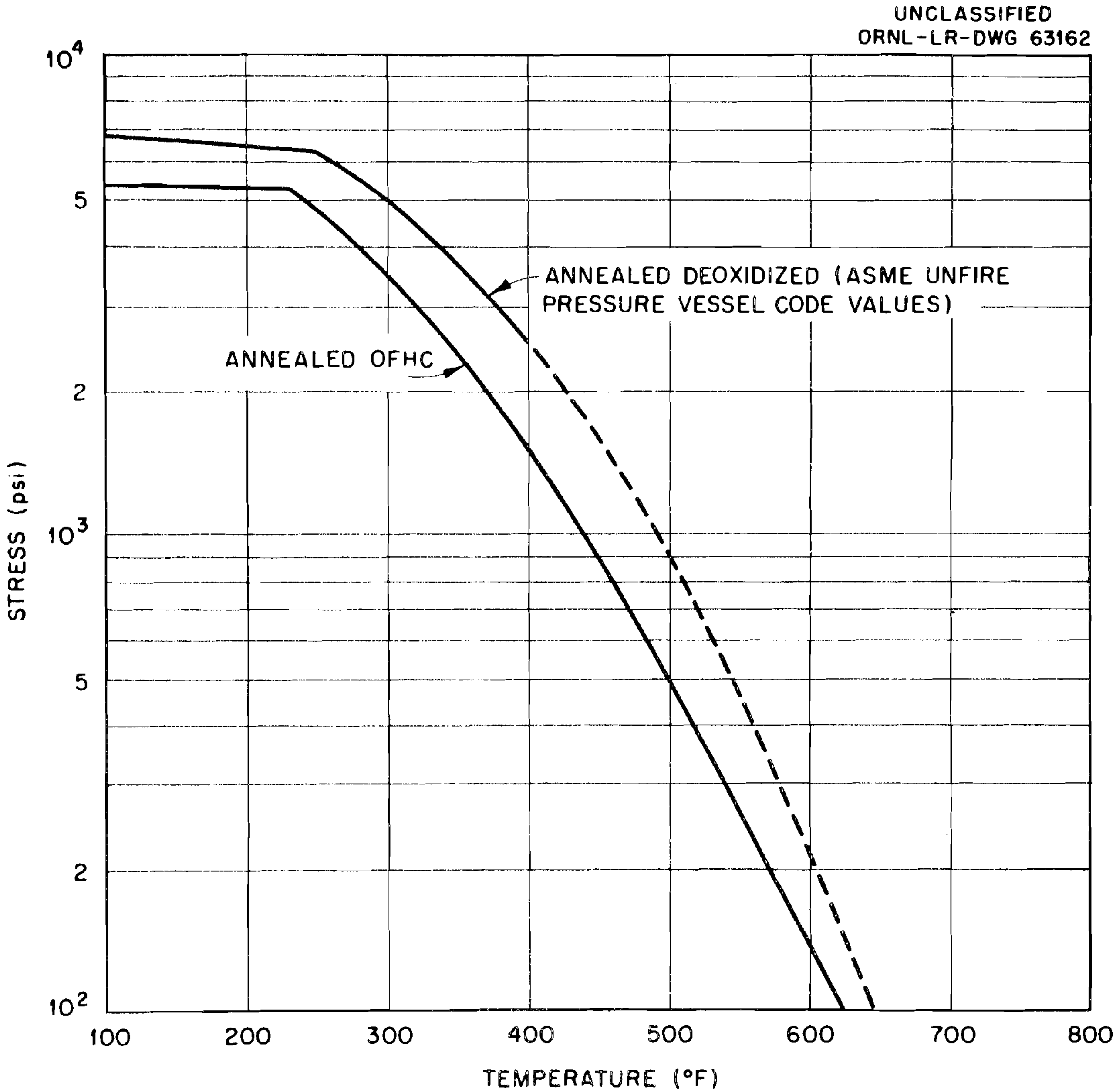

Fig. 5. Design Stresses for Annealed OFHC Copper Determined by ASME Unfired Pressure Vessel Code.

Vessel Collapse by External Pressure

A design problem, probably greater than that posed by excessive deformation, is that the vessel may collapse by an unavoidable differential pressure. The collapse of the vessel may be considered to occur in two ways: (1) an instantaneous collapse caused by a rapid increase in differential pressure; and (2) a creep collapse caused by a sustained differential pressure over a period of time. Both problems are solved using a single relationship developed by R. von Mises[4] for the critical pressure to cause collapse.

$$\phi = \frac{(1-\gamma^2)P_{cr}a}{E\,h} = \frac{1-\gamma^2}{(n^2-1)\left(1+\frac{n^2\ell^2}{\pi^2a^2}\right)^2} + \frac{h^2}{12a^2}\left(n^2-1+\frac{2n^2-1-\gamma}{1+\frac{n^2\ell^2}{\pi^2a^2}}\right) \qquad (1)$$

where

P_{cr} = critical pressure to cause collapse
a = radius of the vessel
h = wall thickness of the vessel
E = modulus of elasticity
γ = Poisson's ratio
ℓ = length of vessel between ends or supports
n = number of lobes in the collapsed tube.

The results of calculations using Eq. (1) are represented, as done by Timoshenko,[5] in Fig. 6. This is done by taking as abscissas the values of the ratio of ℓ/a and as ordinates the quantity ϕ. Then, for each value of a/h, a line is obtained which is formed by portions of curves constructed for various values of n.

The same set of curves can be used to solve both the instantaneous collapse and the time-dependent collapse problems. The difference is in the use of the proper values for E. The values of E used for the instantaneous case are simply Young's modulus for stresses to the proportional limit.

---

[4] R. von Mises, Z. Ver deut. Ing. 58, 750 (1914).

[5] S. Timoshenko, Theory of Elastic Stability, p 451, McGraw-Hill, New York, 1936.

UNCLASSIFIED
ORNL-LR-DWG 58722

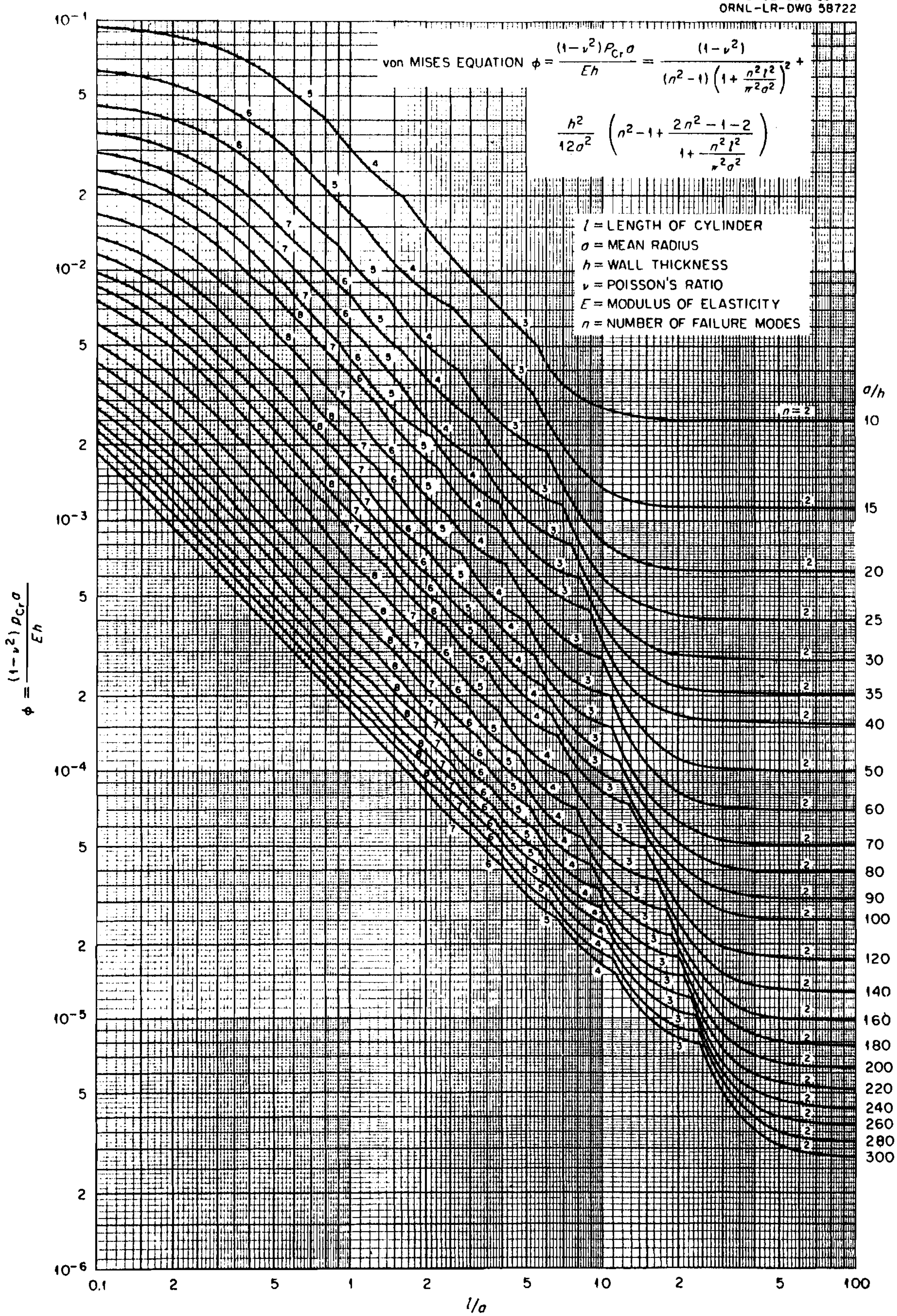

Fig. 6. Master Curve for Critical External Pressure to Collapse Cylindrical Vessels.

$$E = \frac{d\sigma}{d\epsilon} = \text{Constant for stresses to the proportional limit.} \quad (2)$$

In general, the proportional limit is higher than or equal to the highest design stress allowed in a particular case. Thus, for design purposes, a practical upper limit for the external pressure acting on the vessel is determined by:

$$P_{cr} = \frac{\sigma_{PL}\, a}{h} \quad (3)$$

where

σ_{PL} = proportional limit (stress).

Thus, for the case of instantaneous collapse by a rapid loss of vacuum in the outside containment vessel, a single value of E is used. These values have been calculated for copper at 750°F and are shown in Fig. 7. In this case, a limiting upper stress, equal to the proportional limit of 4000 psi, was chosen. It is noted that the short-time strength of OFHC copper does not vary appreciably with temperature; thus, this same set of curves should be fairly applicable down to room temperature. The critical pressures given in Fig. 7 are for short-time collapse only; thus, they are not applicable for cases where pressure remains on the vessel for any period of time.

The more geometrically stable vessels will not necessarily collapse under pressures producing stresses greater than the proportional limit. This stress was chosen arbitrarily as a maximum because, in general, excessive deformation by yielding may be defined as failure as well as collapse. The critical pressure to cause instantaneous collapse can be easily obtained for these vessels with high geometric stabilities; however, to obtain generalized values for these cases, as shown in Fig. 7, requires an impractical amount of effort. The solution of the problem utilizes a reduced modulus of elasticity which considers the redistribution of bending stresses in the vessel wall by yielding. This is,

$$E_R = \frac{4E\, E_\sigma}{\left[\sqrt{E} + \sqrt{E_\sigma}\right]^2}, \quad (4)$$

where

E_R = reduced modulus,

E = Young's modulus,

E_σ = tangent modulus at the stress under consideration.

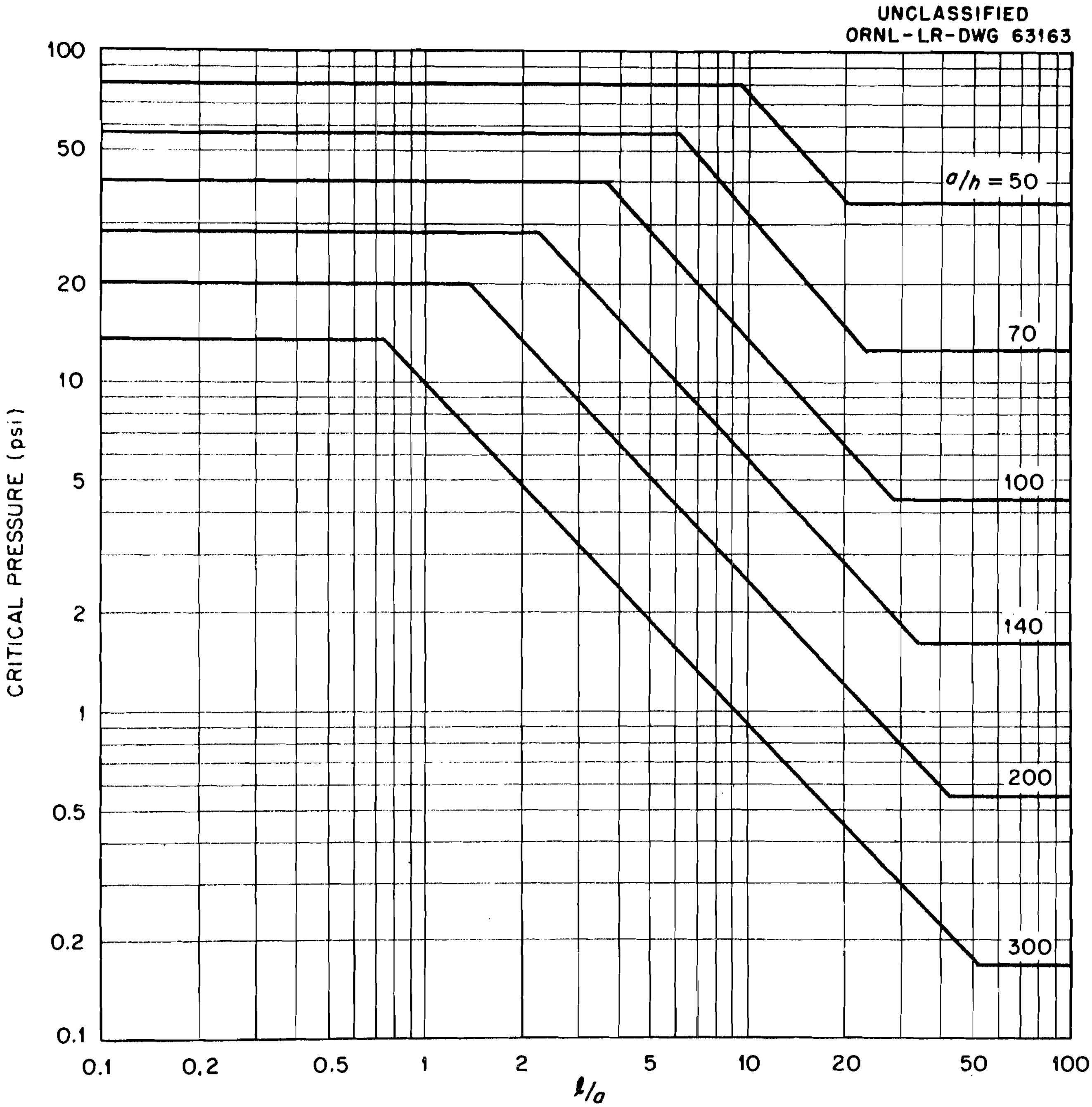

Fig. 7. Critical Pressure for Collapse of Annealed OFHC Copper Vessels for a Rapid Increase in Pressure at 750°F.

Shown in Fig. 8 is a plot of the modulus values for copper which can be used to determine the critical pressure to cause instantaneous collapse for the more stable vessels. The tangent modulus values can be obtained from actual stress-strain curves in a graphical manner as was done to obtain Fig. 8 or in an approximate analytical manner. Most materials obey the general relationship:

$$\epsilon = A\sigma^{\alpha} \tag{5}$$

where

ϵ = strain,

σ = stress,

A and α are material constants.

Differentiating Eq. (5) yields

$$d\epsilon = A\alpha\sigma^{\alpha-1}\, d\sigma . \tag{6}$$

Therefore, from Eq. (2)

$$E_{\sigma} = \frac{d\sigma}{d\epsilon} = \frac{\sigma^{1-\alpha}}{A\alpha} . \tag{7}$$

Thus, the tangent modulus can be obtained for each stress level above the proportional limit if the constants for the stress-strain Eq. (5) are known. For the case of copper at 250°F, the values are

$\alpha = 9$

$A = 1.097 \times 10^{-37}$

For the case of time-dependent collapse under sustained loads, the modulus used may be an isochronous tangent modulus. A general creep law which applies to most materials is:

$$\epsilon_c = \left(\frac{\sigma}{B}\right)^{\beta} t^m \tag{8}$$

where

ϵ_c = creep strain,

t = time,

B, β, m = material constants.

Differentiating Eq. (8) with t = constant

$$\partial\epsilon_c = \frac{\beta\sigma^{\beta-1}}{B^{\beta}}\, t^m\, \partial\sigma \tag{9}$$

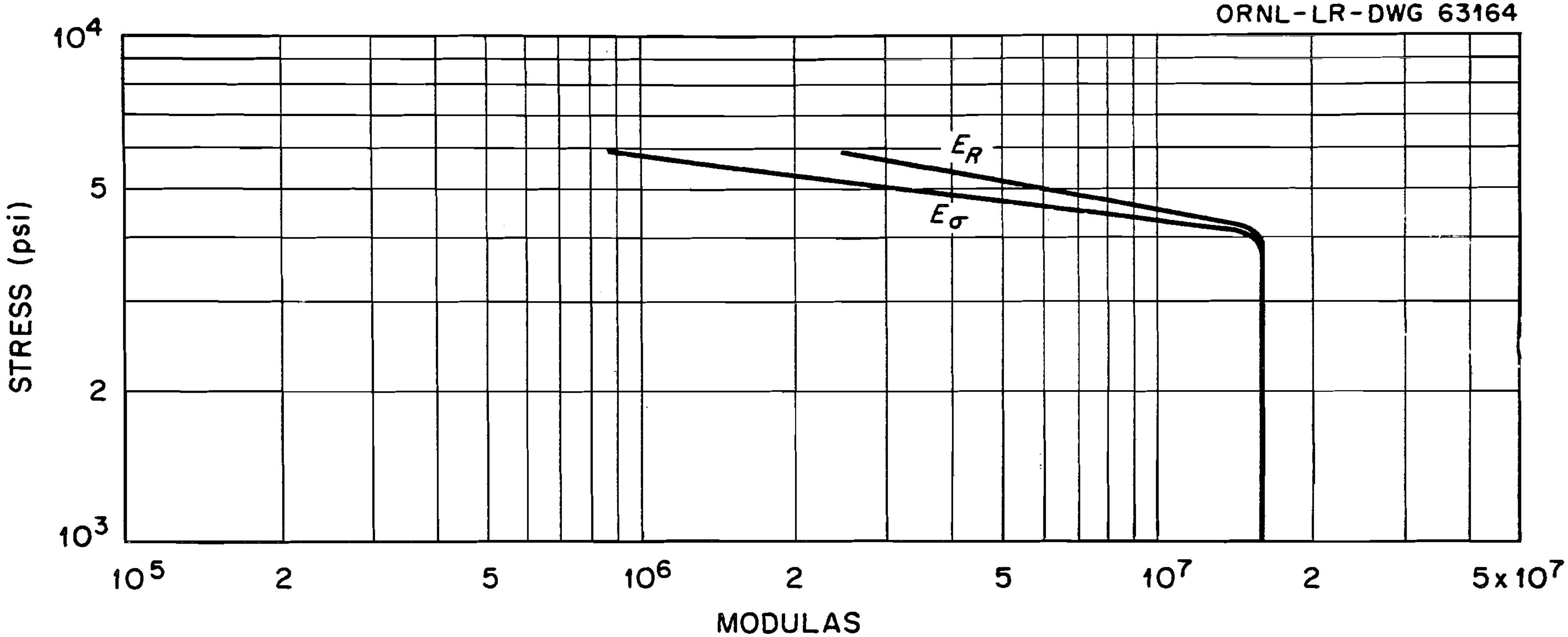

Fig. 8. Stress Versus Modulus for Annealed OFHC Copper at 750°F.

$$\left(\frac{\partial\sigma}{\partial\epsilon}\right)_t = E_\sigma = \frac{B^\beta}{\beta\sigma^{\beta-1}t^m} . \tag{10}$$

The values of E_σ found in Eq. (10) can be substituted into Eq. (1)

$$P_{cr} = \frac{Bh}{a} \left(\frac{\phi}{\beta t^m(1 - \gamma^2)}\right)^{\frac{1}{\beta}} . \tag{11}$$

Thus, if the creep constants are known, the critical pressure to cause collapse at time, t, can be obtained. It is also interesting to note that the collapsing pressure for a vessel with fixed geometry can be related as:

$$P_{cr} = Kt^{-\frac{m}{\beta}} \tag{12}$$

where

$$K = \frac{Bh}{a} \left(\frac{\phi}{\beta\ (1 - \gamma^2)}\right)^{\frac{1}{\beta}} .$$

Thus, critical pressure-time curves using logarithmic coordinates should have the reciprocal slope of the stress-strain rate curve if $m = 1$. The value of m at high temperatures is 1; however, it does decrease with temperature. Since this value is seldom reported in the literature, it is convenient and sufficiently accurate to use $m = 1$.

The values of the critical pressure to cause collapse in 24 hr have been calculated and are given in Fig. 9. The calculations were made using the following values for the constants:

$B = 5.3 \times 10^6$,

$\beta = 1.34$,

$m = 1$.

Again, there is a limiting stress which, in this case, will be that stress which will cause undesirable deformation without collapse in the time period. This limiting stress must be chosen by the designer for that particular application. It should be realized that the time under the pressure is accumulative; thus, the differential pressure-time history should be recorded to anticipate lifetime expectancy.

This type of analysis does satisfy experimental evidence concerning collapse; however, the designer and the operator must be warned that confidence bands about

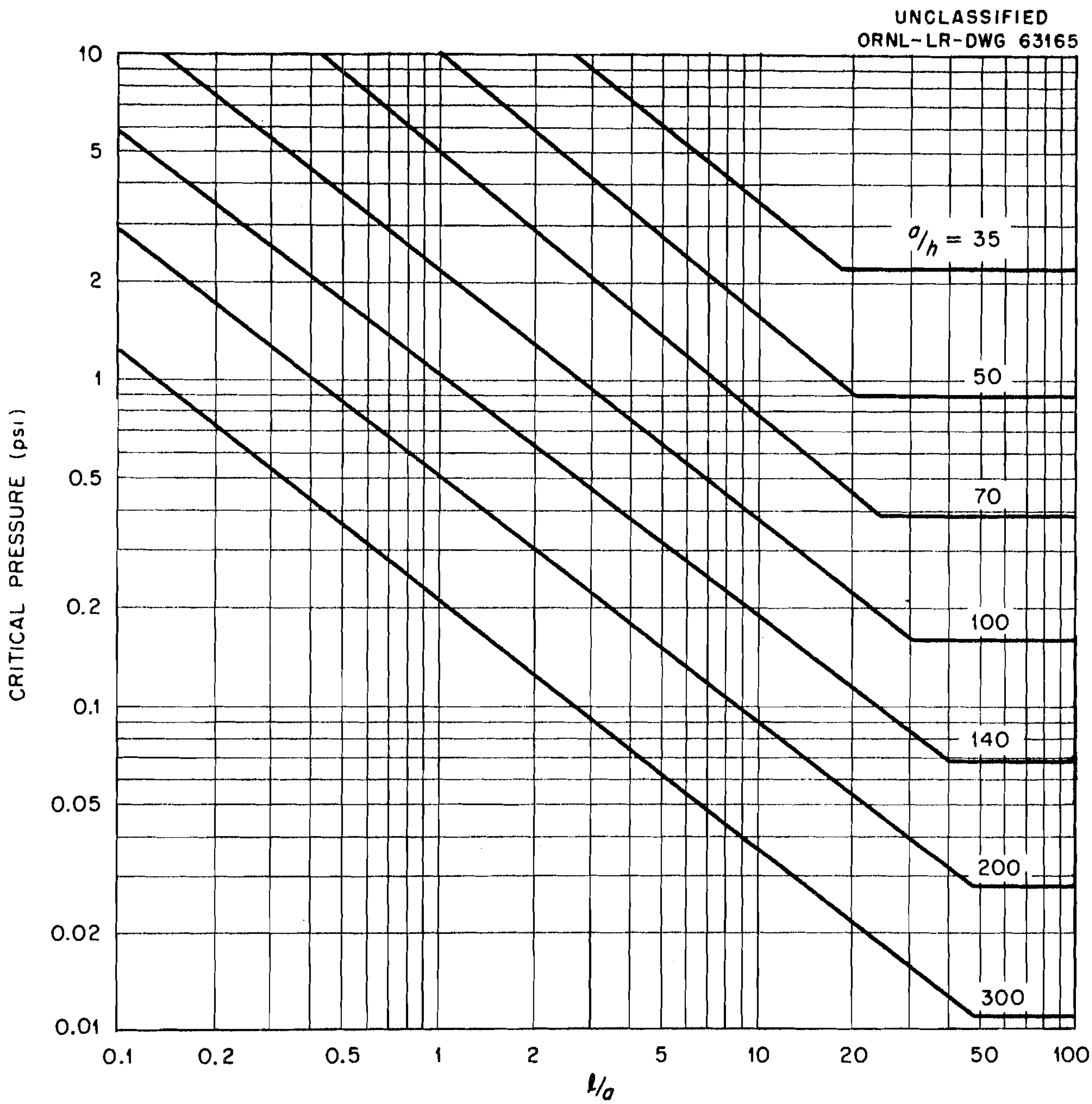

Fig. 9. Critical Pressure for Collapse of Annealed OFHC Copper Vessels after 24 hr at 750°F.

the predicted values are fairly wide. This is, in general, a result of geometric imperfections in the vessel, lack of uniform wall thickness, and high ellipticity values causing very significant losses in the critical pressure for collapse. This, then, infers that either extreme tolerances are built into the vessels or appropriate safety factors are used. For vessels with fairly small wall-thickness variations (less than 5%) and ellipticity ratios ($\frac{\text{major diameter}}{\text{minor diameter}} > 0.99$), a safety factor of two would be in order.

DISTRIBUTION

1. Central Research Library
2. ORNL – Y-12 Technical Library Document Reference Section
3–7. Laboratory Records
8. Laboratory Records, ORNL RC
9. G. M. Adamson, Jr.
10. B. S. Borie
11–13. R. E. Clausing
14. J. M. Corum
15. J. E. Cunningham
16. D. A. Douglas, Jr.
17. H. G. Duggan
18. J. H Frye, Jr.
19. B. L. Greenstreet
20–22. M. R. Hill
23–32. C. R. Kennedy
33. H. G. MacPherson
34. W. D. Manly
35. W. R. Martin
36. H. E. McCoy
37. C. J. McHargue
38. P. Patriarca
39. M. J. Skinner
40. C. O. Smith
41. R. L. Stephenson
42. J. T. Venard
43. J. R. Weir, Jr.
44–58. Division of Technical Information Extension (DTIE)
59. Research and Development Division (ORO)