A Tutorial on Pipe Flow Equations

by
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Preface and Dedication

This paper is a form of plagiarism for it contains few new thoughts! The author is extremely indebted to the following two groups of engineers for developing its concepts fully:

Samuel I. Hyman, Brooklyn Union Gas Company, Brooklyn, New York
Michael A. Stoner, Stoner Associates, Inc, Carlisle, Pennsylvania
, Michael A. Karnitz, Stoner Associates, Inc, Carlisle, Pennsylvania
Gas Flow Formulas – Strengths, Weaknesses and Practical Applications
published in the 1975 AGA Distribution Conference Proceedings
and republished as
Gas Flow Formulas – An Evaluation
in Pipeline and Gas Journal, December1975 and January 1976

J. Christopher Finch, Natural Gas Pipeline Company of America
David W. Ko, Natural Gas Pipeline Company of America
Tutorial – Fluid Flow Formulas
published in the 1988 PSIG Conference Proceedings

In addition, the subject of explicit friction factor equations was definitively covered by:

Garry A. Gregory, Neotechnology Consultants, Ltd., Calgary, Alberta, Canada
Maria Fogarasi, University of Calgary., Calgary, Alberta, Canada
Alternate to Standard Friction Factor Equation
Published in Oil and Gas Journal, April 1, 1985.

They said it all (or mostly all considering some exciting new work published in the year 2000).

Abstract:

The purpose of this paper is to describe the equations which govern the flow of compressible fluids through pipes. Particular emphasis is placed on those used within the natural gas industry in hopes that engineers within that industry can make knowledgeable decisions on how to model pipes. Its thesis is that all practical equations were created to solve intense numerical problems and have been made obsolete by advancing computing technology. It further discusses a new flow formula proposed by the GERG Research project 1.19

A Note Concerning Units:

These equations have generally been published in the English system of units. Where appropriate, the alternate equations in metric units have been included, with the names of the metric units being shown in italic type. Since the Pole, Spitzglass, and Weymouth equations are included only for historical interest, only their original published form is presented

Biographical Sketch:

Don Schroeder, whose present title is Director, Technical Affairs, has been employed by Stoner Associates, Inc. of Carlisle Pennsylvania in various capacities over the past 23 years. During this time he has served as principal author of SAI's gas steady-state and transient offerings as well as being heavily involved in their optimization efforts. Prior to joining Stoner Associates, he worked for the former Columbia Gas System for 12 years: 5 in various engineering capacities within their Pittsburgh Group Companies, and 7 in their Service Corporation's Operations Research Department. Don's academic background includes a Bachelor of Science in Chemical Engineering degree from Carnegie Institute of Technology in its "pre-Melon" days. Don served as secretary of PSIG (Pipeline Simulation Interest Group) from 1971 to 1973, chairman from 1973 to 1975, and has been the immediate past-chairman ever since (this is the longest he ever held a job). He also has served as PSIG treasurer since 1989.

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I. The Fundamental Equation

During the almost two centuries that the natural gas industry has been in existence there has always been a need for workable equations to relate the flow of gas through a pipe to the properties of both the pipe and the gas and to the operating conditions such as pressure and temperature. The usefulness of such equations is obvious: systems must be designed and operated with full knowledge of what pressures will result from required flow rates. The purpose of this paper is to describe the ways that this has been accomplished and to provide some practical insight into what the current practice should be.

Since nearly every text on fluid mechanics, and they are legion, contains some derivation of the fundamental equation governing one dimensional, compressible fluid flow, it is not necessary to repeat that derivation here. Excellent derivations are presented in both the Hyman, Stoner, Karnitz and the Finch, Ko papers referenced in the bibliography. Essentially one begins with the partial differential equations of motion along with the equation of state and then starts assuming and integrating. The end result for flow in a horizontal pipe is the following equation:

$$Q = C\frac{T_b}{P_b}D^{2.5}e\left(\frac{P_1^{\,2} - P_2^{\,2}}{LGT_aZ_af}\right)^{.5}$$

Where:

- C Constant, 77.54 (English units); *.0011493 (Metric units)*
- D Pipe diameter (inches) *(millimeters)*
- e Pipe efficiency (dimensionless)
- f Darcy-Weisbach friction factor (dimensionless)
- G Gas specific gravity (dimensionless)
- L Pipe length (miles) *(kilometers)*
- P_b Pressure base (PSIA) *(Kilopascals)*
- P_1 Inlet pressure (PSIA) *(Kilopascals)*
- P_2 Outlet pressure (PSIA) *(Kilopascals)*
- Q Flow rate (standard cubic feet/day) *(standard cubic meters/day)*
- T_a Average temperature (°R) (°K)
- T_b Temperature base (°R) (°K)
- Z_a compressibility factor (dimensionless)

Let us now examine the components of this equation. Note first that this expresses flow rate in terms of standard cubic feet per day. Although it looks like one, this is not a measure of volume per time; it is rather a measure of mass per time. It is in fact the mass contained in one cubic foot of the stated gravity gas at the standard conditions defined by P_b and T_b. These standard conditions are established by gas sales contracts and are not those conditions usually defined by the scientific community. To further complicate matters, if we permit gas gravity to vary throughout the system, the definition of a standard cubic foot also varies and we wind up comparing quantities in different units as if they were the same. So much for the concept of mass continuity!

This equation shows clearly how flow varies with the pertinent parameters. Obviously bigger, shorter, colder, more efficient pipes containing lighter gasses permit more flow. Of particular interest is Z_a, the equation of state. Since Z_a is a function of pressure and temperature, it must be evaluated at average conditions for the pipe. For temperature an arithmetic average flowing temperature is usually used while the following equation, which accounts for the non-linearity of pressure drop with distance, is generally accepted for determining average pressure:

$$P_{av} = \frac{2}{3}\left[P_1 + P_2 - \left(\frac{P_1 P_2}{P_1 + P_2}\right)\right]$$

This equation is also used for linepack determinations, which represent the amount of gas entrained in the pipe. An alternative and more precise way of treating supercompressibility would be to modify the fundamental equation to incorporate supercompressibility evaluated separately at inlet and outlet conditions. Since it further complicates the computation without offering significant benefits, this has not been generally done for pipes but it is common in reservoir work .

$$Q = C\frac{T_b}{P_b}D^{2.5}e\left(\frac{P_1^{\,2}/Z_1 - P_2^{\,2}/Z_2}{LGT_a f}\right)^{.5}$$

Pipes are usually not horizontal. So long as the slope is not too great, a correction for the static head of fluid may be incorporated and determined as follows:

$$Q = C\frac{T_b}{P_b}D^{2.5}e\left(\frac{P_1^{\,2} - P_2^{\,2} - H_c}{LGT_a Z_a f}\right)^{.5}$$

where:

$$H_c = \frac{0.0375g\,(H_2 - H_1)P_a^{\,2}}{ZT_a} \qquad \text{(English)}$$

$$H_c = \frac{0.06835g\,(H_2 - H_1)P_a^{\,2}}{ZT_a} \qquad \textit{(Metric)}$$

Note also that the flow equation includes an efficiency term for calibration purposes. Many purists, including Dr. Stoner, have argued that using a correct correlation for the friction factor alleviates the need for providing such an adjustment and that pipe roughness alone is sufficient as an adjustment mechanism. This is simply not the case. Roughness is one of these seemingly obvious things that appear to be intuitively obvious until an attempt is made to use them quantatatively. Yes, there is more frictional loss in concrete pipe than in drawn tubing, but reducing it to one measure in anyplace but a laboratory is difficult. While roughness can account for frictional effects such as bends and fittings, pipes also can have various obstructing materials like condensate accumulations, rust, and sediment that behave more like diameter reductions. For these and other reasons that will become more apparent when the friction factor is discussed, pipe roughness alone is an inadequate compensator.

II. The Friction Factor

This brings us to the most interesting and complex part of the equation, the friction factor. The first complication that arises is that there are two common friction factor definitions in standard usage : the Fanning and the Darcy-Weisbach.. In the nineteenth century, two groups approached the fluid flow problem independently and arrived at remarkably similar results. Since the Darcy-Weisbach factor is simply 4 times the Fanning factor, it's mostly a matter of personal choice and what branch of engineering you come from, and the only problem is keeping track of which one is being discussed. Although I am a chemical engineer and therefore should use Fanning, the Darcy-Weisbach friction factor is used exclusively in this paper in deference to the civil engineers who comprise the industry, but beware of reading other papers, particularly the Finch and Ko paper.

Use of the fundamental equation for calculating flow requires the numerical evaluation of f, the friction factor. In general, the friction factor itself is in turn a function of flow rate, thus making the whole flow equation an implicit one. For purposes of determining friction factor, it has been found that fluid flow may be characterized by a dimensionless grouping of variables known as the Reynolds' Number, which is defined as:

$$N_{re} = \frac{Dvr}{m}$$

Where:

N_{re} Reynolds' number (dimensionless)
D pipe diameter (feet) *(meters)*
v fluid velocity (feet/second) *(meters/second)*
ρ fluid density ($lb_m/foot^3$) *($kg/meter^3$)*
μ fluid viscosity (lb_m/second-foot) *(kg/second-meter)*

Note that in this context the units are not the same for they must cancil to produce a dimensionless group. For a compressible fluid, we can determine density from the equation of state to substitute for ρ, and determine velocity as flow rate/area corrected to actual conditions to substitute for v. Conveniently, much of the equation of state cancels-out and we are left with the following expression:

$$N_{re} = \frac{.015379QgP_b}{\mu DT_b} \qquad \text{(English)}$$

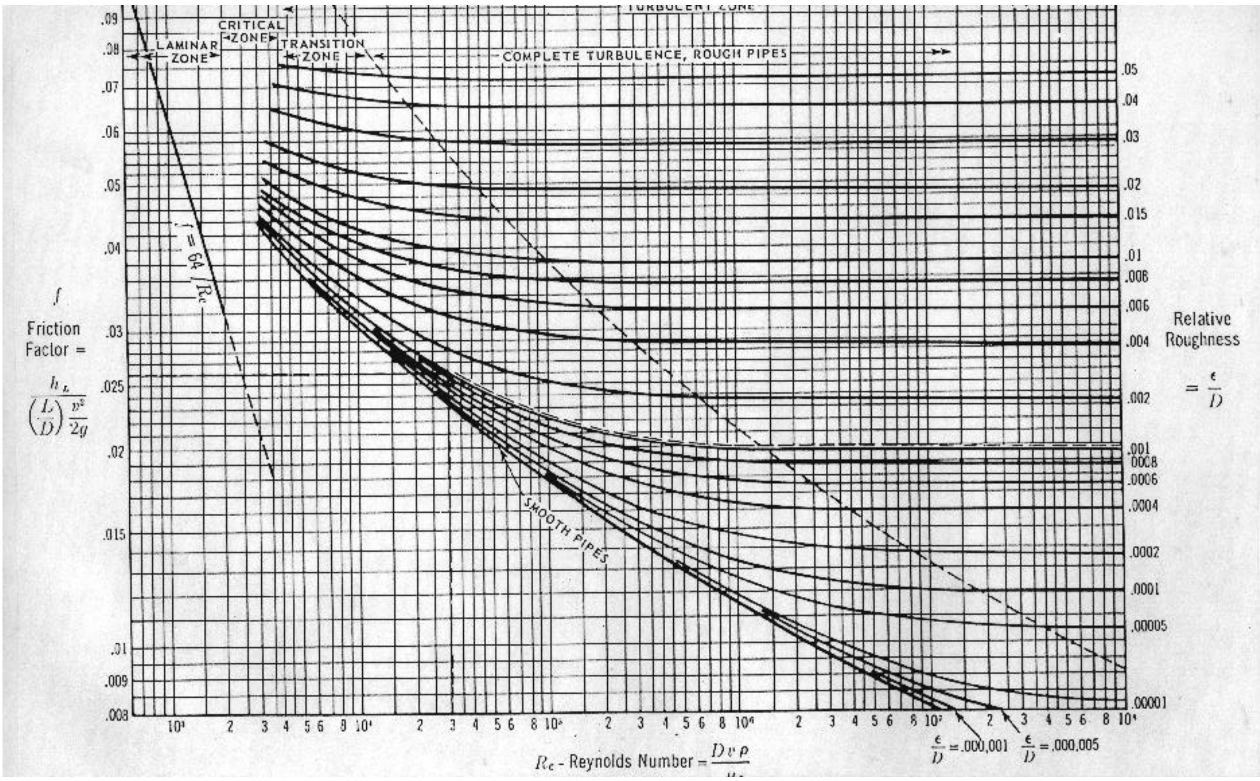
$$N_{re} = \frac{49.44QgP_b}{\mu DT_b} \qquad \textit{(Metric)}$$

Where:

N_{re} Reynolds' number (dimensionless)
D pipe diameter (inches) *(millimeters)*
g Gas specific gravity (dimensionless)
P_b Base Pressure (PSIA) *(Kilopascals)*
Q Gas flow rate (standard ft^3/day) *(standard m^3/day)*
T_b Base Temperature (°R) *(°K)*
μ fluid viscosity (lb_f-sec/ft^2) *(pascal-sec)*

This is a better way to view the Reynolds' number in a gas industry context since it points out that the Reynolds' number is essentially proportional to the flow rate.

The other parameter in the friction factor correlation is pipe roughness. Friction factor may be correlated as a function of the Reynolds' Number and the relative pipe roughness (absolute roughness whatever that means divided by inside diameter). This function is usually presented in the familiar Moody Diagram.



To understand it, the Moody diagram may be broken down into four zones: Laminar, Transition, Partially turbulent, Fully turbulent.

The Laminar zone is the part on the extreme left. In this zone of extremely low flow rate the fluid flows strictly in one direction and the friction factor shows a sharp dependency on flow rate as defined by the Hagen-Poiseuille equation:

$$f = \frac{64}{N_{re}}$$

The Fully Turbulent zone is the part on the extreme right where the lines flatten-out. In this zone of extremely high flow rate the fluid flows laterally within the pipe in complete turbulence as well as in the primary direction and the friction factor shows no dependency on flow rate. Note: the ultimate friction factor is only a function of roughness and an ideally smooth pipe never makes it to this zone. Here the friction factor is given by the rough pipe law of Nikuradse:

$$\frac{1}{\sqrt{f}} = 2\log\frac{D}{e} + 1.14$$

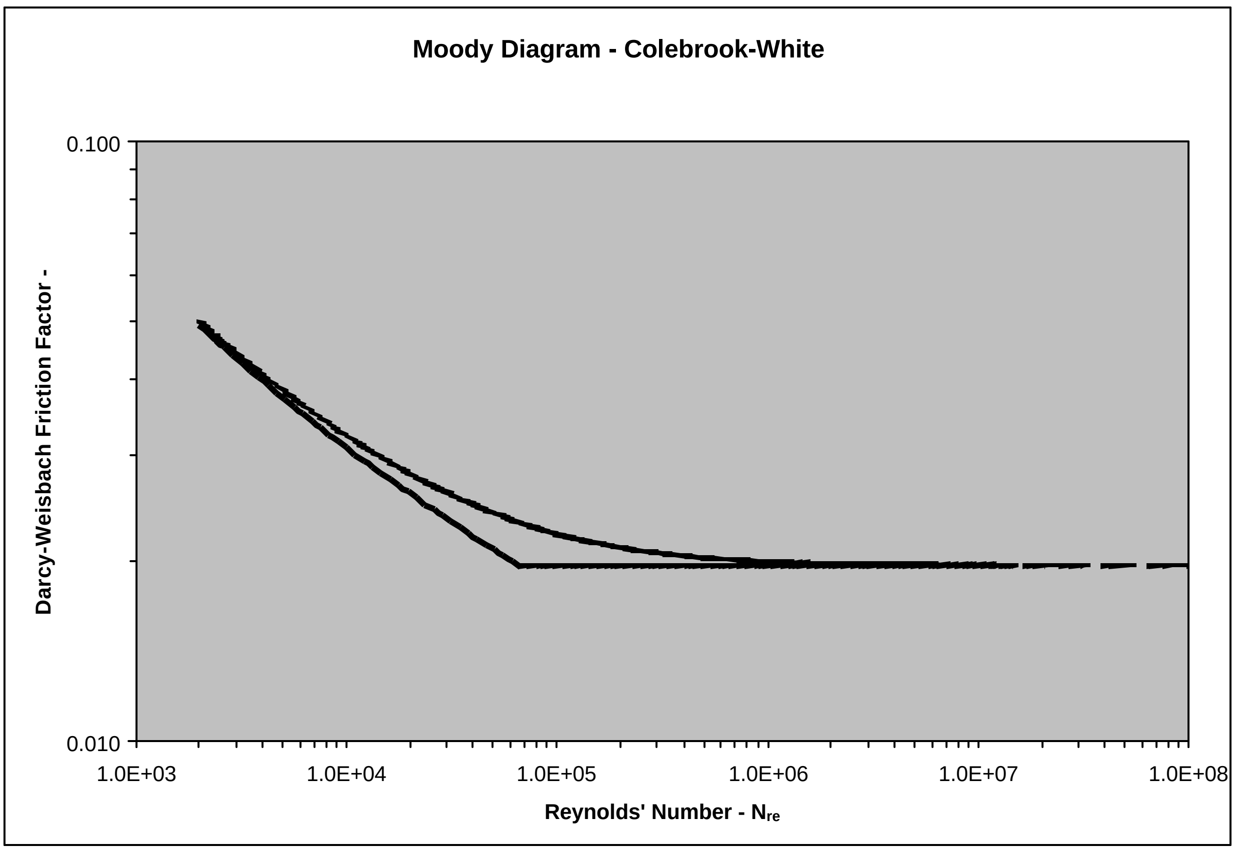
The Partially Turbulent zone is the part in the middle where the curves exist. In this zone of moderately high flow rate the fluid flows laterally within the pipe as well as in the primary direction although some laminar boundary layer outside the zone of roughness still exists. Starting in the left side of this region, flow is governed by the smooth pipe law of von Karman and Prandtl:

$$\frac{1}{\sqrt{f}} = 2\log\left(N_{re}\sqrt{f}\right) - 0.8$$

How the friction factor varies across this region from the smooth pipe law to the rough pipe law is not completely agreed-upon. Some feel that the straight horizontal lines of the rough pipe law should be extended to the smooth pipe law forming a corner at the intersection. Others feel that nature abhors corners and the Colebrook-White equation, which is nothing more than a combination of the two, is the proper method:

$$\frac{1}{\sqrt{f}} = -2\log\left(\frac{e/D}{3.7} + \frac{2.51}{N_{re}\sqrt{f}}\right)$$

This is shown in the following diagram:



Unfortunately the published data are inconclusive so that either method may be considered valid. Recent work by GERG (Groupe European de Recherches Gazieres) has proposed a new flow equation that includes an additional parameter "n". For details please refer to the Gersten *et. al.* paper in the bibliography. "n" is essentially a shape factor that places the friction factor between these two limits (n=1 approximates Colebrook-White while n=10 approximates the sharp intersection). Several observations are important:

1. The Colebrook-White equation always predicts a higher friction factor; hence it is the more conservative.
2. The maximum difference in friction factor is about 17% which translates to a 8.5% difference in flow.
3. This maximum difference occurs at fairly low Reynolds' numbers associated with low pressure drop and lessens with increasing pressure drop where it is more important.
4. There is significant scatter to the data.

While this work is exciting, it is unclear how adding one more factor that is even more physically remote than the roughness term will help clarify the vary murky world of system calibration.

These equations point out the other reason for stating earlier that roughness alone is not a sufficient calibration parameter: roughness only affects half of the equation. Note that the smooth pipe law does not include an effect for roughness. This means that as flow rate decreases, roughness enters in less and less, and not at all if using the sharp intersection technique.

The Critical zone is where there are no lines, hence no function. In this zone of relatively low flow between the Laminar and Partially Turbulent zones no one knows what is happening. This is not due to lack of effort in trying to explain it. The fact is that the upper end of laminar flow is unstable and in this region multiple behaviors are observed.

The connection of these zones works pretty much as follows. As fluid starts flowing from rest, the friction factor decreases at a fairly fast rate. During this time the fluid is in the laminar flow regime in which there is no motion at the wall, the velocity follows a parabolic distribution to the center of the pipe, and there is no non-axial flow. At some point, usually assumed to be at an N_{re} greater than 2000, the laminar flow begins to break up in the center of the pipe and the friction factor sharply and discontinuously increases. Precisely where this happens cannot be determined. Nothing fully predictable happens until an N_{re} of at least 3250 is reached, at which point a partially turbulent regime may be assumed to be established and the friction factor is again decreasing with flow rate in a regular manner according to the smooth pipe law. As flow further increases, the turbulent core increases in diameter at the expense of the annular laminar outer layer. At some point the inherent roughness of the pipe limits further drop in friction factor. As stated previously, how the friction factor approaches its limit is not completely clear.

The mathematical representation of the above has presented serious problems to modelers from the beginning. The most common approach has been to use either the familiar Colebrook-White equation or a combination of the smooth pipe and rough pipe laws which accurately represent the partial and fully turbulent regions, use the Hagen - Poiseuille relationship for laminar flow, and connect the end of laminar flow, assumed at an N_{re} of 2000, and the beginning of the smooth pipe law, assumed at an N_{re} of 3250 with a straight line. This is without question the best that can be done. Its only questionable area is that it has reduced the ambiguity of the Critical zone to a representative equation, but that operation is essential for modeling.

A performance issue does arise however. The fact that neither the Colebrook - White equation nor the smooth pipe law are explicit in the partially turbulent zone (**f** appears on both sides of the equation) requires an iterative method for computation. Since the function is very well behaved and predictor-corrector techniques work very well, this only means that the computing requirements increase slightly. Alternative methods have been developed to provide an explicit, hence faster performing, method. An excellent comprehensive discussion of the available equations may be found in the Gregory and Fogarasi paper. Note that these equations are only defined for the partially and fully turbulent zones. Critical flow is still a problem where some appropriate modifications must be made. As an example, the equation of Chen, which is the most precise, is presented here:

$$\frac{1}{\sqrt{f}} = -4\log\left\langle \frac{e/D}{3.7065} - \frac{5.0452}{N_{re}}\log\left[\frac{\left(e/D\right)^{1.1096}}{2.8257} + \left(\frac{7.149}{N_{re}}\right)^{.8961}\right]\right\rangle$$

The critical zone presents another more serious numeric problem. First it must be stated that normal pipes through which any significant amount of fluid is flowing and which have any measurable headlosses will have $\mathbf{N_{re}}$ much greater than 3250. Still, very low flow pipes do occur in both liquid and gas systems. Since the friction factor can over double very quickly and go in the "wrong' direction in the $\mathbf{N_{re}}$ range of 2150-3250, gradient methods for solving a network using this equation simply may not converge to a unique solution.

One solution to this problem is to cut-off the laminar zone at the point where $\mathbf{f}$ drops to its value at the beginning of the partially turbulent zone and use that value as a constant for all points in between. This will make the fundamental pipe element inherently stable and introduce a maximum friction factor error of 45% at an $\mathbf{N_{re}}$ of 2300, which translates to a 20% error in flow, but so what? There is no flow or pressure drop in the first place! This "error" disappears completely outside the $\mathbf{N_{re}}$ range of 1400-3250 and should introduce no discernable change in simulation results for gas systems.

III. The "Practical" Equations

The fundamental flow equation as described above are universally accepted as the full and complete statement of how fluid flow works. Why then are there other equations in some use to describe this phenomenon? To understand the reasons for this we must review some history. Gas lines date back to England in the early 19th century and engineers have needed to determine their capacity ever since. It is difficult for many to recall the time as recently as the 1960's when there were no personal computers or even desk-top calculators that could do much more than add or subtract. Pocket calculators with extended mathematical functions? Forget it! As a personal note, I participated in the first computing class offered by Carnegie Institute of Technology in 1961. Slide rules and mental arithmetic were the orders of the day! In that environment an implicit relationship such as Colebrook-White, which was well-known then, was impractical and some simplification was essential. Specialized slide rules and nomographs were the order of the day for manually solving gas networks within a Hardy-Cross framework.

In the following discussions I will adopt the way of looking-at things established in the Hyman, Stoner, Karnitz paper since it provides the best common ground for comparison. Although the equations may appear to be very different, any real equation may be equated to the fundamental equation and the result solved for the friction factor. In this process many of the common factors cancel and the friction factor relationship is much simpler. The friction factor equation may then be superimposed on the Moody diagram to view the comparison, remembering that any difference in f affects flow as vf

One starting point for simplification is to recall that originally gas was distributed in low-pressure systems measured in inches of water column. Under these conditions the flow can be considered non-compressible and the fundamental equation may be reduced by factoring the $(P_1^2 - P_2^2)$ term into $(P_1 - P_2)(P_1 + P_2)$ and assuming an appropriate average pressure. The equation then becomes:

$$Q = \frac{C}{2}\frac{T_b}{P_b}D^{2.5}e\left(\frac{(P_1 - P_2)P_a}{LGT_a Zf}\right)^{.5}$$

That may not look like much but it's a help. In 1851 a Dr. Pole submitted one of the first flow equations to the natural gas industry. His equation is:

$$Q = 1350D^{2.5}\left(\frac{P_1 - P_2}{LG}\right)^{.5}$$

Where:

D	Pipe diameter (inches)
G	Gas specific gravity (dimensionless)
L	Pipe length (yards)
P_1	Inlet pressure ("$H_2$0)
P_2	Outlet pressure ("$H_2$0)
Q	Flow rate (standard cubic feet/hour)

The equivalent friction factor for this is:

$$f = .0291$$

Not bad for 1851! Note that .0291 is just about what one would expect for low pressure pipe under moderately low flow conditions.

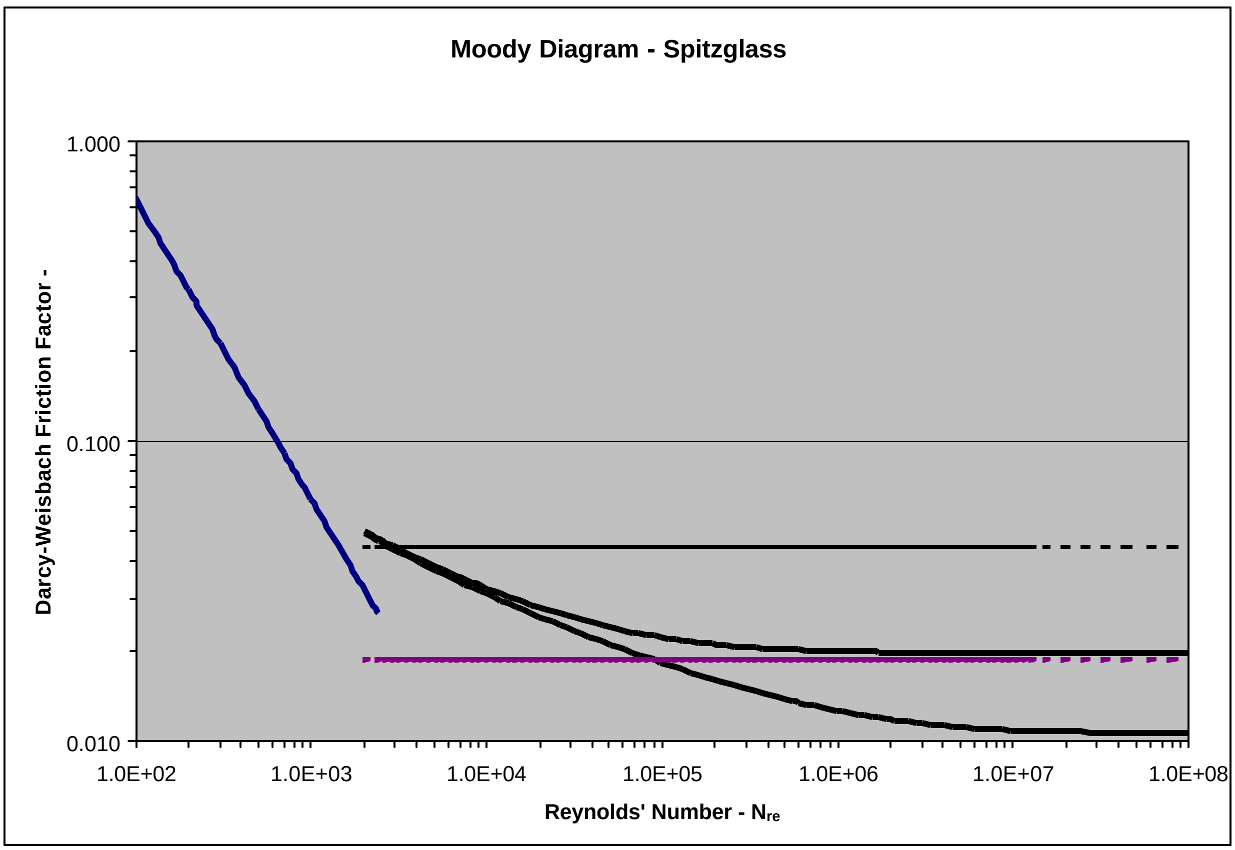
Constant friction factors are certainly not the most realistic. At the very least one would expect the friction factor to vary with the pipe diameter. Probably the oldest equation still in current use is the Spitzglass equation, first published in 1912, which comes in two flavors:

$$Q = 1172D^{3}e\left(\frac{(P_1 - P_2)P_a}{LG\,(D + 3.6 + .03D^{2})}\right)^{.5}$$ Low pressure ("H_2O)

$$Q = 1128D^{3}e\left(\frac{(P_1^{2} - P_2^{2})}{LG\,(D + 3.6 + .03D^{2})}\right)^{.5}$$ High pressure (PSIA)

The equivalent friction factor for this is:

$$f = \frac{4\left(1 + \frac{3.6}{D} + .03D\right)}{354}$$



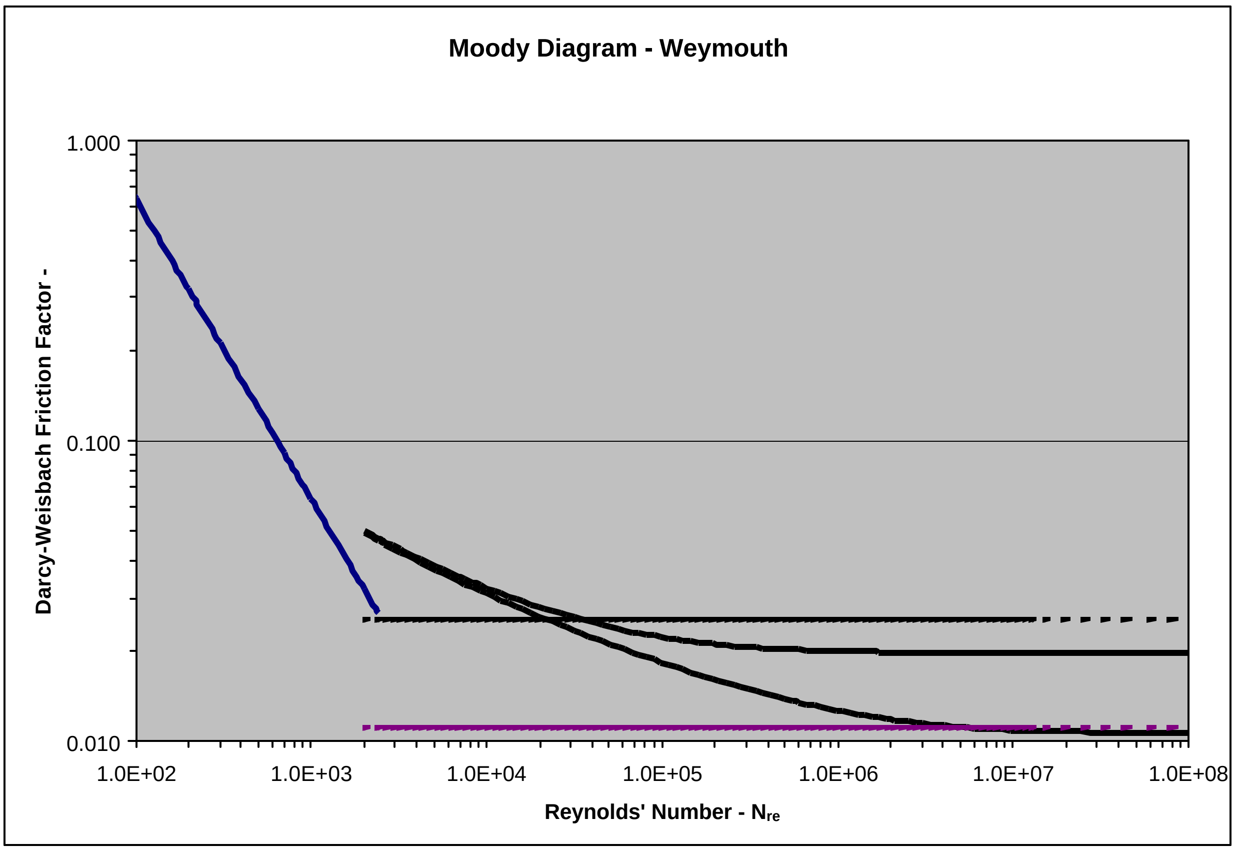
This equation is a minor improvement in that it allows the friction factor to vary with pipe size. Note however that the friction factor decreases with increasing pipe size, which is what one would expect for a constant roughness, but only to a diameter of about 10.95 inches, then it starts increasing! This behavior obviously is counter to what should happen although the range over which it may be observed, .0187 - .0246, is not too broad.

Turning to more transmission-like conditions, the Weymouth equation is another diameter varying friction equation, also dating from 1912, as follows:

$$Q = 433.49 \quad \frac{T_b}{P_b} D^{8/3} e \left(\frac{P_1^{\,2} - P_2^{\,2} - H_c}{LGT_a Z_a} \right)^{.5}$$

The equivalent friction factor for this is:

$$f = \frac{4}{\left(11.18D^{\,1/6}\right)^2}$$



Note that the friction factor decreases consistently with increasing pipe size, and it does so in a range of .008 - .02 that is more appropriate to high flow situations. This was one of the earlier transmission equations that has survived to this day. Proper tuning of efficiencies can make it work well, particularly in a design mode.

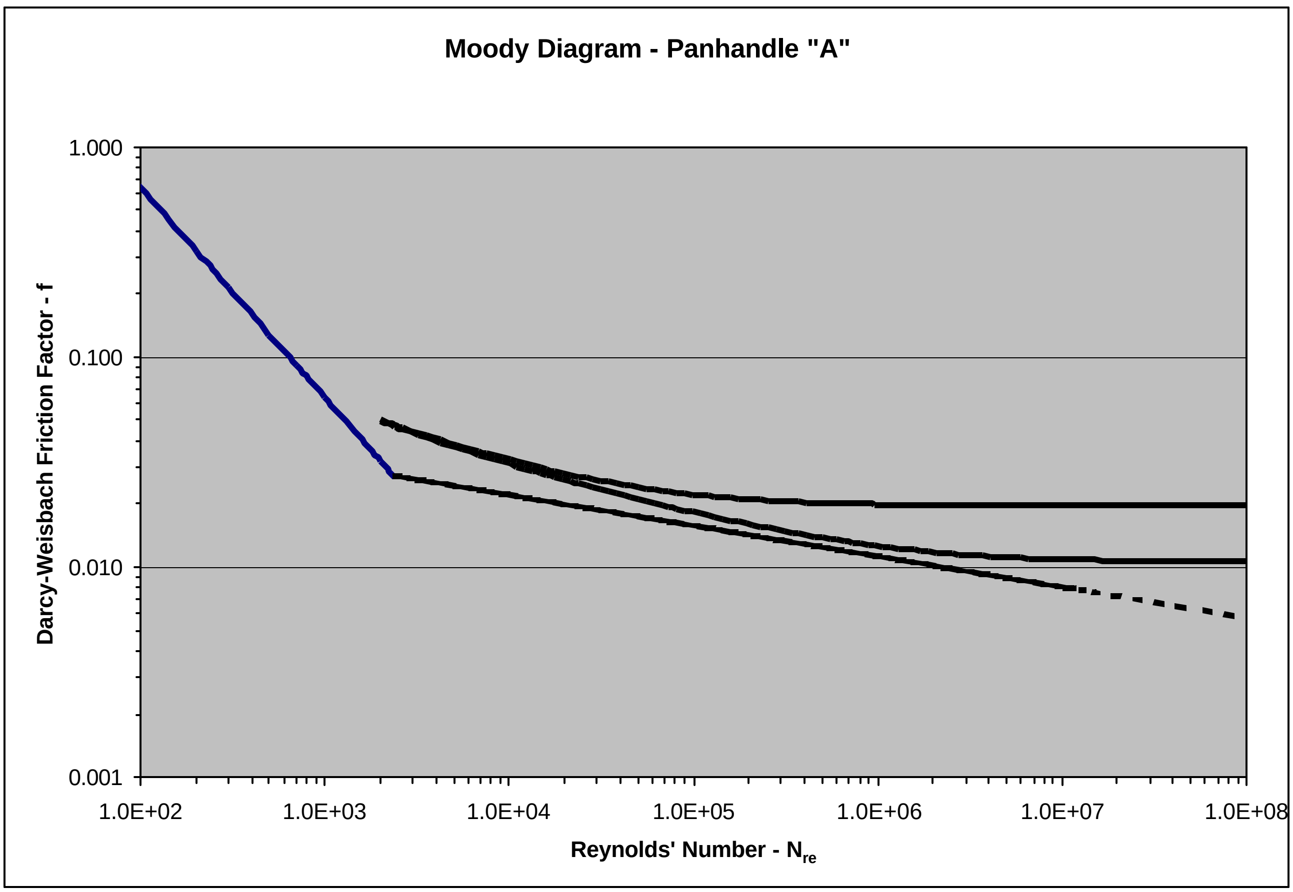
The next improvement comes in the form of equations that show a friction factor that varies with Reynolds' Number and hence flow, but do so in an explicit form. The first of these is the Panhandle A equation:

$$Q = 435.87 \left(\frac{T_b}{P_b}\right)^{1.0788} D^{2.6182}\ e\left(\frac{P_1^{\ 2} - P_2^{\ 2} - H_c}{LG^{.8538}\ T_a Z_a}\right)^{.5394} \qquad \text{(English)}$$

$$Q = .0045965 \left(\frac{T_b}{P_b}\right)^{1.0788} D^{2.6182}\ e\left(\frac{P_1^{\ 2} - P_2^{\ 2} - H_c}{LG^{.8538}\ T_a Z_a}\right)^{.5394} \qquad \textit{(Metric)}$$

The equivalent friction factor for this is:

$$f = \frac{4}{\left(6.87 N_{re}^{\ .07305}\right)^2}$$



Note that this line falls slightly below the Colebrook-White lines but that it can be moved up or down by using an appropriate efficiency. Its slope is appropriate for relatively low Reynolds' numbers.

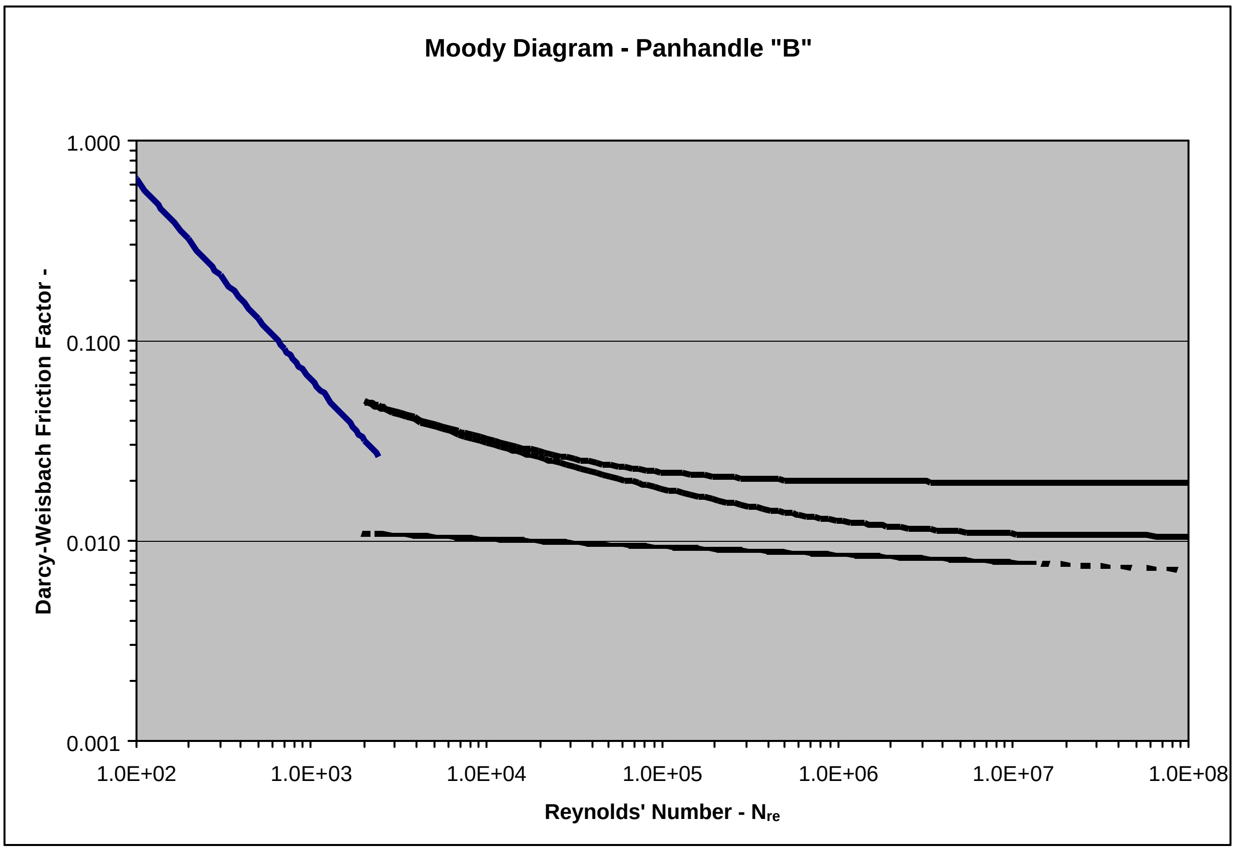
After recognizing that flow rates were increasing beyond those used in developing the correlation, the Panhandle B equation was proposed:

$$Q = 737 \left(\frac{T_b}{P_b}\right)^{1.02} D^{2.53}\, e\left(\frac{P_1^{\,2} - P_2^{\,2} - H_c}{LG^{.961}\, T_a Z_a}\right)^{.51} \qquad \text{(English)}$$

$$Q = .010019 \left(\frac{T_b}{P_b}\right)^{1.02} D^{2.53}\, e\left(\frac{P_1^{\,2} - P_2^{\,2} - H_c}{LG^{.961}\, T_a Z_a}\right)^{.51} \qquad \textit{(Metric)}$$

The equivalent friction factor for this is:

$$f = \frac{4}{\left(16.49 N_{re}^{\,.01961}\right)^2}$$



Note that this line falls below the Colebrook-White lines but that it can be moved up or down by using an appropriate efficiency. Its slope is appropriate for higher Reynolds' numbers than Panhandle "A".

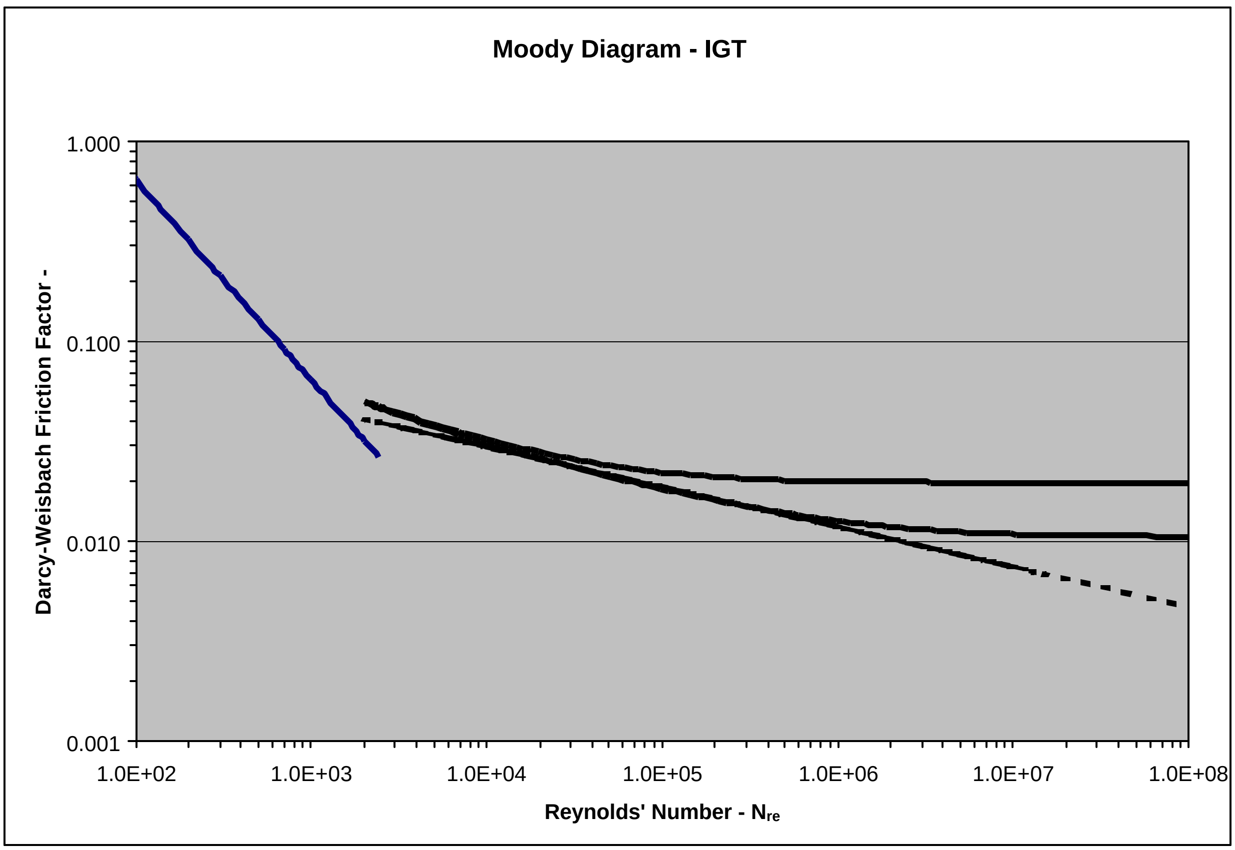
Not wanting to miss the boat, the Institute of Gas Technology also developed the IGT equation:

$$Q = 92.66\ \frac{T_b}{P_b}\frac{D^{8/3}}{g^{4/9}\mu^{1/9}}e\left(\frac{P_1^{\,2} - P_2^{\,2} - H_c}{LT_aZ_a}\right)^{5/9} \qquad \text{(English)}$$

$$Q = .0012753\ \frac{T_b}{P_b}\frac{D^{8/3}}{g^{4/9}\mu^{1/9}}e\left(\frac{P_1^{\,2} - P_2^{\,2} - H_c}{LT_aZ_a}\right)^{5/9} \qquad \textit{(Metric)}$$

The equivalent friction factor for this is:

$$f = \frac{4}{\left(4.619N_{re}^{\ .1}\right)^2}$$



All of these flow dependent equations share one common attribute: at low flow they are conservative and under-predict flow while at high flow overly optimistic and over-predict flow. The main difference is where low and high flow are defined.

IV. Some Conclusions and Observations

What does any of this mean? If we accept the Moody diagram as the definitive statement of fluid flow, which is seems like nearly everyone does, then all of the practical equations are simply ways of arriving at wrong answers more quickly. Of course, like the stopped clock that is right twice a day, they can give correct answers at some conditions particularly when tuned through proper use of efficiency, but why bother? Modern computing technology has rendered the computational effort a non-issue and one cannot use any of these without carefully computing Reynolds' Numbers and checking each pipe in each system to determine when the clock is right. Their proper place is on the bookshelf with the slide rule!

In the original 1975 paper, Sam Hyman went out of his way to state that "anyone continuing to use a fixed friction factor equation is not properly addressing the problem." I would rephrase that as "anyone not using a Moody Diagram based friction factor equation is not properly addressing the problem." Probably the worst thing that can be done in to use either the Panhandle "A" or IGT equations blithely without checking Reynolds' Numbers since the friction factor, and hence the associated pressure drops, can decrease without bound. The most common question asked when converting from these equations is "why did my pressure drops increase so much?" Since most distribution systems are rarely fully stressed (how often does design temperature really occur?), this is a potentially dangerous situation. To me, the proper way to approach an empirical problem is to go to the theory to get the correct shape of the solution and then to go to the data to refine the answers. This is precisely what the explicit friction factor relationships (Chen, Shacham, et. al) do, and what the empirical equations do not do.

A few words about performance are in order since this ultimately becomes a balancing act between precision and performance. To attempt to measure this I chose a fairly typical large integrated natural gas distribution system having somewhere in excess of 200,000 nodes and 500 regulators. By choosing this kind of network, the friction factor computations will be exercised over a wide range of flows and pressures. The following table shows the computational performance of this network relative to the most rigorous method, Darcy-Weisbach:

Friction Method	Relative Computational Time	Savings
Darcy-Weisbach	1.000	-
Chen	.913	9%
Shacham	.844	16%
IGT	.842	16%

This indicates that there is not a wide range in performance over all, and virtually no difference between the Shacham equation, which has the correct shape, and the IGT, which does not. Any differences become much less significant when we consider that this problem only requires nominally three minutes to solve on a fairly slow 333 megahertz Pentium II. Clearly we are talking about a difference of a few seconds on a more typical machine, not a high price for peace-of-mind.

This leads to four sensible alternatives for approaching flow equations:

1. Use the full Moody Diagram complete with the Hagen-Poiseuille equation for laminar flow, the Colebrook-White equation or the GERG approach for the rest of the flow and make some assumptions to traverse the critical zone. This is the most defendable course of action.

2. Use the Colebrook-White equation or the GERG approach for the flow above N_{re} of 3250 and do something smooth with f for N_{re} < 3250. In gas flow, misstating f in this range of N_{re} makes no practical difference and does eliminate some potential convergence problems.
3. Use the Chen or Shacham equation instead of Colebrook-White. No significant difference in results should occur but performance will improve as shown above.
4. Use the smooth pipe law of von Karman and Prandtl (or an explicit fit thereof) intersected with the rough pipe law of Nikuradse. This will generally produce a somewhat lower friction factor than Colebrook-White. Since the difference is not large and disappears at the extremes, this is a perfectly safe approach that may follow observed data more closely.

I suspect that using the new equation proposed by GERG as a means of interpolating between Colebrook-White and alternative 4 may be much more computationally intensive that any other, and I'm not sure what the basis for picking the degree of interpolation is. This is a wonderful area for more research and it is refreshing to see reexamination of the earlier work.

My personal choice for gas systems is alternative 3 with the misstated laminar region of alternative 2. This should give good conservative answers with the least amount of computational problems. For liquid systems alternative 1 is preferred since the laminar region may be significant.

Some further comments on flow equation migration are in order. Ultimately pipeline efficiency should be used to calibrate a model to reality. As stated earlier, roughness is really not much help and going much beyond the published tables is pointless. It is good for getting a correct shape to the flow function, but not much else. Matching reality involves three separate issues:

1. Accounting for problems with the flow equation,
2. Accounting for problems with the pipe such as bends, fittings, junk inside, and the like, and
3. Accounting for operational problems such as the relationship between daily and instantaneous flow.

The efficiencies used by many companies are a composite of these items. When considering a change in flow equation it is important to attempt to separate issue 1 from the others. It is reasonable to expect that the efficiency contributions from items 2 and 3 to remain constant whatever flow equation is used while issue 1 should change. For example, an efficiency of 85% could comprise:

1. A component because the friction factor is too low, and
2. A component because someone left a rag in the pipe, and
3. A component because the only flows we have are peak day and we need to design for peak hour.

Since the calibration is an overall process, this may not be a simple matter. What it means is that this is not a simple procedure of changing flow equations and using either the same efficiencies as before or ignoring efficiency and using 100%. Efficiency must be reviewed on a more local basis.

Appendix A. Nomenclature and Units

Symbol	Definition	English Unit	Metric Unit	Factor
D	Pipe diameter	inches	*mm*	.03937
e	Pipe efficiency	-	-	-
f	Darcy-Weisbach friction factor	-	-	-
g	Gas specific gravity	-	-	-
H_c	Elevation correction	$PSIA^2$	*Kpa^2*	.021034
H_1	Inlet elevation	feet	*meters*	3.2808
H_2	Outlet elevation	feet	*meters*	3.2808
L	Pipe length	miles	*Km*	.62137
N_{re}	Reynolds' number	-	-	-
P_{av}	Average pressure	PSIA	*KPascals*	.14503
P_b	Pressure base	PSIA	*KPascals*	.14503
P_1	Inlet pressure	PSIA	*KPascals*	.14503
P_2	Outlet pressure	PSIA	*KPascals*	.14503
Q	Standard Flow rate	ft^3/day	*m^3/day*	35.315
T_a	Average temperature	°R	*°K*	1.8
T_b	Temperature base	°R	*°K*	1.8
v	fluid velocity	ft/sec	*m/sec*	3.2808
Z_a	compressibility factor	-	-	-
Z_1	compressibility factor at inlet conditions	-	-	-
Z_2	compressibility factor at outlet conditions	-	-	-
ε	Pipe roughness	inches	*mm*	.03937
μ	Gas viscosity	lb_f-sec/ft^2	*pascal-sec*	.0020886
ρ	fluid density	lb/ft^3	*Kg/m^3*	.062417

The stated factor multiplies the Metric unit value to reach the English unit value. For example:

1 meter X 3.2808 = 3.2808 feet

Appendix B. Bibliography

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