Optimisation of autofrettage in thick-walled cylinders

G.H. Majzoobi *, A. Ghomi
Mechanical Engineering Department, Faculty of Engineering, Bu-Ali Sina University, Hamadan, Iran
* Corresponding author: E-mail address: gh_majzoobi@yahoo.co.uk

Received 15.11.2005; accepted in revised form 15.04.2006

ABSTRACT

Purpose: The purpose of this paper is optimization of the weight of compound cylinder for a specific pressure. The variables are shrinkage radius and shrinkage tolerance.

Design/methodology/approach: SEQ technique for optimization, the finite element code, ANSYS for numerical simulation are employed to predict the optimized conditions. The results are verified by testing a number of closed end cylinders with various geometries, materials and internal pressures.

Findings: The weight of a compound cylinder could reduce by 60% with respect to a single steel cylinder. The reduction is more significant at higher working pressures. While the reduction of weight is negligible for \( k < 2.5 \), it increases markedly for \( 2.5 < k < 5.5 \). The stress at the internal radii of the outer and inner cylinders become equal to the yield stresses of the materials used for compound cylinders. The experimental results showed higher bursting pressure for optimized cylinders.

Research limitations/implications: The research must be done for non-linear material models and for multiple compound cylinders.

Practical implications: The results can be used for high pressure vessels such as artillery tubes, gun barrels and son on.

Originality/value: The numerical results indicated that for an optimum condition, the stress at the internal radii of the outer and inner cylinders become equal to the yield stresses of the materials used for compound cylinders.

Keywords: Analysis and modelling; Numerical techniques; Compound cylinder; Optimisation; Finite element method

1. Introduction

Autofrettage is an elastic-plastic techniques to increase the pressure capacity of thick-walled cylinders. In this technique, the cylinder is subjected to an internal pressure so that its wall becomes partially plastic. The pressure is then released and the resulting residual stresses increase the pressure capacity of the cylinder in the next loading stage [1 & 2]. The analysis of residual stresses and deformation in an autofretaged thick-walled cylinder has been given by Chen [3] and Franklin and Morrison [4].

In autofrettage, the key problem is to determine the appropriate the optimum of the radius of elasto-plastic junction to be called \( r_{opt} \) in this paper and the autofrettage pressure, \( p_{opt} \).

In the results reported by Harvey [6], no detailed result but only a concept about autofrettage was given. Brownell and Young [7], and Yu [8] proposed a repeated trial calculation method to determine \( r_{opt} \) which were a bit too tedious and inaccurate; moreover this method is based on limiting only hoop stress and is essentially based on the first strength theory which is in agreement with brittle materials, while pressure vessels are made generally from ductile materials [9 & 10] which are in excellent agreement with the third or the fourth strength theory [11 & 13]. The graphic method presented by Kong [12] was also a bit too tedious and inaccurate. The purpose of the present paper is to find out a simple, applicable,
and reasonable approach to determine $r_{opt}$, which does not appear to be available in the existing literature, and to study problems about autofrettage.

Based on the third and the fourth strength theory, Zhu and Yang [14] presented an analytic equation for optimum radius of elastic-plastic juncture, $r_{opt}$ in autofrettage technology. Their work presented the influence of autofrettage on stress distribution and load-bearing capacity of a cylinder and optimum pressure in autofrettage technology was studied. The work by Zhu and Yang considers only elastic-perfectly material model which obviously is not the case for most of the applications.

According to work given by Zhu and Yang [14], the optimum elasto-plastic radius, $r_{opt}$, and optimum autofrettage pressure, $p_{opt}$, are calculated from the relations given below:

(a) In view of the third strength theory:

$$r_{opt} = r \exp \left( \frac{p}{\sigma_y} \right)$$

$$p_{opt} = \frac{\sigma_y}{2} \left[ 1 - \left( 1 - \frac{2p}{\sigma_y} \right) \exp \left( \frac{2p}{\sigma_y} \right) \right] + p$$

(b) In view of the fourth strength theory:

$$r_{opt} = r \exp \left( \sqrt[3]{\frac{3p}{\sigma_y}} \right)$$

$$p_{opt} = \frac{\sigma_y}{\sqrt[3]{3}} \left[ 1 - \left( 1 - \sqrt[3]{\frac{3p}{\sigma_y}} \right) \exp \left( \sqrt[3]{\frac{3p}{\sigma_y}} \right) \right] + p$$

In the present work, the optimization techniques, numerical simulation and experiments are employed to predict the optimized autofrettage radius. SEQ technique is used for optimization purposes. A finite element code, is employed for numerical simulations and finally, the results are verified by testing a number of closed end cylinders at various autofrettage pressures. The results are then compared with those given by Zhu and Yang [14].

2. Analytical relations

The distribution of radial and hoop stresses within the elastic region and plastic core can be described as follow:

For elastic perfectly plastic material:

In elastic region, $a \leq r \leq c$ :

$$\sigma_r = \frac{k^2 \rho_a - p_c}{1 - k^2} \left( \frac{b}{r} \right)^2$$

$$\sigma_\theta = \frac{k^2 \rho_a - p_c}{1 - k^2} \left( \frac{b}{r} \right)^2$$

In plastic region, $c \leq r \leq b$ :

$$\sigma_r = \frac{\sigma_c}{2} \left[ \frac{c^2}{b^2} - 1 - \ln \left( \frac{c}{r} \right) \right]$$

For elastic-plastic material with linear strain hardening:

In plastic region, $a \leq r \leq c$ , for an internal pressure $P_i$ :

$$\sigma_r = \frac{\sigma_c}{2} \left[ 1 + \frac{c^2}{b^2} \ln r \right] + \frac{1}{k^2 - 1} \left( \ln \frac{c^2}{a^2} + 1 - \frac{c^2}{b^2} \right)$$

$$\sigma_\theta = \frac{\sigma_c}{2} \left[ 1 + \frac{c^2}{b^2} \ln r \right] + \frac{1}{k^2 - 1} \left( \ln \frac{c^2}{a^2} + 1 - \frac{c^2}{b^2} \right)$$

When the cylinder is pressurized to the autofrettage pressure and the pressure is removed, the residual stress distribution across the wall of the cylinder can be expressed as follow [15]:

For elastic-perfectly plastic material:

Residual stresses in plastic region $a \leq r \leq c$ :

$$\sigma_r = \frac{\sigma_c}{2} \left[ 1 + \frac{c^2}{b^2} \ln r \right] + \frac{1}{k^2 - 1} \left( \ln \frac{c^2}{a^2} + 1 - \frac{c^2}{b^2} \right)$$

Residual stresses in elastic region $c \leq r \leq b$ :

$$\sigma_r = \frac{\sigma_c}{2} \left[ 1 + \frac{c^2}{b^2} \ln r \right] + \frac{1}{k^2 - 1} \left( \ln \frac{c^2}{a^2} + 1 - \frac{c^2}{b^2} \right)$$

For elasto-plastic material:

Residual stresses in plastic region $a \leq r \leq c$ :

$$\sigma_r = \frac{\sigma_c}{2} \left[ 1 + \frac{c^2}{b^2} \ln r \right] + \frac{1}{k^2 - 1} \left( \ln \frac{c^2}{a^2} + 1 - \frac{c^2}{b^2} \right)$$

$$\sigma_\theta = \frac{\sigma_c}{2} \left[ 1 + \frac{c^2}{b^2} \ln r \right] + \frac{1}{k^2 - 1} \left( \ln \frac{c^2}{a^2} + 1 - \frac{c^2}{b^2} \right)$$
\[ \sigma_{\text{eq}} = -\sigma_r \left( \frac{b^2}{r^2} + 1 \right) - \frac{c^2}{b^2} + \frac{2 \ln \frac{c}{a} + (1 - \nu^2) \frac{E_p}{E} \left( \frac{c^2}{a^2} - \frac{c^2}{b^2} \right)}{2 \left( k^2 - 1 \right) + 1} \]

\[ \sigma_{\text{eq}} = \frac{c \sigma_r}{2} + \frac{b^2}{r^2} - 1 - \frac{1}{k^2 - 1} \left( \ln \frac{a^2}{b^2} + 1 \right) + p \frac{b^2}{r^2} - 1 \]

Overall stresses in elastic region \( a \leq r \leq c \):

\[ \sigma_{\text{eq}} = \sigma_c + \sigma_r = \frac{c^2}{b^2} - 1 - \frac{1}{k^2 - 1} \left( \ln \frac{a^2}{b^2} + 1 \right) + p \frac{b^2}{r^2} - 1 \]

For elastic-plastic material:

Overall stresses in plastic region \( a \leq r \leq c \):

\[ \sigma_{\text{eq}} = \frac{c^2}{b^2} - 1 - \frac{1}{k^2 - 1} \left( \ln \frac{a^2}{b^2} + 1 \right) + p \frac{b^2}{r^2} - 1 \]

General solution for optimization problem:

\[ \min f(x) \]

s.t. \( g(x) \leq 0 \]

To find the optimal solution, the problem is solved using the Sequential Quadratic Programming (SQP) technique. The results are presented in Tables 1 and 2 and illustrated graphically in Figures 2 and 3 for aluminum and steel, respectively.

Table 1:

<table>
<thead>
<tr>
<th>P</th>
<th>( b(m) )</th>
<th>( c(m) )</th>
<th>k</th>
<th>r</th>
</tr>
</thead>
<tbody>
<tr>
<td>58</td>
<td>0.0184</td>
<td>0.0162</td>
<td>1.84</td>
<td>2.02</td>
</tr>
<tr>
<td>46</td>
<td>0.0163</td>
<td>0.0147</td>
<td>1.63</td>
<td>1.41</td>
</tr>
<tr>
<td>42</td>
<td>0.0157</td>
<td>0.0142</td>
<td>1.57</td>
<td>1.24</td>
</tr>
<tr>
<td>34</td>
<td>0.0144</td>
<td>0.0133</td>
<td>1.44</td>
<td>0.91</td>
</tr>
</tbody>
</table>

Fig. 1: Two specimens before and after testing
\( X = (x_1, x_2, x_3)^T \)

In which \( x_1 = a \) is the inner radius of the internal cylinder, \( x_2 = c \) is the radius of elasto-plastic junction and \( x_3 = b \) is the outer radius of the cylinder. The objective function, \( f(x) \), is the weight of compound cylinder. The constraints of the problem are defined as follow:

1- \( g_1(X) = \sigma_{eq} - \sigma_y \leq 0 \). This implies that equivalent stress at the inner surface of the cylinder should not exceed the yield stress, \( \sigma_y \).

2- \( g_2(X) = \sigma_{eq} - \sigma_y \leq 0 \). This implies that equivalent stress at the outer surface of the cylinder should not exceed the yield stress, \( \sigma_y \).

3- \( g_3(X) = u_{r1} - u_a \leq 0 \). This is an optional constraint implying that the radial displacement at the inner radius of compound cylinder must remain less than a specific value, \( u_a \).

4- \( x_1 \) and \( x_3 \) have always to be positive and are forced to remain within the range \( a \leq x \leq a_2 \).

5- \( x_2 \) must be less than the outer radius and greater than the inner one.

The optimization problem can be summarized as follows:

Minimize: \( f(x) = \pi \gamma (x_1^2 - x_2^2) \)

Subject to:

\[
g_1(x_1, x_2, x_3) = \sigma_{eq} - \sigma_y \leq 0
\]

\[
g_2(x_1, x_2, x_3) = \sigma_{eq} - \sigma_y \leq 0
\]

\[
g_3(x_1, x_2, x_3) = u_{r1} - u_a \leq 0
\]

\[
g_4(x_1, x_2, x_3) = x_1 - x_2 \leq 0
\]

\[
g_5(x_1, x_2, x_3) = x_2 - x_3 \leq 0
\]

\[
g_6(x_1, x_2, x_3) = x_3 - a_2 \leq 0
\]

Where, \( \gamma \) is the specific weight of the cylinder.

SQP technique [17 & 18] was employed for optimization process which was performed using MATLAB software.

### 4. Material and specimens

Two materials, aluminum and steel were used for optimization purposes. The material’s properties are exactly the same as those used by Majzoobi and Ghomi [21].

The material’s properties and the geometry of the specimens (only aluminum alloy) used for burst tests were different from those used for optimization purposes. Figure 1 illustrates the shape of the specimens adopted from the work of Manning [19 & 20]. The ratio of inner to outer radii was 2.2, \( k = b/a = 2.2 \), for all specimens. The material had a yield stress of \( \sigma_y = 190 \text{MPa} \). Figure 1 shows two specimens, one before testing and one after bursting. The gauge length of the specimens was 70 mm for all specimens.

### 5. Optimization results

The optimization problem defined in section 4 was performed for aluminum and steel cylinders with \( a = 0.01 \text{ m}, b = 0.022 \text{ m} \) under the following conditions:

#### Table 1: The results for aluminum cylinders

<table>
<thead>
<tr>
<th>( P_1 ) (MPa)</th>
<th>( b ) (m)</th>
<th>( c ) (m)</th>
<th>( k ) (kg/m)</th>
<th>( t_{j(opt)} ) (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>30</td>
<td>0.0138</td>
<td>0.0129</td>
<td>1.38</td>
<td>0.77</td>
</tr>
<tr>
<td>34</td>
<td>0.0144</td>
<td>0.0133</td>
<td>1.44</td>
<td>0.91</td>
</tr>
<tr>
<td>38</td>
<td>0.0150</td>
<td>0.0138</td>
<td>1.50</td>
<td>1.06</td>
</tr>
<tr>
<td>42</td>
<td>0.0157</td>
<td>0.0142</td>
<td>1.57</td>
<td>1.24</td>
</tr>
<tr>
<td>46</td>
<td>0.0163</td>
<td>0.0147</td>
<td>1.63</td>
<td>1.41</td>
</tr>
<tr>
<td>50</td>
<td>0.0170</td>
<td>0.0152</td>
<td>1.70</td>
<td>1.60</td>
</tr>
<tr>
<td>54</td>
<td>0.0177</td>
<td>0.0157</td>
<td>1.77</td>
<td>1.81</td>
</tr>
<tr>
<td>58</td>
<td>0.0184</td>
<td>0.0162</td>
<td>1.84</td>
<td>2.02</td>
</tr>
<tr>
<td>62</td>
<td>0.0191</td>
<td>0.0167</td>
<td>1.91</td>
<td>2.25</td>
</tr>
<tr>
<td>66</td>
<td>0.0199</td>
<td>0.0173</td>
<td>1.99</td>
<td>2.51</td>
</tr>
<tr>
<td>70</td>
<td>0.0206</td>
<td>0.0178</td>
<td>2.06</td>
<td>2.75</td>
</tr>
<tr>
<td>74</td>
<td>0.0214</td>
<td>0.0184</td>
<td>2.14</td>
<td>3.04</td>
</tr>
<tr>
<td>76.5</td>
<td>0.0219</td>
<td>0.0187</td>
<td>2.19</td>
<td>3.22</td>
</tr>
</tbody>
</table>

For elastic-perfectly plastic material:

The autofrettage pressure varied between 30 to 75.5 MPa for aluminum cylinders and between 300 to 600 MPa for steel cylinders. From the optimizations, the optimum value of the radius of elasto-plastic junction, \( c \), and weight of the cylinder were obtained. The results and their comparison with those predicted by analytical approach given by Zhu and Yang [14] are summarized in Tables 1 and 2 and illustrated graphically in figures 2 & 3 for aluminum and steel, respectively.
It can be seen from the tables and the figures that, in the first place, c increases as the working pressure increases, and in the second place, the average difference between the results obtained in this work and those given by Zhu and Yang is only 6.8% for aluminum and 7.4% for steel cylinders, respectively.

(b) elastic-plastic with linear strain hardening:
The results in this case are summarized in Tables 3 and 4 and depicted in figures 4 & 5 for aluminum and steel, respectively.

Table 2:
The results for steel cylinders

<table>
<thead>
<tr>
<th>P (MPa)</th>
<th>b(m)</th>
<th>c(m)</th>
<th>k</th>
<th>W (kg/m)</th>
<th>r_{j(opt)} (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>300</td>
<td>0.0151</td>
<td>0.0138</td>
<td>1.51</td>
<td>3.16</td>
<td>0.0145</td>
</tr>
<tr>
<td>350</td>
<td>0.0162</td>
<td>0.0146</td>
<td>1.62</td>
<td>4.01</td>
<td>0.0154</td>
</tr>
<tr>
<td>400</td>
<td>0.0172</td>
<td>0.0154</td>
<td>1.72</td>
<td>4.83</td>
<td>0.0164</td>
</tr>
<tr>
<td>450</td>
<td>0.0184</td>
<td>0.0162</td>
<td>1.84</td>
<td>5.88</td>
<td>0.0174</td>
</tr>
<tr>
<td>500</td>
<td>0.0195</td>
<td>0.0170</td>
<td>1.95</td>
<td>6.91</td>
<td>0.0186</td>
</tr>
<tr>
<td>550</td>
<td>0.0208</td>
<td>0.0179</td>
<td>2.08</td>
<td>8.20</td>
<td>0.0197</td>
</tr>
<tr>
<td>600</td>
<td>0.0220</td>
<td>0.0188</td>
<td>2.20</td>
<td>9.47</td>
<td>0.0210</td>
</tr>
</tbody>
</table>

Fig. 2. Variation of c versus working pressure for aluminum cylinders

Fig. 3. Variation of c versus working pressure for steel cylinders

Table 3:
The results for aluminum cylinders

<table>
<thead>
<tr>
<th>P (MPa)</th>
<th>b(m)</th>
<th>c(m)</th>
<th>k</th>
<th>W (kg/m)</th>
<th>r_{j(opt)} (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>30</td>
<td>0.0138</td>
<td>0.0129</td>
<td>1.38</td>
<td>0.77</td>
<td>0.0133</td>
</tr>
<tr>
<td>34</td>
<td>0.0144</td>
<td>0.0134</td>
<td>1.44</td>
<td>0.91</td>
<td>0.0139</td>
</tr>
<tr>
<td>38</td>
<td>0.0150</td>
<td>0.0138</td>
<td>1.50</td>
<td>1.06</td>
<td>0.0144</td>
</tr>
<tr>
<td>42</td>
<td>0.0156</td>
<td>0.0143</td>
<td>1.56</td>
<td>1.22</td>
<td>0.0150</td>
</tr>
<tr>
<td>46</td>
<td>0.0163</td>
<td>0.0148</td>
<td>1.63</td>
<td>1.41</td>
<td>0.0156</td>
</tr>
<tr>
<td>50</td>
<td>0.0169</td>
<td>0.0152</td>
<td>1.69</td>
<td>1.57</td>
<td>0.0162</td>
</tr>
<tr>
<td>54</td>
<td>0.0176</td>
<td>0.0158</td>
<td>1.76</td>
<td>1.78</td>
<td>0.0168</td>
</tr>
<tr>
<td>58</td>
<td>0.0183</td>
<td>0.0163</td>
<td>1.83</td>
<td>1.99</td>
<td>0.0175</td>
</tr>
<tr>
<td>62</td>
<td>0.0190</td>
<td>0.0168</td>
<td>1.90</td>
<td>2.21</td>
<td>0.0182</td>
</tr>
<tr>
<td>66</td>
<td>0.0197</td>
<td>0.0173</td>
<td>1.97</td>
<td>2.44</td>
<td>0.0189</td>
</tr>
<tr>
<td>70</td>
<td>0.0204</td>
<td>0.0179</td>
<td>2.04</td>
<td>2.68</td>
<td>0.0196</td>
</tr>
<tr>
<td>74</td>
<td>0.0212</td>
<td>0.0184</td>
<td>2.12</td>
<td>2.96</td>
<td>0.0204</td>
</tr>
<tr>
<td>78</td>
<td>0.0219</td>
<td>0.0190</td>
<td>2.19</td>
<td>3.22</td>
<td>0.0212</td>
</tr>
</tbody>
</table>

Table 4:
The results for steel cylinders

<table>
<thead>
<tr>
<th>P (MPa)</th>
<th>b(m)</th>
<th>c(m)</th>
<th>k</th>
<th>W (kg/m)</th>
<th>r_{j(opt)} (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>300</td>
<td>0.0151</td>
<td>0.0139</td>
<td>1.51</td>
<td>3.16</td>
<td>0.0145</td>
</tr>
<tr>
<td>350</td>
<td>0.0161</td>
<td>0.0146</td>
<td>1.61</td>
<td>3.93</td>
<td>0.0154</td>
</tr>
<tr>
<td>400</td>
<td>0.0172</td>
<td>0.0154</td>
<td>1.72</td>
<td>4.83</td>
<td>0.0164</td>
</tr>
<tr>
<td>450</td>
<td>0.0183</td>
<td>0.0162</td>
<td>1.83</td>
<td>5.79</td>
<td>0.0174</td>
</tr>
<tr>
<td>500</td>
<td>0.0194</td>
<td>0.0171</td>
<td>1.94</td>
<td>6.82</td>
<td>0.0186</td>
</tr>
<tr>
<td>550</td>
<td>0.0206</td>
<td>0.0180</td>
<td>2.06</td>
<td>8.00</td>
<td>0.0197</td>
</tr>
<tr>
<td>600</td>
<td>0.0218</td>
<td>0.0189</td>
<td>2.18</td>
<td>9.25</td>
<td>0.0210</td>
</tr>
<tr>
<td>610</td>
<td>0.0220</td>
<td>0.0190</td>
<td>2.20</td>
<td>9.47</td>
<td>0.0213</td>
</tr>
</tbody>
</table>

Fig. 4. Variation of c versus working pressure for aluminum cylinders

Again, the difference between the results obtained in this work and those given by Zhu and Yang is only 6.4% for aluminum and 7.1% for steel cylinders, respectively. Therefore, it may be concluded that (a) there is not significant difference...
between elastic perfectly plastic and elastic plastic with strain
hardening material models for prediction of optimum radius of
elastic plastic junction in autofrettage process and (b) the
analytical results given by Zhu and Yang is confirmed by
optimization results obtained in this investigation.

The results given in Table 5 clearly show a distinct
discrepancy between the optimum pressures predicted by Zhu
and Yang and the numerical results obtained in this work. As
seen in the Table 5, the numerical results for optimum
autofrettage pressures vary between $P_{y1}=347$ MPa and
$P_{y2}=555$ MPa as calculated above. This is more consistent with
experimental results given in the next section.

6. Numerical results

Single cylinders with the dimensions; $a=0.1$ m, $b=0.2$ m and
an elastic perfectly plastic material’s model with $\sigma_y = 800$ MPa
were used for numerical modeling. The two pressure limits $P_{y1}$
and $P_{y2}$ can be computed as follows [1 & 21]:

$$P_{y1} = \frac{\sigma_y}{\sqrt{3}}(1-1/k^2) = \frac{800}{\sqrt{3}}(1-1/2^2) = 347 \text{ MPa}$$

$$P_{y2} = \sigma_y \ln(k) = 800 \ln(2) = 555 \text{ MPa}$$

The cylinders were subjected to autofrettage pressures ranging
from 350 MPa to 650 MPa. After removing the autofrettage
pressure (AP), the cylinders were subjected to the working
pressures of 100, 200 and 400 MPa. From the numerical
simulations, the curve of von-Mises stress distribution was
obtained for each autofrettage and working pressure (WP). From
the curve, the value and the position of maximum von-Mises
(MVS) stress were extracted. This stress and its position were
then plotted versus autofrettage pressure for each working
pressure. The results are shown in figure 6.

It is observed that for each working pressure, the MVS
remains constant up to an autofrettage pressure which is nearly
equal to $P_{y1}$. The curve then begins to decline to a certain point
thereafter begins to rise or remain constant. It can be seen that for
all WPs, the rising portion of the curves end at a point which is
nearly equal to $P_{y2}$.

From the numerical results, it can be concluded that: (i) the
MVS depends on the working pressure and for any WP, the best
AP lies between $P_{y1}$ and $P_{y2}$; (ii) for autofrettage pressures
lower than $P_{y1}$ and higher than $P_{y2}$ the MVS remains
unchanged; (iii) the position of MVS moves towards the outer
radius as AP increases, and for autofrettage pressures higher than $P_{y2}$ does not change.

Fig. 6. Variation of maximum von-Mises stress versus autofrettage
pressure at three working pressures

Now, let examine the accuracy of equations for computation of optimum autofrettage pressure (Eq. 4) given by Zhu and Yang. For the material used in the numerical simulations, $\sigma_y = 800$ MPa , and the working pressures, 100, 200 and 400
MPa, the optimum autofrettage pressures calculated from

$$P_{opt} = \frac{800}{\sqrt{3}}(1-1/2^2) = 347 \text{ MPa}$$

Table 5

<table>
<thead>
<tr>
<th>Working pressure, $p$</th>
<th>100</th>
<th>200</th>
<th>400</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_{opt}$ (equation 4)</td>
<td>113</td>
<td>258</td>
<td>714</td>
</tr>
<tr>
<td>$P_{opt}$ (figure 6)</td>
<td>440</td>
<td>450</td>
<td>550</td>
</tr>
</tbody>
</table>

The results given in Table 5 clearly show a distinct
discrepancy between the optimum pressures predicted by Zhu
and Yang and the numerical results obtained in this work. As
seen in the Table 5, the numerical results for optimum
autofrettage pressures vary between $P_{y1}=347$ MPa and
$P_{y2}=555$ MPa as calculated above. This is more consistent with
experimental results given in the next section.
The results for elastic-plastic material with linear strain hardening are plotted in Figure 7 for working pressure of 200 MPa. As the figure suggests, the two material models, yield nearly the same results for the minimum von-Mises stress and differ only at the autofrettage pressures higher than $P_{1,2}$ which are not of concern in this work. This trend was observed for other working pressures.

7. Experimental results

The experiments were carried out using a high pressure pump with a pressure capacity ranging from 1 to 275 MPa (40000 psi). The diag gauge of the pump had a minimum division of 3.5 MPa. Therefore an error of about ±1.75 MPa was expected for each reading. The bursting pressure for autofrettaged cylinders are given in Table 6. The results have been adopted from the work by Majzoobi et al [2]. In order to see the differences more clearly, the pressures are given in psi in parenthesis.

Table 6

<table>
<thead>
<tr>
<th>Autofrettage pressure (psi)</th>
<th>Bursting pressure (psi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(22500)</td>
<td>(18000)</td>
</tr>
<tr>
<td>115.1</td>
<td>124.1</td>
</tr>
<tr>
<td>148.2</td>
<td>151.7</td>
</tr>
</tbody>
</table>

The values of $P_{1,2}$ and $P_{2,2}$ as calculated from equations quoted above, are $P_{1,2} = 115.2$ MPa (16700 psi) and $P_{2,2} = 150$ MPa (21700 psi). It can be observed from Table 6 that the bursting pressure increases as autofrettage pressure rises but remains less than $P_{1,2}$.

However, when the pressure exceeds $P_{2,2}$, the bursting pressure begins to decrease. This is quite consistent with the results extracted from figure 6. As it was explained in section 6, the minimum von-Mises stress lies between $P_{1,2}$ and $P_{2,2}$. As working pressure increases, autofrettage pressure also increases and for high working pressures it approaches $P_{2,2}$.

8. Conclusions

From the optimization, numerical and experimental results the following conclusion can be derived:
1. The optimum radius of elastic-plastic junction in autofrettaged cylinder does not differ significantly for elastic-perfectly plastic material compared with those obtained using an elastic-plastic with strain hardening material’s model.
2. The results verify the analytical solution for optimizing the radius of elastic-plastic junction given by Zhu and Yang.
3. The optimum pressure predicted by Zhu and Yang are significantly different from the numerical predictions in this work.
4. The numerical results revealed that the optimum autofrettage pressure is obtained when von-Mises equivalent stress across the wall of the cylinder attains its minimum value.
5. The numerical predictions are verified by experimental results.

Acknowledgements

The authors wish to thank Dr Maleki, the Dean of Faculty of Engineering, Bu-Ali Sina University for his support. They also like to thank Mr. Heidari for his contribution in preparation of specimens and conducting the experiments.

References

However, when the pressure exceeds a certain value, the bursting pressure for autofrettaged cylinders are significantly different from the numerical predictions in this work. The values of bursting pressure are tabulated in Table 6.

Table 6

<table>
<thead>
<tr>
<th></th>
<th>Bursting Pressure</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(MPa)</td>
</tr>
<tr>
<td>1</td>
<td>115.2 (16700 psi)</td>
</tr>
<tr>
<td>2</td>
<td>150.0 (21700 psi)</td>
</tr>
<tr>
<td>3</td>
<td>148.2 (21500 psi)</td>
</tr>
</tbody>
</table>

Therefore, an error of about 1.75 MPa was expected for each material compared with those obtained using an elastic-plastic analysis. The diag gauge of the pump had a minimum division of 3.5 MPa. As the figure suggests, the two material models, yield hardening and isotropic hardening are plotted in figure 7 for working pressure of 200 MPa. The results for elastic-plastic material with linear strain hardening are also plotted for working pressure of 200 MPa.

The optimum radius of elastic-plastic junction in autofrettaged cylinders does not differ significantly for elastic-perfectly plastic and perfectly plastic behaviour of material. The authors wish to thank Dr Maleki, the Dean of Faculty of Mathematical Sciences and Computer Engineering, Bu-Ali Sina University for his support. They also like to thank Mr. Heidari for his contribution in preparation of this project.