Xenon diffusion through seals ***DRAFT*** D. Shuman, 6/17/2011

first some units and constants

$R := 8.314 J \cdot mol^{-1} \cdot K^{-1}$ $M_{a_Xe} := 136 gm \cdot mol^{-1}$

1 standard cc is: $scc := \frac{1 \cdot atm \cdot 1 \cdot cm^3}{R \cdot 273K}$ $scc = 4.464 \times 10^{-5} mol$

1 standard cubic meter is: $scm := 10^6 scc$ $scm = 44.642 mol$

Next, some possible self-diffusion coefficients:

from Diffusion Coefficients of Xenon in Polystyrene determined by Xe-129 NMR Spectroscopy, Inglefield, et. al (1996) Macromolecules, diffusion coefficient for Xe in polystyrene (similar to acrylic) is:

$D_{Xe_PS} := 1.9 \cdot 10^{-9} \frac{cm^2}{s}$ $D_{Xe_PS} = 1.9 \times 10^{-13} \frac{m^2}{s}$ (at 25C, 10-15 atm Xe pressure)

for polypropylene, from in Organic Polymers by Pulsed Field Gradient NMR Spectroscopy, Junker & Veeman

$D_{Xe_PP} := 4 \cdot 10^{-12} \frac{m^2}{s}$

for liquid crystal plastic

$D_{lcd} := 2 \cdot 10^{-10} m^2 s^{-1}$ from Ruohonen, et al

for helium through PEEK film, amorphous , from Gas permeability reduction in PEEK films, Amanat, et al

flux is given: for dimensions and pressure

$J_{He_PEEK} := 8 \cdot 10^{-7} \frac{mol}{m^2 s}$ $t_{PEEK} := .25mm$ $delP := .020bar$

Concentration gradient is then

$dCdx := \frac{delP}{R \cdot 273K \cdot t_{PEEK}}$ $dCdx = 3.525 \times 10^3 \frac{mol}{m^4}$

and permeation rate is:

$C_{He_PEEK} := J_{He_PEEK} \cdot dCdx^{-1}$ $C_{He_PEEK} = 2.27 \times 10^{-10} \frac{m^2}{s}$

From Parker O-ring handbook, permeation rates for He through butyle/nitrile (lowest of all elastomers) are ~3-4x higher than for Xe at RT, so assuming PEEK behaves similarly:

$C_{Xe_PEEK} := .3 C_{He_PEEK}$ $C_{Xe_PEEK} = 6.809 \times 10^{-11} \frac{m^2}{s}$

From Vacuum Technology, O'Hanlon, appendix C.6, kapton (polyimide)

$C_{He_Kap} := 1.9 \cdot 10^{-12} \frac{m^2}{s}$ nylon is only 0.3 but has huge outgassing (H20?)

again, for Xe relative to He:

$C_{Xe_Kap} := 0.3 C_{He_Kap}$ $C_{Xe_Kap} = 5.7 \times 10^{-13} \frac{m^2}{s}$

It appears PEEK is unusually high permeability, so we choose kapton (polyimide), assume square gasket cross section for now. We use an O-ring for a back up only seal, to seal in case of loss of contact pressure

Xe concentration in vessel

$C_i := \frac{1}{v}$ $\quad v := \frac{RT}{P}$ $\quad C_i := \frac{P_{MOPa}}{R \cdot 293K}$ $\quad C_i = 615.763 \frac{mol}{m^3}$

Flux/unit area, J through seals, from Fick's Law for square cross section (what we have at present, simplify to lineal "area"

$J := -D \cdot \left(\frac{d}{dx} C\right)$ $\quad Q_{Xe} := -D \cdot C_i \cdot \frac{A}{dx}$ $\quad L_a := \frac{A}{dx}$ $\quad Q_{Xe} := J \cdot L_a$

Exposed (lineal) area, total (60 cans x 2 O-rings)

$r_{O_ring} := 4cm$ $\quad L_a := 2 \cdot 60 \cdot 2\pi r_{O_ring}$ $\quad L_a = 30.159\,m$

$Q_{Xe} := D_{Xe_PP} \cdot C_i \cdot L_a$ $\quad Q_{Xe} = 7.428 \times 10^{-8} \frac{mol}{s}$ $\quad Q_{Xe} \cdot (R \cdot 293K) = 1.357 \times 10^{-3}\, torr \cdot L \cdot s^{-1}$

In mass terms:

$M_{a_Xe} \cdot Q_{Xe} = 0.01 \frac{mg}{s}$ $\quad M_{a_Xe} \cdot Q_{Xe} = 0.319 \frac{kg}{yr}$

For the O-ring, from Parker O-ring handbook, leakage, L, is approximated by the following formula:

$L := 0.7 FDPQ(1 - S)^2$ in std. cc/sec; for quantities given:

$F := 3 \cdot 10^{-8} \frac{cc(std)\,cm}{cm^2 s \cdot bar}$ (.6-3.0) for Xe through nitrile or butyl rubber, 25C NASA, via Parker hdbk

$D := 3$ in

$P := 225$ psi

$Q := 1.5$ dimensionless squeeze factor , from fig 3-11

$S := .2$ squeeze, percentage expressed as decimal

Leak rate, per O-ring

$L := 0.7 \cdot F \cdot D \cdot P \cdot Q \cdot (1 - S)^2$ $\quad L = 1.361 \times 10^{-5}$ std cc/sec per O-ring

Molar flow

$Q_{Xe_O_ring} := 2 \cdot 60 L \cdot scc \cdot s^{-1}$ $\quad Q_{Xe_O_ring} = 7.29 \times 10^{-8} \frac{mol}{s}$

Mass flow:

$M_{Xe_O_ring} := Q_{Xe_O_ring} \cdot M_{a_Xe}$ $\quad M_{Xe_O_ring} = 0.313 \frac{kg}{yr}$

So, it appears that diffusion through (nitrile, butyl) O-rings would be much the same as through kapton (Nylon, perhaps, might be lower), with gaskets assumed square. This could be reduced by ~5-10x using thin, flat gaskets, but window breakage might become an issue. Even so, the amount is non-negligible and should be recovered. An Helicoflex gasket instead of the O-ring should be tried as an R&D project, to see if surface damage results. Also, epoxied-in windows might show significantly less leakage, and my also be worthy of an R&D project. Issues of reliability could arise however; there is some flexing of the window under pressure (44 micronfor 4mm window thk.).

Gas conductance through cable conduit

We desire to lows pressure rise in central manifold upon window break (viscous or turbulent flow regime) sufficiently to allow a burst disk to open, yet have a gas conductance that allows for leak checking with good time response (molecular flow regime). Even if we use a gas fill (N2 or other) inside the PMT's, we still will want to pull a temporary vacuum for leak checking.

Requirement for safety: we want central manifold (CM) pressure rise time to 2 barg max. to be 10 ms or greater (burst disk responds "in millisec" according to Fike, a burst disk mfr.

$t_{2bar} > 10ms$

Central manifold volume:

$V_{cm} := 30cm \cdot \pi 5cm^2 \qquad V_{cm} = 471.239\,cm^3$

Amount of gas present at 2 barg (3 bara):

$N_{cm_STP} := 3bar \cdot V_{cm} \qquad N_{cm_STP} = 1.06 \times 10^3\,torr \cdot L$

Maximum Flow rate

$Q_{v_max} := \frac{N_{cm_STP}}{.01s} \qquad Q_{v_max} = 1.06 \times 10^5\,torr \cdot L \cdot s^{-1}$

For the following:

specific heat ratio	gas density, upstream	discharge coeff., max
$\kappa := 1.667$	$\rho := 0.1 \frac{gm}{cm^3}$	$C_d := 1$

Gas flow will be sonically choked at entrance to conduit as upstream pressure is greater than 2x back pressure, per eq.

$$\left(\frac{\kappa + 1}{2}\right)^{\frac{\kappa}{\kappa - 1}} = 2.053$$

Mass flow rate equation for choked condition:

$$\dot{m} = C\,A\sqrt{k\,\rho\,P\left(\frac{2}{k+1}\right)^{(k+1)/(k-1)}}$$

wikipedia: choked flow eq.

For a conduit made with 3/16 inch copper refrigeration tubing:

$d_{i_con} := .125in \qquad d_{cable} := 1mm$

$A_{flow} := .7854 \cdot \left(d_{i_con}^2 - d_{cable}^2\right) \qquad A_{flow} = 7.132\,mm^2$

$$dMdt := C_d \cdot A_{flow} \cdot \sqrt{\kappa \cdot \rho \cdot 15bar \cdot \left(\frac{2}{\kappa + 1}\right)^{\frac{\kappa+1}{\kappa-1}}}$$

$dMdt = 0.063 \frac{kg}{s} \qquad \frac{dMdt \cdot R \cdot 293K}{M_{a_Xe}} = 8.5 \times 10^3\,torr \cdot L \cdot s^{-1}$ compare --> $Q_{v_max} = 1.1 \times 10^5\,torr \cdot L \cdot s^{-1}$

this is an order of magnitude lower than required, so even for a step function pressure rise in can, burst disk has plenty of time to open. Note that the reverse flow condition (from central manifold into adjacent cans, will not start out as choked flow, and may not progress

Requirement for leak check sensitivity: for starters, lets say we want a maximum can pressure of 0.1 millitorr for a manifold pressure of 1microtorr (100x pressure drop min.) (this is just an estimate)

Given average velocity (Maxwell's RMS eq.):

$T := 293K$

$v_a := \sqrt{\frac{3R \cdot T}{M_{a_Xe}}}$ $\qquad v_a = 231.8 \frac{m}{s}$

Tube gas conductance, molecular flow, from Practical Vacuum Techniques , Brunner & Batzer, 1974

Cable conduit length: $l_{con} := 0.1m$

$P_f := 10^{-6} torr$

$C_{m_cond} := \frac{v_a \cdot (d_{i_con} - d_{cable})^3}{0.5 l_{con}}$

$C_{m_cond} = 0.048 \frac{L}{s}$

Assume bends are negligible compared to length of tube

Assume that we have a leak rate of:

$Q_l := \frac{Q_{Xe} \cdot R \cdot 293K}{60}$ $\qquad Q_l = 2.262 \times 10^{-5} torr \cdot L \cdot s^{-1}$

Then pressure in can is

$P_{Xe_v} := P_f + \frac{Q_l}{C_{m_cond}}$ $\qquad P_{Xe_v} = 4.752 \times 10^{-4} torr$

pressure in can is dominated by leakage for manifold pressures 10^-6 torr and lower

This pressure is higher than what we initially desire. However, it is still low enough that the idea of using vacuum insulation for the PMT pins may be feasible, as it is well to the left of the Paschen minimum of 0.5 torr-cm. We would still need resistor cooling; probably passive heatsinking to the can would allow convective cooling from the Xe. Without active cooling, we will need to verify that the Xe flow rate is high enough to remove this heat. A horizontal orientation of the detector may benefit from having Xe flow ports on the main cylindrical vessel to avoid convective cells. It may be possible to mount the cans to a copper plate for heatsinking to the vessel.