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Coherent Synchrotron Radiation in the Recirculating Linac X-ray Source

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Here we present preliminary estimates for a coherent synchrotron radiation (CSR) of electrons in the recirculating linac x-ray source. As it is described elsewhere [1-3], CSR causes energy loss of electrons and emittance increase in the orbit plane that is caused by this energy loss. Since we have a relatively large horizontal emittance, small additional emittance growth seems not to be harmful. Therefore, we mainly concern in the energy loss because of its potentially adverse effect on production of short x-ray pulses through increased energy spread in the bunch.

In the recirculating linac we have electron bunches of various lengths and energies. We also use several types of dipole magnets. It turns out that in all cases we are in a regime of a so-called steady-state CSR [3] defined as follows:

$$\varphi_m \geq \left(24\frac{l_b}{\rho}\right)^{1/3}. \tag{1}$$

Here φ_m is the magnet arc angle, ρ is the bending radius, and l_b is the electron bunch length. The above condition means that the magnet has a sufficient length that the radiation of the tail particles surpasses the head particles before the electron bunch leaves the magnet. The CSR energy loss per unit length of trajectory is then written [1-3]:

$$\frac{dE(s)}{dz} \cong -\frac{2}{3^{1/3}\sqrt{2\pi}}\frac{Ne^2}{\sigma_0\rho^{2/3}}\int_{-\infty}^{s}\left(e^{-\frac{s'^2}{2\sigma_0^2}} - e^{-\frac{(s'-l_b)^2}{2\sigma_0^2}}\right)\frac{ds'}{(s-s')^{1/3}}, \tag{2}$$

where N is a number of particles per bunch, and e is the electron charge. Deriving (2), we assume a uniform longitudinal density distribution $\lambda(s) = N/l_b$ in interval $0 < s < l_b$ with smooth transitions at the edges with a characteristic length c_0 as shown in Figure 1.

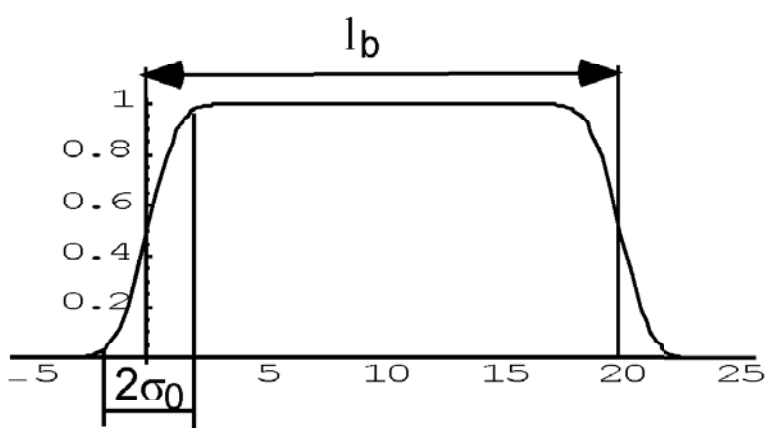

Figure 1. The longitudinal density of electrons.

Integral (2) can be evaluated in analytical functions and resulting plot of $dE(s)/dz$ is shown in Figure 2.

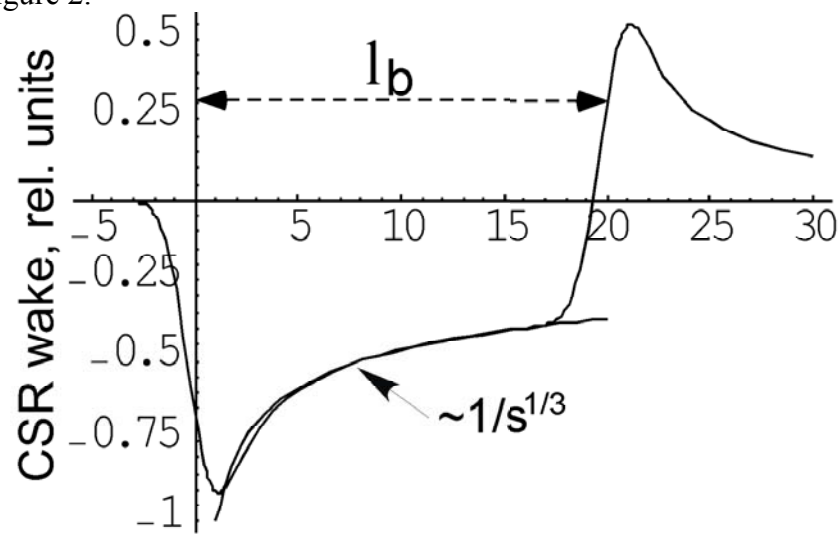

Figure 2. CSR wake function $dE(s)/dz$ for an electron longitudinal density distribution shown in Figure 1.

One can notice that $dE(s)/dz \sim 1/s^{1/3}$ over the entire length of the bunch excluding edges. For this functional dependence one can consider partial compensation of the energy variation within the electron bunch induced by CSR using off-crest acceleration in the linac. Using $1/s^{1/3}$ dependence for CSR wake function one can calculate the average energy loss per electron due to CSR in the magnet of the length L_m for main core particles with the expression [3]:

$$\Delta E = \frac{L_m}{l_b}\int_0^{l_b}\frac{dE(s)}{dz}ds \cong \frac{3^{2/3}Ne^2}{\rho^{2/3}l_b^{4/3}}L_m . \qquad (3)$$

Table 1 lists all magnets of the recirculating linac at all beam energies and bunch lengths. The amount of the energy loss in each magnet calculated using above formula is given in the Table 1 in a row named as "dE". The row named as "# of magnets" gives a number of identical magnets with identical beam conditions. The next row gives a total energy loss summed over all identical magnets. This is so-called a free space radiation. In practice, the electron bunch moves inside the vacuum chamber that acts as the waveguide for the radiation. Not all spectral components of the CSR propagates in the waveguide and therefore an actual radiated energy is less than in the free space environment. For an estimation of the shielding effect of vacuum chamber we follow recipe suggested in [4]:

$$\Delta E_{\text{shielded}} / \Delta E_{\text{free space}} \cong 4.2(n_{th}/n_c)^{5/6}\exp(-2n_{th}/n_c), \qquad (n_{th} > n_c) \qquad (4)$$

Here $n_{th} = \sqrt{2/3}(\pi\rho/h)^{3/2}$ is the threshold harmonic number for a propagating radiation, h is the height of the vacuum pipe, $n_c = \rho/c_c$ is the characteristic harmonic number for a Gaussian longitudinal density distribution with the rms value of c_c. The meaning of n_c is that the spectral component of the radiation with harmonic numbers beyond n_c is

incoherent. We define $c_c = l_b / 3.22$. This gives us the closest approximation of spectra for the uniform stepped density distribution with the spectra for the Gaussian distribution. Two bottom rows in Table 1 marked as "Shielded dE" show CSR losses for two cases that include shielding. There we used h equal to 0.9 cm and 0.7 cm.

Table 1.

Beam energy, MeV	120	120	120	720	1320	1920	2520	2520	2520
Bunch length, ps	20	2	2	2	2	2	2	2	2
Magnet length, cm	125.66	125.66	20.4	80	80	80	80	20	25.4
Magnet field, kG	3.33	3.33	6.4	11.12	10.23	9.93	10.56	9.54	20
dE, keV	-19	-409	-103	-176	-111	-85	-74	-17	-36
# of magnets	1	2	2	22	40	58	34	12	6
Total dE, keV	-19	-818	-205	-3874	-4447	-4925	-2509	-207	-215
h, cm	0.90	0.90	0.90	0.90	0.90	0.90	0.90	0.90	0.90
Shielded, keV	0	-419	-147	-1209	-557	-291	-47	-6	-28
h, cm	0.90	0.70	0.70	0.70	0.70	0.70	0.70	0.70	0.70
Shielded, keV	0	-212	-98	-436	-116	-40	8	-1	-6

References

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2. J.S. Nodvick and D.S. Saxon, Phys. Rev. 96(1954)180.
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