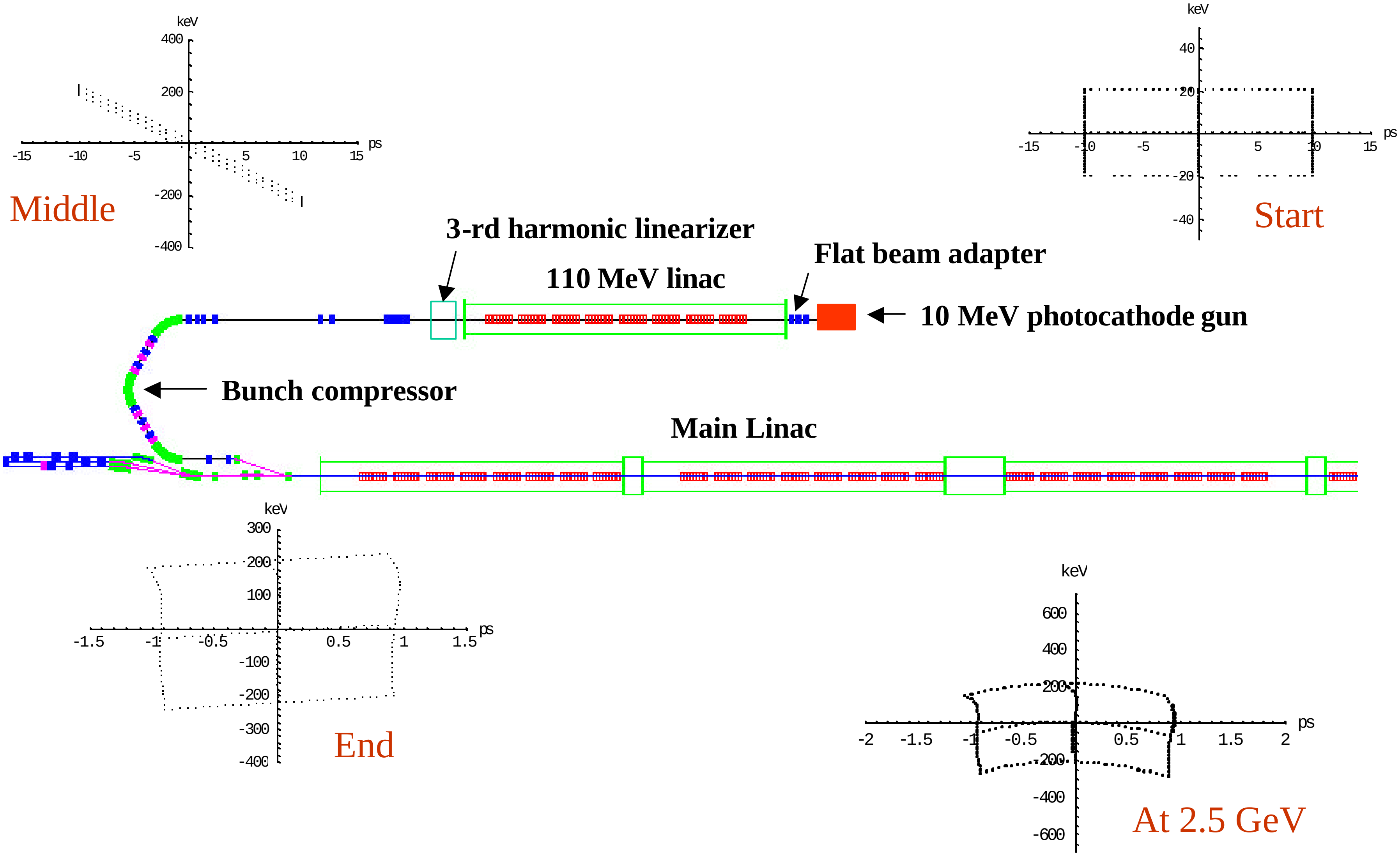
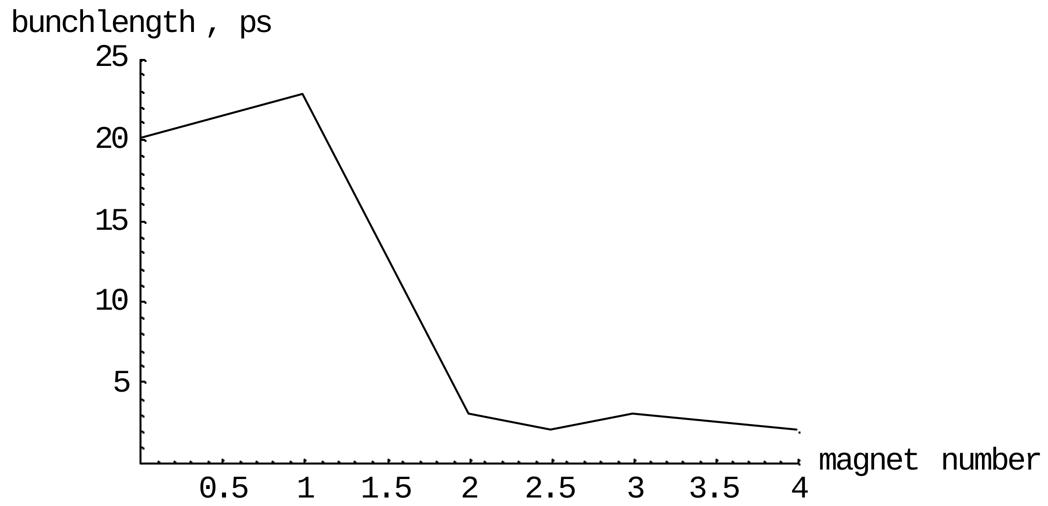
Studies of longitudinal dynamics and bunch compression including CSR and longitudinal wake , 11/5/02



Variation of the bunch length along bunch compressor



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October 28, 2002

Coherent Synchrotron Radiation in the Recirculating Linac X-ray Source

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Here we present preliminary estimates for a coherent synchrotron radiation (CSR) of electrons in the recirculating linac x-ray source. As it is described elsewhere [1-3], CSR causes energy loss of electrons and emittance increase in the orbit plane that is caused by this energy loss. Since we have a relatively large horizontal emittance, small additional emittance growth seems not to be harmful. Therefore, we mainly concern in the energy loss because of its potentially adverse effect on production of short x-ray pulses through increased energy spread in the bunch.

In the recirculating linac we have electron bunches of various lengths and energies. We also use several types of dipole magnets. It turns out that in all cases we are in a regime of a so-called steady-state CSR [3] defined as follows:

$$\varphi_m \geq \left(24\frac{l_b}{\rho}\right)^{1/3}. \tag{1}$$

Here φ_m is the magnet arc angle, ρ is the bending radius, and l_b is the electron bunch length. The above condition means that the magnet has a sufficient length that the radiation of the tail particles surpasses the head particles before the electron bunch leaves the magnet. The CSR energy loss per unit length of trajectory is then written [1-3]:

$$\frac{dE(s)}{dz} \cong -\frac{2}{3^{1/3}\sqrt{2\pi}}\frac{Ne^2}{\sigma_0\rho^{2/3}}\int_{-\infty}^{s}\left(e^{-\frac{s'^2}{2\sigma_0^2}} - e^{-\frac{(s'-l_b)^2}{2\sigma_0^2}}\right)\frac{ds'}{(s-s')^{1/3}}, \tag{2}$$

where N is a number of particles per bunch, and e is the electron charge. Deriving (2), we assume a uniform longitudinal density distribution $\lambda(s) = N/l_b$ in interval $0 < s < l_b$ with smooth transitions at the edges with a characteristic length c_0 as shown in Figure 1.

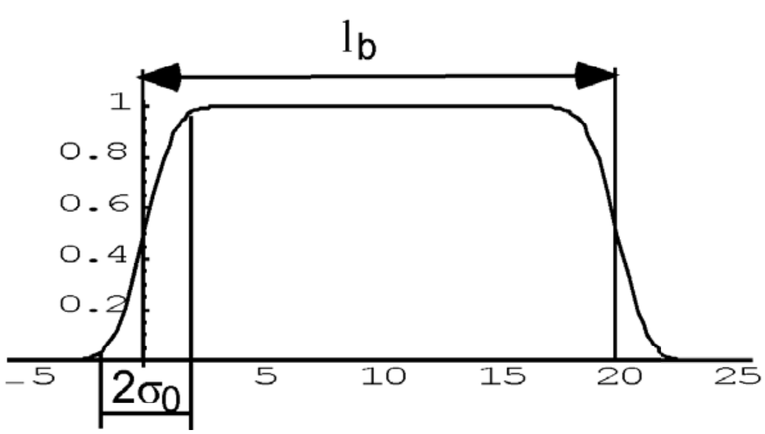


Figure 1. The longitudinal density of electrons.

Integral (2) can be evaluated in analytical functions and resulting plot of $dE(s)/dz$ is shown in Figure 2.

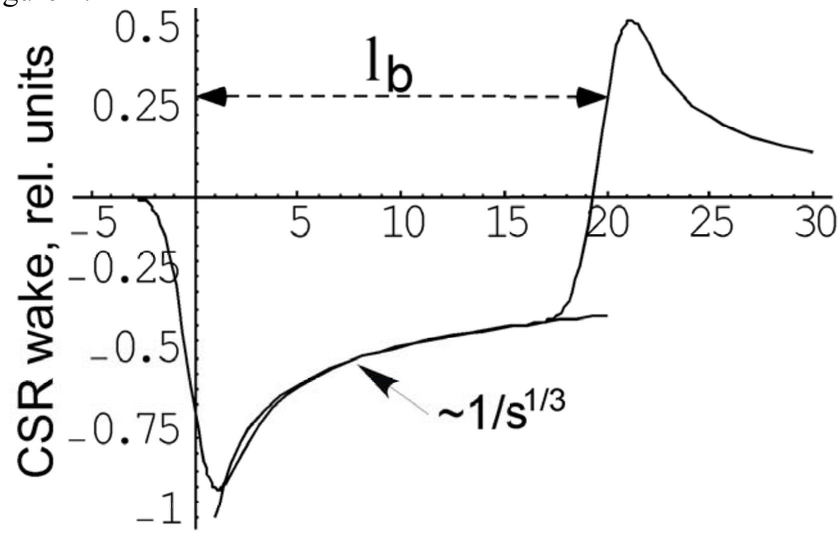


Figure 2. CSR wake function $dE(s)/dz$ for an electron longitudinal density distribution shown in Figure 1.

One can notice that $dE(s)/dz \sim 1/s^{1/3}$ over the entire length of the bunch excluding edges. For this functional dependence one can consider partial compensation of the energy variation within the electron bunch induced by CSR using off-crest acceleration in the linac. Using $1/s^{1/3}$ dependence for CSR wake function one can calculate the average energy loss per electron due to CSR in the magnet of the length L_m for main core particles with the expression [3]:

$$\Delta E = \frac{L_m}{l_b}\int_0^{l_b} \frac{dE(s)}{dz} ds \cong \frac{3^{2/3} Ne^2}{\rho^{2/3} l_b^{4/3}} L_m . \qquad (3)$$

Table 1 lists all magnets of the recirculating linac at all beam energies and bunch lengths. The amount of the energy loss in each magnet calculated using above formula is given in the Table 1 in a row named as "dE". The row named as "# of magnets" gives a number of identical magnets with identical beam conditions. The next row gives a total energy loss summed over all identical magnets. This is so-called a free space radiation. In practice, the electron bunch moves inside the vacuum chamber that acts as the waveguide for the radiation. Not all spectral components of the CSR propagates in the waveguide and therefore an actual radiated energy is less than in the free space environment. For an estimation of the shielding effect of vacuum chamber we follow recipe suggested in [4]:

$$\Delta E_{\text{shielded}} / \Delta E_{\text{free space}} \cong 4.2\left(n_{th}/n_c\right)^{5/6} \exp\left(-2n_{th}/n_c\right), \qquad (n_{th} > n_c) \qquad (4)$$

Here $n_{th} = \sqrt{2/3}\left(\pi\rho/h\right)^{3/2}$ is the threshold harmonic number for a propagating radiation, h is the height of the vacuum pipe, $n_c = \rho / c_c$ is the characteristic harmonic number for a Gaussian longitudinal density distribution with the rms value of c_c. The meaning of n_c is that the spectral component of the radiation with harmonic numbers beyond n_c is incoherent. We define $c_c = l_b / 3.22$. This gives us the closest approximation of spectra for the uniform stepped density distribution with the spectra for the Gaussian distribution. Two bottom rows in Table 1 marked as "Shielded dE" show CSR losses for two cases that include shielding. There we used h equal to 0.9 cm and 0.7 cm.

Table 1.

Beam energy, MeV	120	120	120	720	1320	1920	2520	2520	2520
Bunch length, ps	20	2	2	2	2	2	2	2	2
Magnet length, cm	125.66	125.66	20.4	80	80	80	80	20	25.4
Magnet field, kG	3.33	3.33	6.4	11.12	10.23	9.93	10.56	9.54	20
dE, keV	-19	-409	-103	-176	-111	-85	-74	-17	-36
# of magnets	1	2	2	22	40	58	64	12	6
Total dE, keV	-19	-818	-205	-3874	-4447	-4925	-4723	-207	-215
h, cm	0.90	0.90	0.90	0.90	0.90	0.90	0.90	0.90	0.90
Shielded, keV	0	-419	-147	-1209	-557	-291	-174	-6	-28
h, cm	0.90	0.70	0.70	0.70	0.70	0.70	0.70	0.70	0.70
Shielded, keV	0	-212	-98	-436	-116	-40	-18	-1	-6

References

1. L.I. Schiff, Rev. Sci. Instr. 17 (1946)6.
2. J.S. Nodvick and D.S. Saxon, Phys. Rev. 96(1954)180.
3. E.L. Saldin, E.A. Schneidmiller, and M.V. Yurkov, NIM A 398, (1997)373.
4. R.Li, C.L. Bohn, J.J. Bisongnano, Particle Accelerator Conference, (1997)1644.

A. Zholents, 10/31/02

Longitudinal Wake Function

As the electron bunch travels through the linac the longitudinal and transverse wakefields are excited and act back on the bunch itself. The longitudinal wakefields cause correlated energy variation along the bunch and we are going to calculate them for a uniform longitudinal charge distribution of electrons defined as $l(s) = Q/l_b$ in interval $0 < s < l_b$, where Q is the bunch charge and l_b is the bunch length. We use wake function:

$$w(s)\left[\frac{\mathrm{V}}{\mathrm{pC\,m}}\right] = -38.1\left(1.165\exp\left(-\sqrt{\frac{\mathrm{s}}{3.65\,\mathrm{mm}}}\right) - 0.165\right) \qquad (1)$$

given in [1] for a point charge steady state wake. The use of the steady state wake is justified for a main linac, but it is only approximately correct in the case of the injector linac with much fewer accelerating structures.

Using (1) and above defined charge distribution we calculate energy loss of electrons as a function of their position in the bunch using the following expression:

$$\frac{1}{Q}\frac{dE(s)}{dz}\left[\frac{e\mathrm{V}}{\mathrm{pC\,m}}\right] = -19.05\int_{-1}^{2s/l_b}\left(1.165\exp\left(-\sqrt{\frac{\mathrm{s} - x\,(l_b/2)}{3.65\,\mathrm{mm}}}\right) - 0.165\right)dx \qquad (2)$$

We found that the right hand side of Eq. (3) can be accurately described by a quadratic polynomial. For a an injector linac with the bunch length of 20 ps we obtain:

$$\frac{1}{Q}\frac{dE(s)}{dz}\left[\frac{e\mathrm{V}}{\mathrm{pC\,m}}\right] = -9.32 - 6.34\left(\frac{2s}{l_\mathrm{b}}\right) + 2.42\left(\frac{2s}{l_\mathrm{b}}\right)^2, \qquad (3)$$

and for the main linac with the bunch length of 2 ps we obtain:

$$\frac{1}{Q}\frac{dE(s)}{dz}\left[\frac{e\mathrm{V}}{\mathrm{pC\,m}}\right] = -15.25 - 13.72\left(\frac{2s}{l_\mathrm{b}}\right) + 1.33\left(\frac{2s}{l_\mathrm{b}}\right)^2 \qquad (4)$$

Figure 1 shows the plot of two functions given by Eq. (3) and Eq. (4). We also directly evaluate Eq. (3) and placed the result on the same plot. No visible differences from the approximations were found over the entire plot range.

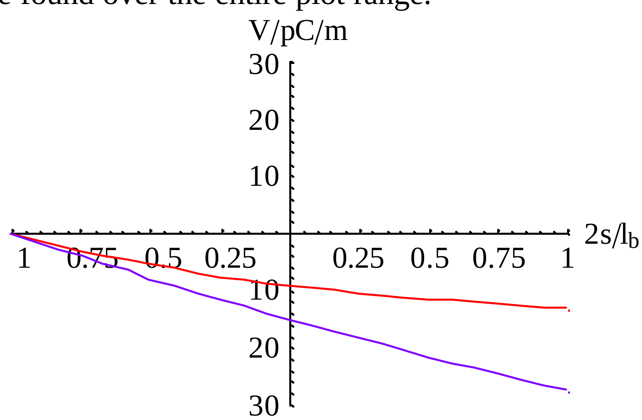


Figure 1. Energy loss of electrons due to the longitudinal wake for 2 ps (blue line) and 20 ps (red line) bunches.

References

1. TESLA, Technical Design Report, March 20001, DESY–011, 2001.

A. Zholents, 11/3/02

Injection system

In this chapter we consider a part of the injection system for the recirculating linac beginning form the end of the photocathode gun and ending at the end of the main linac. In the present analysis we are mainly concerning in a preservation of the longitudinal phase space during the acceleration of the 20 ps long electron bunch in the injector linac and its compression to 2 ps. We prefer not to go below 2 ps because stronger coherent synchrotron radiation (CSR) and less effective shielding of the CSR by the vacuum pipe for shorter bunches. At the same time a 2 ps long electron bunch is short enough that a cosine-like energy variation along the bunch due to the acceleration on the crest of the accelerating field is small and. does not degrade generation of femtosecond x-ray pulses. The soft x-ray FEL application may require shorter than 2 ps electron bunches and this option is being studied.

Figure 1 shows a schematic of the injection system. The injector linac consists of seven 1.3 GHz TESLA superconducting RF cavities combined in one cryomodule. The electron beam supplied by the photocathode gun propagates the linac and gains approximately 110 MeV. After this linac it passes through another short linac operated at 3.9 GHz (third harmonic of 1.3 GHz). The second linac acts as a linearizer for a longitudinal phase space [1] and produces a linear head-to-tail energy variation along the bunch for bunch compression. We use the following considerations to define the amplitude of the accelerating field, u_3 and equilibrium RF phase in the linearizer, j_3. First we write the total field acting on the electron at a position s relative to the bunch center:

$$U(s) = u_1 \mathrm{Cos}(k_{RF}s) + u_3 \mathrm{Cos}(j_3 + 3k_{RF}s) + a(k_{RF}s) + b(k_{RF}s)^2 , \qquad (1)$$

where k_{RF} and u_1 are the RF wave vector and amplitude of the accelerating fields for the injector linac, and zero equilibrium phase in the injector linac is assumed. We have shown in the section "long. wake" that in the case of the uniform "top-hat" charge density distribution the longitudinal wake is a quadratic polynomial with respect to s. Therefore we include it in (1) using coefficients a and b that can be easily obtained from Eq.3 of "long. wake" section.

The field acting on the electron at the equilibrium phase is:

$$U(0) = u_1 + u_3 \mathrm{Cos}(j_3) . \qquad (2)$$

Thus the difference $\Delta U(s) = U(s) - U(0)$ shows the variation of the electron energy along the bunch. For a relatively small $k_{RF}s$ we can make it linear by requesting that the sum of all quadratic terms $(k_{RF}s)^2$ in ΔU to be zero. This gives us a condition:

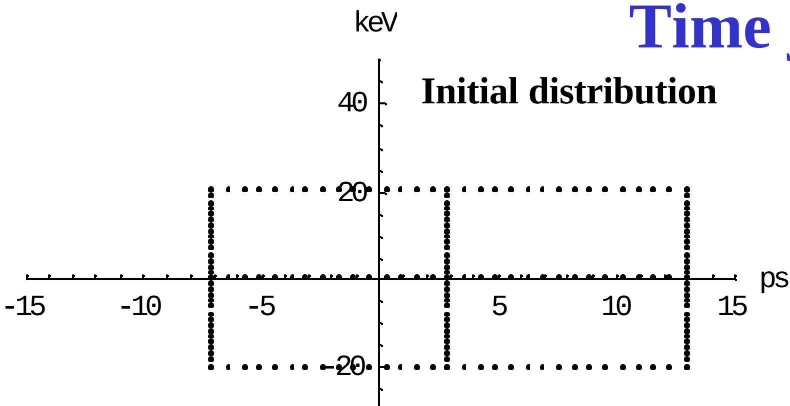
$$u_3 \mathrm{Cos}(j_3) = \frac{u_1 - 2b}{9} . \qquad (3)$$

If the bunch length is going to be compressed in a factor of M and initial energy spread of electrons is $d E$, than the linearizer should give $Md E$ energy variation from head to tail of the electron bunch. This implies the following condition:

$$u_3 \mathrm{Sin}\left(j_3\right) = -\left(\frac{Md\,E}{3k_{RF}\,l_b} + \frac{a}{3}\right), \tag{4}$$

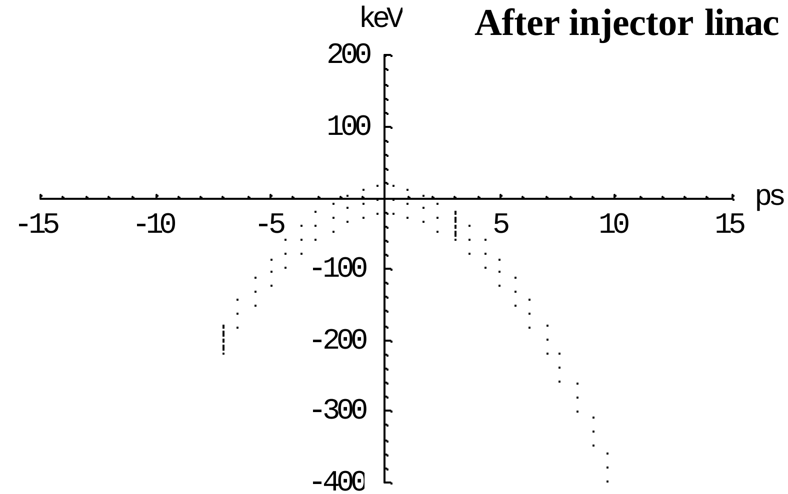
where l_b is the bunch length.

Time jitter studies

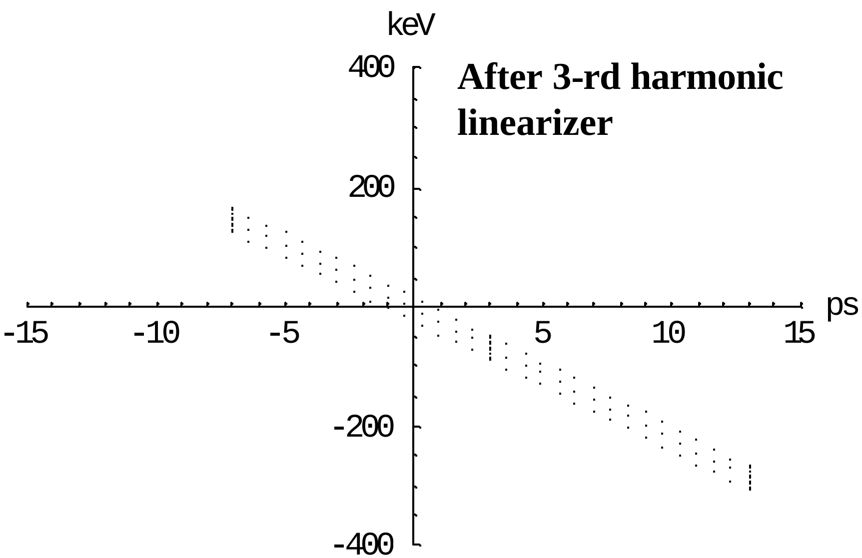


Initial distribution

+3 ps of time

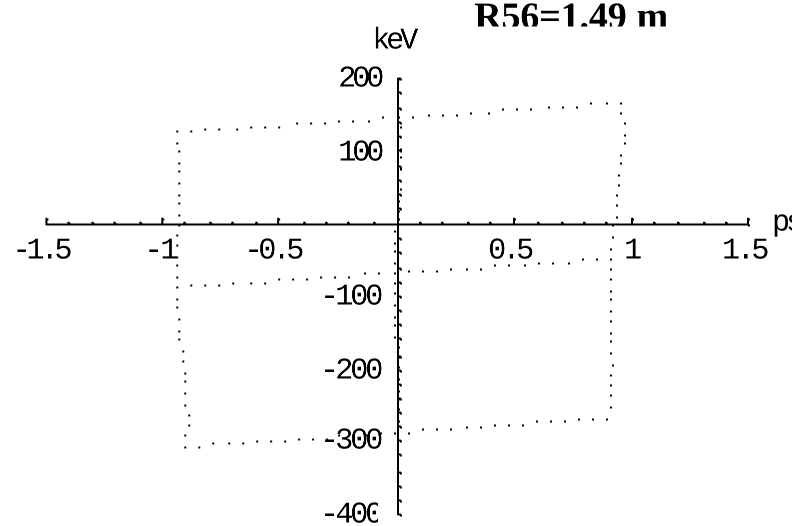


After injector linac

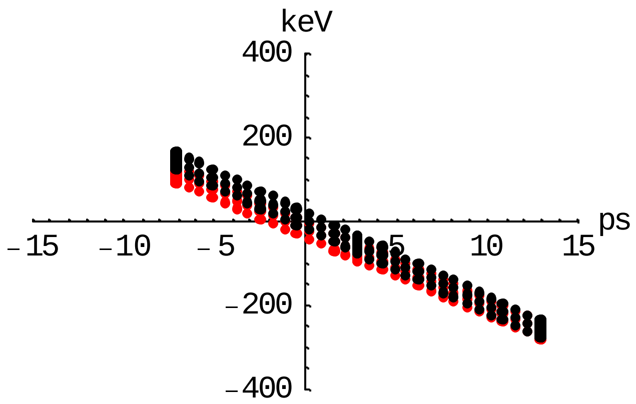


After 3-rd harmonic linearizer

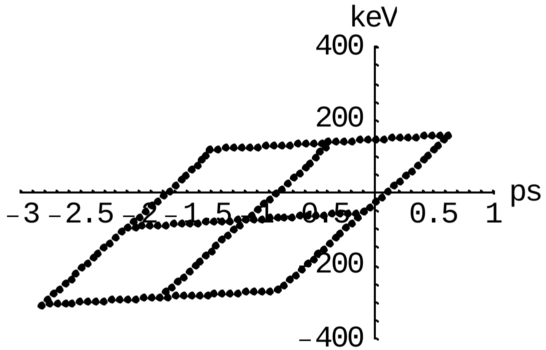
After compressor: R56=1.49 m



Jitter + longitudinal wake

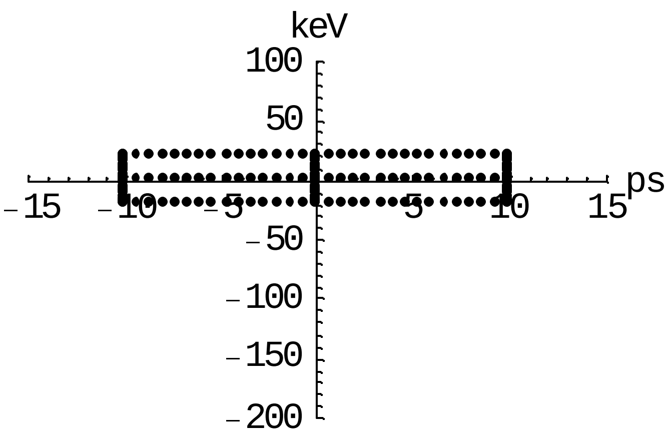


After linac and linearizer without (black dots) and with (read dots) longitudinal wake

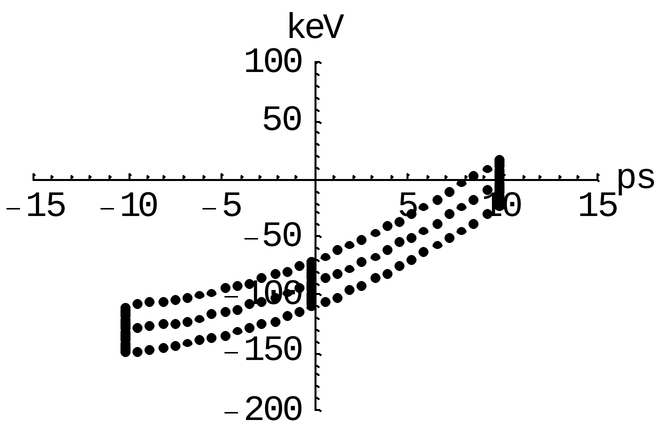


After compression

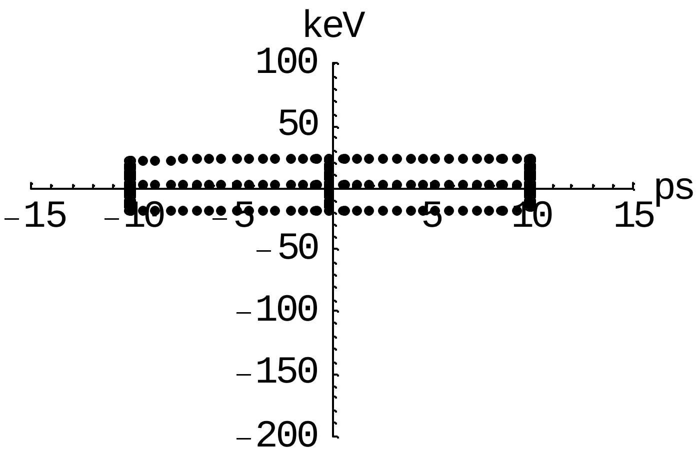
Separated 3-rd harmonic linearizer to compensate the longitudinal wake



At the beginning of the linac

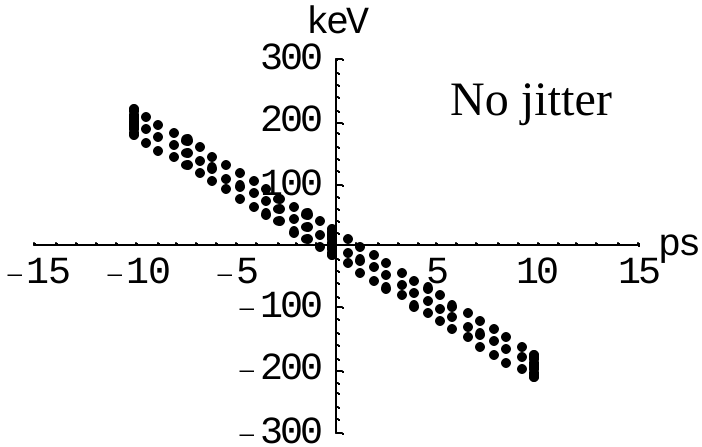


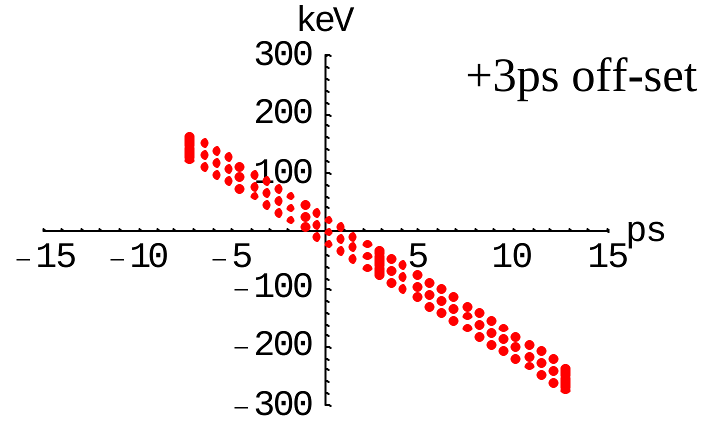
At the end of the linac if only the longitudinal wake

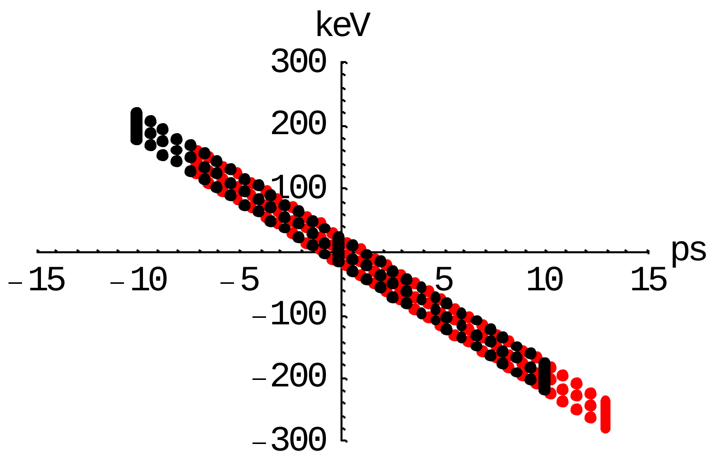


After the linearizer

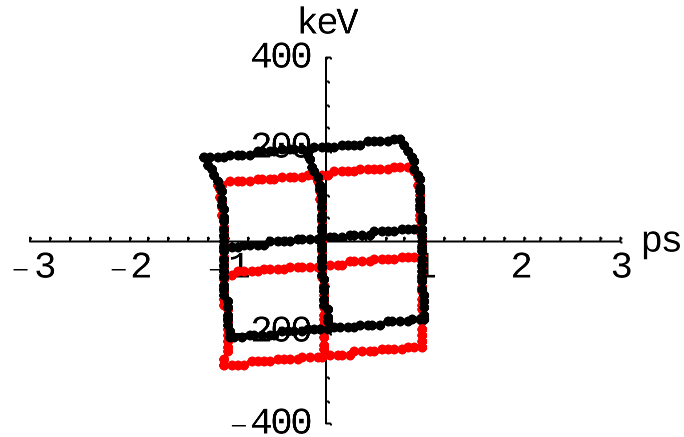
Two 3-rd harmonic linearizers





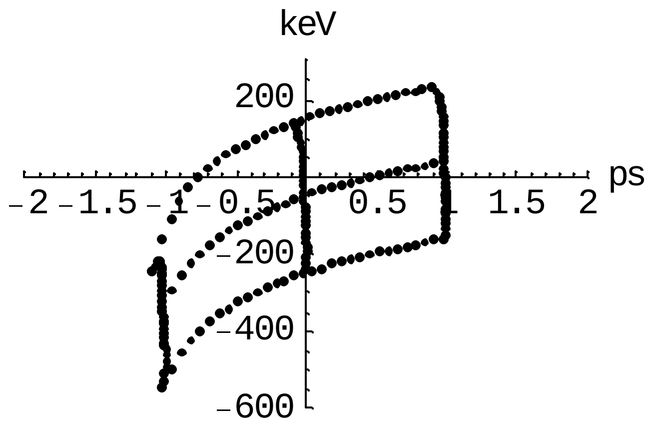


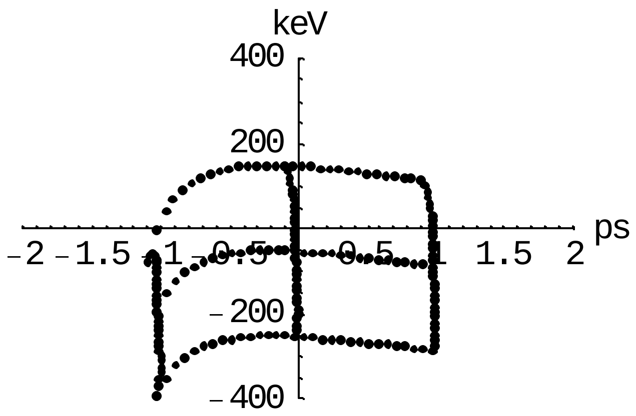
Two plots together



After compression

Effect of CSR in the bunch compressor





First 600 MeV path at 7° phase for CSR compensation

After bunch compressor (CSR included)