VALIDATING FAST SIMULATION OF AIR DAMPING IN MICROMACHINED DEVICES

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ABSTRACT

Optimization of the dynamic performance of air-packaged micromachined devices requires accurate estimation of viscous drag forces, but such simulations can be prohibitively time-consuming if the finite element method (FEM) or the finite difference method (FDM) is used. The recently developed precorrected-FFT accelerated boundary element method (BEM) solver FastStokes has made substantial changes to this situation. To further determine the accuracy of the FastStokes solver, the simulation results of the ADXL76 accelerometer and a micro-mirror are discussed in this paper. Close matches between simulation results and testing results prove the efficiency and broad applicability of this fast Stokes flow solver.

Keywords: Stokes flow, BEM, fluid, MEMS, simulation.

INTRODUCTION

Calculating the viscous drag forces on geometrically complicated micro-machined devices is a challenging job. Semi-analytical approaches have the advantage of being simple and easy to implement [1, 2], but such approaches are only good for certain geometries and require deep understanding of the assumptions used for the simplifications. For these reasons semi-analytical approaches usually fail to achieve good accuracy in general cases. The well-known FEM based or FDM based commercial Navier-Stokes equation solvers are fast enough for simple geometries, but the cost becomes prohibitively high for complicated geometries. To solve this problem, we have developed a Precorrected-FFT accelerated boundary element method based Stokes flow solver. When applied to analyzing lateral motion in structures as complicated as a full comb drive (See Figure-1), the FastStokes program was able to compute accurate drag forces, as verified by comparisons with experiment, in under 20 minutes on a workstation[3,4,5].

A general question that arises is how reliable and how accurate is the FastStokes solver. To answer this question, this paper briefly summarizes the critical aspects of the major algorithms used in the FastStokes program. The rest part of this paper focus on the simulations of ADXL76 accelerometer and a micromirror, and compares the results to measured data.

FASTSTOKES SOLVER

The FastStokes programs solves the steady-state incompressible Stokes equation. Given the isothermal condition, continuity assumption, and low Reynolds number assumption, which are usually good for air-packaged MEMS devices, the steady incompressible Stokes equation is derived from isothermal incompressible Navier-Stokes equation by neglecting the nonlinear convective term :

$$\begin{cases} -\nabla P + \mu \nabla^2 \vec{u} = 0 \\ \nabla \bullet \vec{u} = 0 \end{cases} \quad (1)$$

Here u is the velocity of the fluid, μ is the viscosity and P is the pressure. The corresponding single layer velocity integral equation is

$$u_i(\vec{x}) = -\frac{1}{8\pi\mu} \int G_{ij}(\vec{x}, \vec{y}) f_j(\vec{y}) ds(\vec{y}) \quad (2)$$

f_j is the surface force distribution, G_{ij} is the velocity kernel. In the case when the device feature size gets very small or the device is packaged in vaccum, the continuity assumption may not be applicable. But, for general air-packaged oscillating devices where damping is nonnegligible, the above assumptions usually work well.

Based on the Precorrected-FFT accelerated boundary element method, the FastStokes solver is much faster than traditional volume discretization based 3-D solvers. The surface discretization used by FastStokes generates fewer unknowns than the volume discretization used by the FDM or the FEM. The Precorrected-FFT algorithm, together with the well-known Krylov subspace iterative solver GMRES, forms the backbone of FastStokes. The GMRES algorithm reduces the cost of solving the BEM-

generated linear system from $O(n^3)$ to $O(n^2)$. The Precorrected-FFT algorithm uses projection on to a uniform grid plus the FFT to handle far field interactions, and reduce the computational cost of the matrix vector multiplication step of GMRES to *O(nlog(n))*. The final computation cost of FastStokes is only *O(nlog(n))* comparing with the expensive $O(n^3)$ of traditional direct BEM solvers. FastStokes is not only fast, but also accurate. An important feature of FastStokes is the analytical kernel integration algorithm [6], which guarantees the accuracy of the direct kernel integration and dramatically reduces global error. During the development of FastStokes solver, it was also found that the discretized form of the Stokes BEM integral operator is indeed a singular matrix due to the pressure derivative term in the Stokes equation. An efficient modification of the GMRES algorithm has been made and a pressure pinning method has been introduced to efficiently solve this singular BEM operator problem [7].

A SPHERE EXAMPLE

For the simple sphere geometry, an analytical solution of the Stokes equation exists. Given the radius of the sphere, R_0, and constant velocity $\vec{U}$, the drag force on the sphere is given by:

$$\vec{F} = 6\pi\mu R_0 \vec{U} \tag{3}$$

For our computational experiment, we assumed $\mu = 1, R_0 = 1, U_x = 1$ and used FastStokes to calculate the X-direction drag force. Note the error is mostly due to geometry error of using a flat-panel discretization, see Figure-2. The CPU time for using FastStokes and traditional Gaussian Elimination is compared in Figure-3. If large numbers of panels are used in the discretization, the FastStokes solver is several orders of magnitude faster than Gaussian Elimination. A 500-Mhz dual-processor computer running Alpha-Linux system was used for the simulation.

A MACRO MODEL

Decoupled problems are usually much easier to simulate than coupled domain problems. The rigid body assumption has been used extensively in MEMS modeling to simplify problems, especially for large proof masses supported on thin tethers. For example, mechanical structure, fluid, and electrostatic problems must be considered in modeling an electrostatically actuated air-packaged micro-mirror. The mirror, which is usually thick to avoid having a large curvature, is stiff enough to be modeled as a rigid body. The deformation of the mirror is negligibly small even when large load is applied. A second order spring-mass-damper system is a good macro model for the device and the decoupled problems can be solved separately.

$$J\ddot{\theta} + b\dot{\theta} + k\theta = \Gamma_{electro-static} \tag{4}$$

Where J is the moment of inertia, b is the damping coefficient, k is the stiffness and $\Gamma_{electro-static}$ is the electro-static torque. The quality factor of the system is given by

$$Q = \frac{\sqrt{Jk}}{b} \tag{5}$$

The next critical step is to accurately calculate the coefficients using full 3-D device level simulation programs. The macro model of the device can be further plugged into system level simulators to test the performance of the whole system.

ADXL76

A picture of ADXL76 accelerometer is shown in Figure-4. The device is fabricated by Micromachined Product Division of Analog Devices using *i*MEMS process. Some basic dimensions are listed in the following table.

ADXL 76

Finger overlap	104 um
Air gap between finger	1.3 um
Air gap between substrate	1.6 um
Number of cells	28

It is very clear that the aspect ratio of air gaps between fingers are large enough to generate strong viscous damping forces. This is indeed the dominate source of damping. The substrate is also very close to the finger and proof mass, and the shear damping forces between the substrate and beam (movable comb) should not be neglected. In such cases, a simple model based on semi-analytical approach will not yield accurate results. However, full 3-D simulation of entire device is very difficult even for a fast solver like FastStokes. Those very close fingers cannot be accurately simulated without using a very fine discretization, and this generates large numbers of unknowns. To solve this problem efficiently and to figure out a good method for modeling even more complicated accelerometers, cells instead of the whole devices are simulated (see Figure-5). Simulation results show that the damping forces increase linearly with the number of cells (Figure-6). An extrapolation yields a quality factor of 6.46, which is very close to the tested mechanical damping of the devices.

An even more exciting part of ADXL76 simulation is the simulation of Q-factor drifts due to geometry variations such as beam curvature and positional offsets. Testing a

single device or a batch of wafers doesn't yield much useful results since curvature and offsets are usually coupled. But the simulation results clearly indicate the percentage of damping changes due to geometry variations.

MICRO-MIRROR

As a critical part of the all-optical network, the MEMS micro-mirror must satisfy very high performance requirements. Damping is important for designing a mechanically stable system, but too much damping will negatively affect the performance of the micro-mirror. Both simulation and testing results of two mirror designs are given in this paper. The mirror is electrostatically actuated, four tethers support the mirror and a gimbal. Two important modes of the mirror are the "mirror rotation" motion and "mirror+gimbal rotation" motion. The device is packaged in air.

The Z-direction surface force of a simulation result is shown in Figure-7. Both mirror and gimbal rotate about X-axis, Only half of the structure is shown to give a clear view of force distribution. Figure-8 shows the convergence of the simulation as discretization is refined.

A Polytec Scanning Vibrometer was used to excite and measure two rotation modes of the mirror, and the damping was calculated from the decaying oscillation curves of the step response. Table 1 compares the simulated and measured Q, the two match within 10%.

SUMMARY

This paper compares simulation results with testing results to show the accuracy and efficiency of the FastStokes simulation program. Based on Precorrected-FFT accelected boundary element method, the FastStokes simulation porgram makes fast full 3-D fluid simulation of micromachined devices possible.

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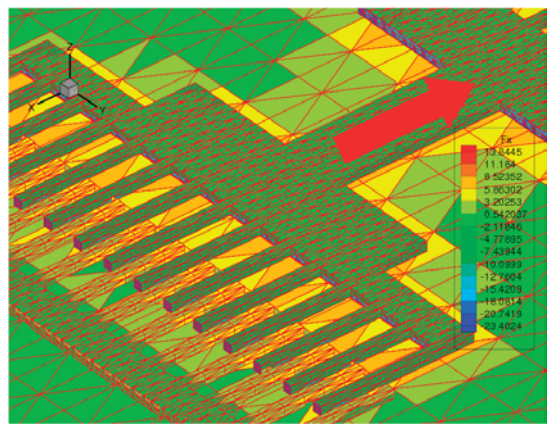

Figure-1 X-direction drag force distribution (Pa) on a lateral resonator

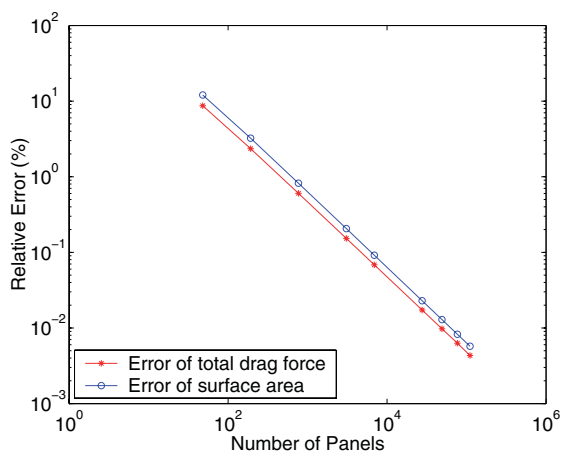

Figure-2 Relative error of sphere vs the number of panels

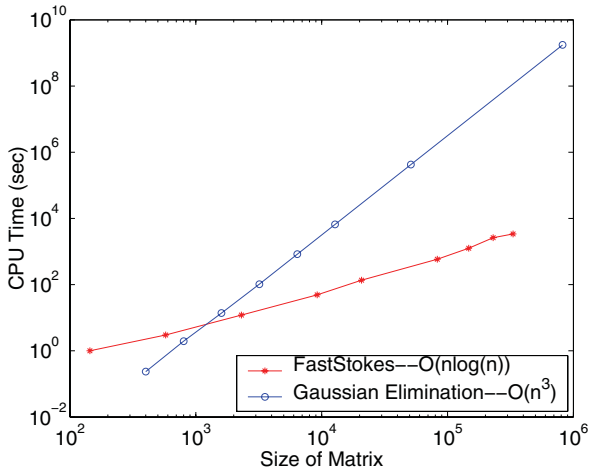

Figure-3 CPU time of FastStokes and Gaussian Elimination

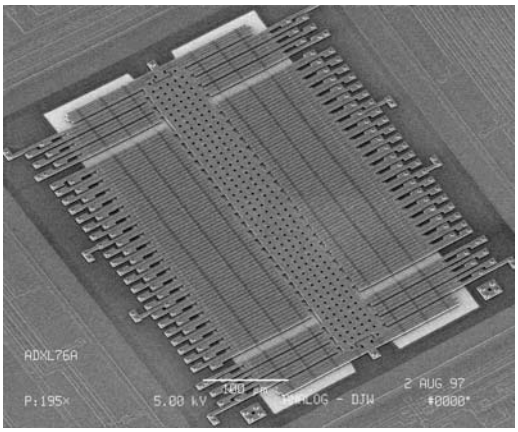

Figure-4 ADXL76 accelerometer

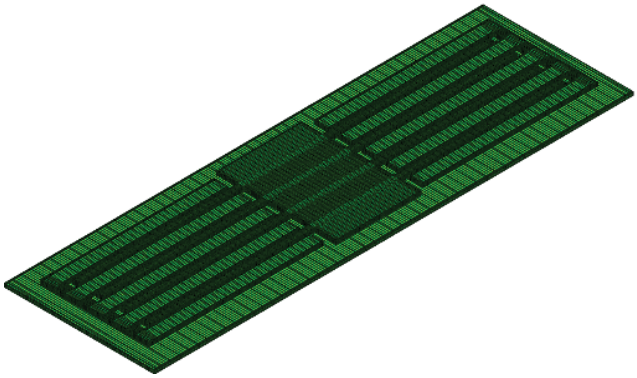

Figure-5 4 cells used in ADXL76 simulation

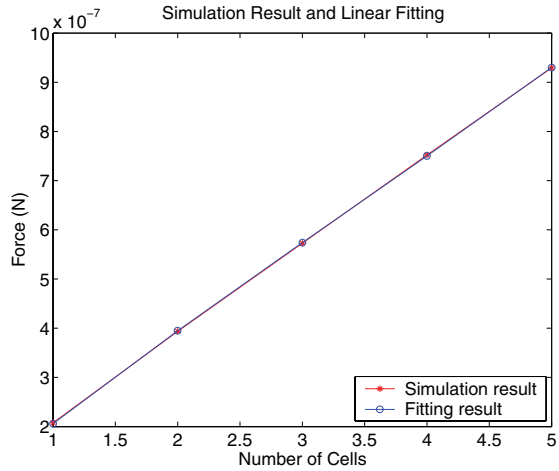

Figure-6 Drag forces on cells and linear data fitting

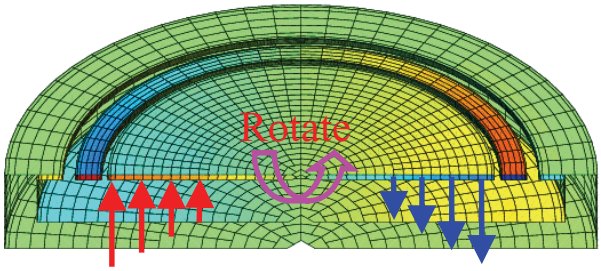

Figure-7 Z-direction drag force on a micro-mirror

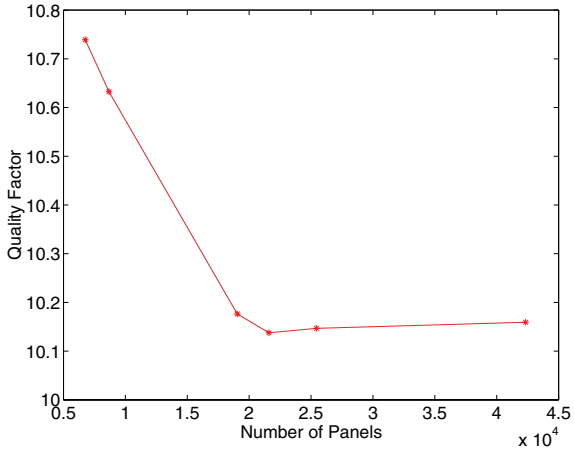

Figure-8 Convergence of micro-mirror simulation

Table-1 Quality factors--Simulations and measurements

	Mode	Q		Error
		Simulated	Measured	(%)
Mirror 1	Mirror+Gimbal	2.36	2.31	2.16
	Mirror	3.14	3.45	8.99
Mirror 2	Mirror+Gimbal	4.69	4.27	9.84
	Mirror	10.16	10.63	4.42