H^- Stripping Equations and Application to the High Intensity Neutrino Source.*

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Abstract

This document reviews the equations of H^- stripping by black-body radiation, magnetic field and residual gas with an application to the 8 GeV H-minus beam produced by the Fermilab High Intensity Neutrino Source. This work is a preamble to the implementation of these stripping effects in the beam dynamics code `TRACK` [1].

1 Introduction

The High Intensity Neutrino Source under development at FNAL is an 8 GeV H^- superconducting linac with primary mission of increasing the intensity of the Main Injector for the FNAL neutrino program. In the current design, the accelerating section of ~ 674 meters brings the beam kinetic energy to 8 GeV and a transfer line of ~ 1 km transports the beam from the accelerating section to the Main Injector (MI10).

H^- ions have two electrons, one tightly bound with a binding energy of 13.6 eV and another one slightly bound at 0.75 eV of binding energy. During the acceleration and transport of the H^- beam, the ions suffer from black-body radiation, magnetic field and residual gas which can strip the slightly bound electron. We review in this document the equations ruling these three types of stripping.

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2 Black-body Radiation Stripping

2.1 Photo-detachment cross-section

During the transport at 8 GeV in the ~ 1 km transfer line from the last accelerating section of the linac to the Main Injector, the ions suffer from the black-body radiation of the beam-pipe which can strip the slightly bound electron, as depicted in Figure 1 :

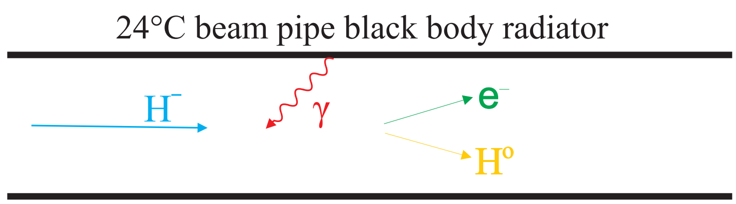


Figure 1: Stripping by black-body radiation of a beam-pipe.

The spectral density of the thermal photons per unit volume emitted by the beam-pipe is given by the Planck formula [2] :

$$E(\omega; T)d\omega = \frac{\hbar}{\pi^2 c^3} \frac{\omega^3}{\exp(\hbar\omega/kT - 1)} d\omega \tag{1}$$

which after integration (see Appendix A for variables definitions) gives a total number of photons of $n_\gamma \approx 2.02 \cdot T[\mathrm{K}]^3$ per $[\mathrm{m}^3]$. For a 300 K beam-pipe : $n_\gamma \approx 5.47 \times 10^{14}$ photons per $[\mathrm{m}^3]$.

In the beam rest frame, the relativistic doppler effect red-shifts the spectral density energy of the photons. The Lorentz transformation relates the photon energy on the beam rest frame ω_r to the photon energy on the laboratory frame ω_l by the equation :

$$\omega_r = \frac{\omega_l}{\sqrt{\frac{1-\beta}{1+\beta}}} \tag{2}$$

with β the Lorentz factor of the beam. For an 8 GeV beam, $\beta \simeq 0.9945$ and Equation 2 reports a Doppler shift of factor ~ 19. Figure 2 reports the photon spectral density energy

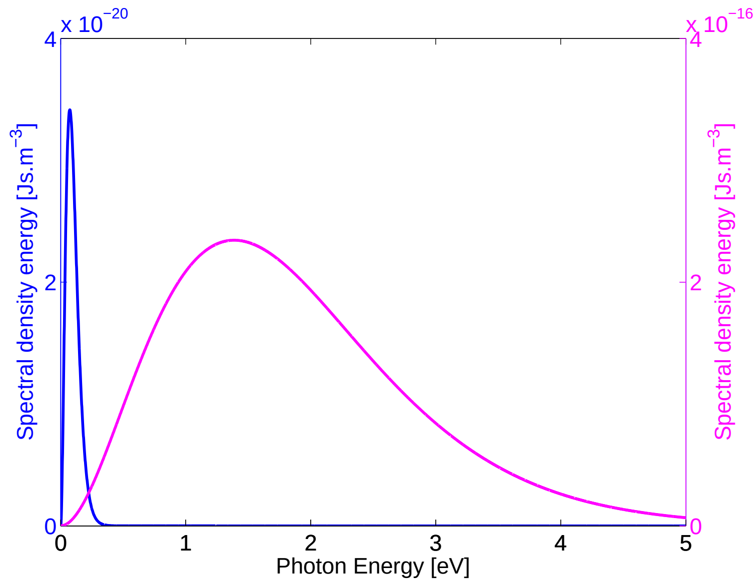


Figure 2: Photons spectral density energy in the laboratory frame (300 K) and Doppler shifted to 8 GeV H^- rest frame.

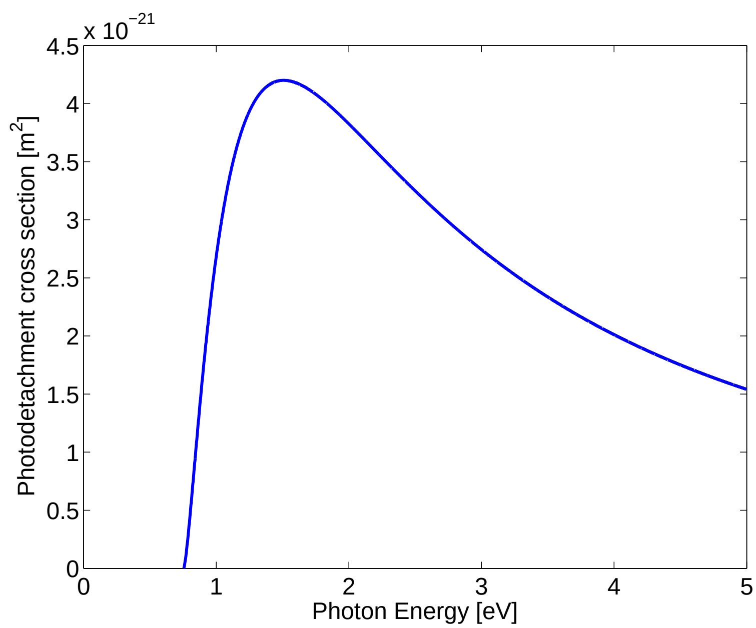


Figure 3: H^- photo-detachment cross-section.

distribution in the laboratory frame and doppler shifted in the 8 GeV beam rest frame. Figure 3 reports the photo-detachement cross-section in the abscence of electric-fields, as reported in Reference [3] :

$$\sigma(E) = 8\sigma_{max}\frac{E_0^{3/2}(E-E_0)^{3/2}}{E^3} \tag{3}$$

with $\sigma_{max} = 4.2\times10^{-21}$ m^2 and $E_0 = 0.7543$ eV. The overlap of the photon spectral distribution in the beam rest frame at 8 GeV presented in Figure 2 and the photo-detachment cross-section given by Figure 3 yields to the black-body radiation stripping.

2.2 Total loss per unit length : Hill-Bryant-Herling Equations

Equations of beam loss due to stripping of H^- by black-body radiation are published by Bryant-Herling in Reference [4] and in detail by C. Hill in Reference [5]. The analytics presented below are inspired by both references.

Bryant-Herling's starts with the fraction lost per unit length (Equation (8) of Ref. [4]) :

$$\frac{\mathrm{d}^3r}{\mathrm{d}\Omega\mathrm{d}\nu\mathrm{d}l} = \frac{(1+\beta\mathrm{cos}\alpha)n(\nu)\sigma(\nu')}{4\pi\beta} \tag{4}$$

where β is the usual Lorentz factor, α the angle between the light beam and the atomic beam, $n(\nu)$ the density of the thermal photons and $\sigma(\nu')$ the cross-section in the beam rest frame. Taking $\epsilon = h\nu/E_0$, $\mathrm{d}\Omega = 2\pi\mathrm{sin}\alpha\mathrm{d}\alpha$ where E_0 is the electron affinity, $n(\nu) = 2\pi n(\omega)$ with :

$$n(\omega) = \frac{1}{\pi^2c^3}\frac{\omega^2}{[\exp{(\hbar\omega/kT)}-1]} \tag{5}$$

and $\omega^2 = \frac{\epsilon^2E_0^2}{\hbar^2}$ then Equation 4 becomes :

$$\frac{\mathrm{d}^3r}{\mathrm{d}\epsilon\mathrm{d}\alpha\mathrm{d}l} = \frac{4\pi E_0^3\epsilon^2\sin\alpha(1+\beta\cos\alpha)\sigma(\epsilon')}{(hc)^3\beta[\exp(\epsilon E_0/kT)-1]} \tag{6}$$

The total loss per unit length is given by (Equation (11) of Reference [4]) :

$$\frac{1}{\mathrm{L}} = \int_0^\infty \mathrm{d}\epsilon \int_0^\pi \mathrm{d}\alpha \frac{\mathrm{d}^3 r}{\mathrm{d}\epsilon \mathrm{d}\alpha \mathrm{d}l} \tag{7}$$

which using Equation 6 becomes :

$$\frac{1}{\mathrm{L}} = \int_0^\infty \mathrm{d}\epsilon \int_0^\pi \mathrm{d}\alpha \frac{4\pi E_0^3 \epsilon^2 \sin\alpha (1+\beta\cos\alpha)\sigma(\epsilon')}{(hc)^3 \beta [\exp(\epsilon E_0/kT) - 1]} \tag{8}$$

The Lorentz transform relates ϵ in the laboratory frame to ϵ' in the beam rest frame by :

$$\epsilon' = \gamma(1+\beta\mathrm{cos}\alpha)\epsilon \tag{9}$$

leading to :

$$\mathrm{d}\epsilon = \frac{\mathrm{d}\epsilon'}{\gamma(1+\beta\cos\alpha)} \tag{10}$$

Therefore Equation 8 becomes :

$$\frac{1}{\mathrm{L}} = \frac{4\pi E_0^3}{\gamma^3\beta(hc)^3} \int_0^\infty \mathrm{d}\epsilon' \int_0^\pi \mathrm{d}\alpha \frac{\epsilon'^2}{(1+\beta\cos\alpha)^2} \cdot \sin\alpha \cdot \sigma(\epsilon') \cdot \frac{1}{[\exp(\epsilon' E_0/kT\gamma(1+\beta\cos\alpha)) - 1]} \tag{11}$$

Using Equation 3 :

$$\sigma(E') = 8\sigma_{max} E_0^{3/2} \frac{(E'-E_0)^{3/2}}{E'^3} \tag{12}$$

and taking $u = \cos\alpha$, $\mathrm{d}u = -\mathrm{d}\alpha \sin\alpha$, $\epsilon' = \frac{E'}{E_0}$, $h = 2\pi\hbar$, Equation 11 becomes :

$$\frac{1}{\mathrm{L}} = \frac{8\sigma_{max} E_0^{3/2}}{2\pi^2\beta\gamma^3(\hbar c)^3} \int_0^\infty \mathrm{d}E' \int_{-1}^{+1} \mathrm{d}u \cdot \frac{1}{(1+\beta u)^2} \cdot \frac{(E'-E_0)^{3/2}}{E'} \frac{1}{[\exp(E'/kT\gamma(1+\beta u)) - 1]} \tag{13}$$

2.3 Attenuation length and Fraction Lost Vs Energy

Integration of Equation 13 is presented in Figure 4 for different beam kinetic energies and for a beam-pipe radiating at 300 K :

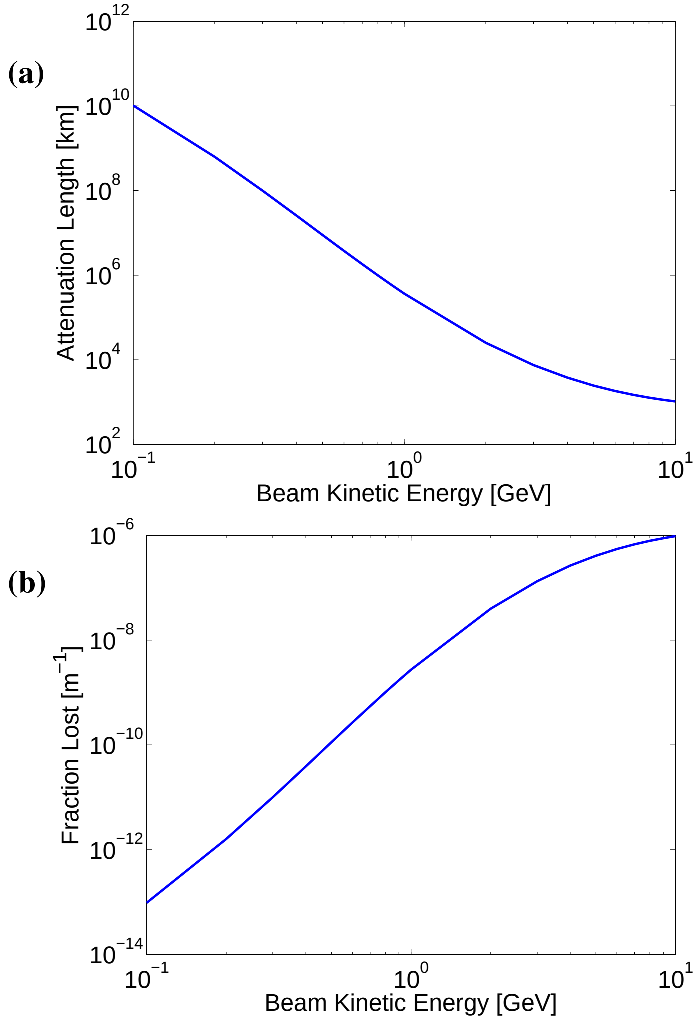


Figure 4: (a) Attenuation length and (b) Fraction Lost Vs. beam kinetic energy for a beam-pipe radiating at 300 K. From Equation 13.

2.4 Attenuation length and Fraction Lost Vs Temperature

Integration of Equation 13 is presented in Figure 5 for different beam-pipe temperatures and a beam kinetic energy of 8 GeV :

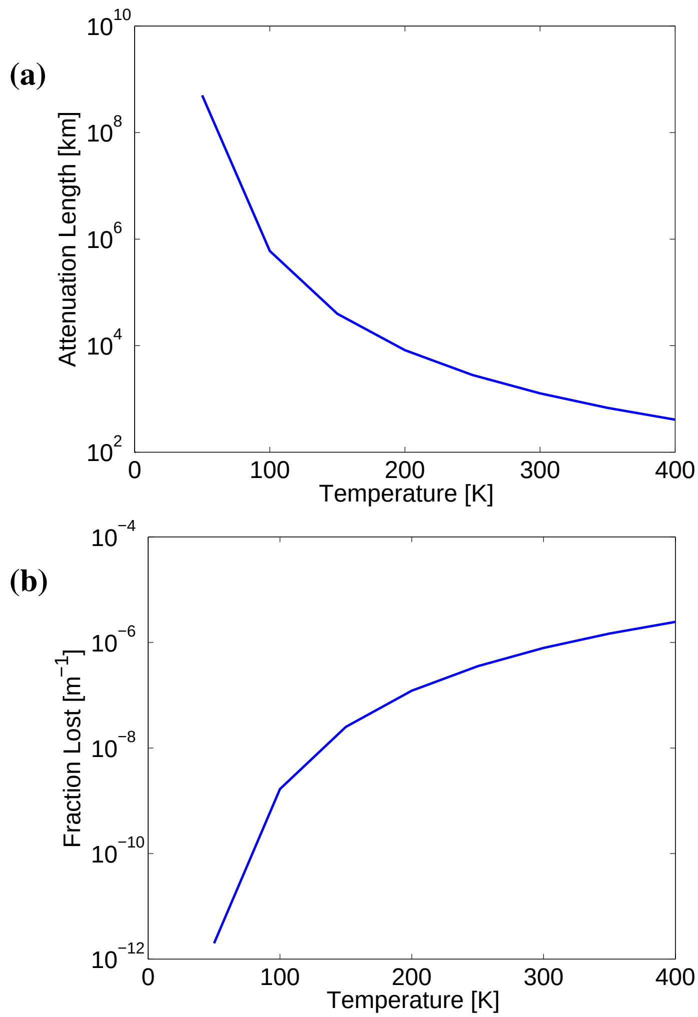


Figure 5: (a) Attenuation length and (b) Fraction Lost Vs. beam-pipe temperature for a beam kinetic energy of 8 GeV. From Equation 13.

2.5 Application to the FNAL HINS Transfer Line

If the ~ 1 km transfer line of the High Intensity Neutrino Source is kept at 300 K, Equation 13 predicts that the loss rate for stripping from collisions with the beam-pipe radiation is $\sim 7.86 \times 10^{-7}$ m^{-1}. Considering the linac particle per macropulse of 1×10^{14}, losses due to beam-pipe radiation are therefore ~ 0.1 W$\cdot$m^{-1} which will cause untolerable activation of the beam-pipe. Cooling of the beam-pipe at 150 K as illustrated in Figure 6 will reduce the loss rate to $\sim 2.52 \times 10^{-8}$ m^{-1} and the beam losses to $\sim 3.2 \times 10^{-3}$ W $\cdot$ m^{-1}, a negligeable level.

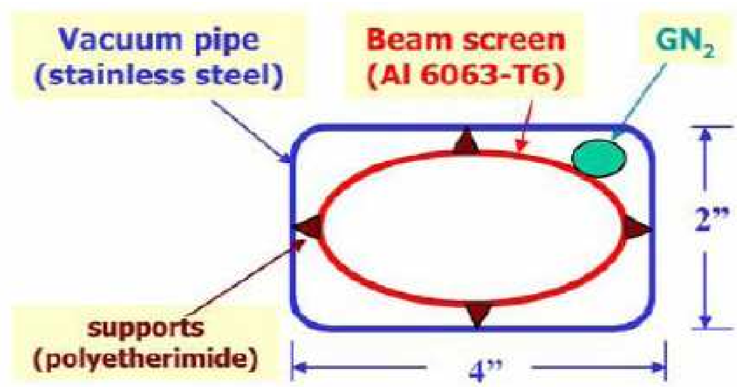


Figure 6: Illustration of a possible 150 K gas nitrogen beam screen inside the vacuum pipe for reducing the black-body radiation. From [6].

3 Magnetic Field Stripping

3.1 Total loss per unit length by Lorentz Stripping

When a H^- ion is bent in a magnetic field, the electrons and proton are bent in opposite directions. If the magnetic field is strong enough, the slightly bound electron can be stripped. As presented in Reference [7], in the ion rest frame the field E is the Lorentz transformation of the magnetic field B :

$$E[\text{MV/cm}] = 3.197 \cdot p[\text{GeV/c}] \cdot B[\text{T}] \tag{14}$$

The ion's lifetime τ_0 in its rest frame is given by the 2 parameters formula [7]:

$$\tau_0 = \frac{A}{E} \exp\left(\frac{C}{E}\right) \tag{15}$$

with A and B two constants given in Table 1 :

Table 1: H^- ion lifetime measurement. From Ref. [8].

Experiment	**Energy** [MeV]	**E** [MV/cm]	**A** [10^{-14} s-MV/cm]	**B** [MV/cm]	**Reference**
Stinson et al.	50	1.87 - 2.14	7.96	42.56	[9] (1969)
Jason et al.	800	1.87 - 7.02	2.47	44.94	[10] (1981)
Keating et al.	800	1.87 - 7.02	3.073	44.14	[11] (1995)

The ions's lifetime τ in the laboratory frame is related to the ion's lifetime in the rest frame τ_0 by the Lorentz transformation :

$$\tau = \gamma \cdot \tau_0 \tag{16}$$

with γ the Lorentz factor. The mean decay length in the laboratory frame is given by the relation :

$$\lambda = c\beta\gamma\tau_0 \tag{17}$$

with β the velocity of the ion and is linked to the fraction lost per meter by the relation :

$$\frac{1}{L} = \frac{1}{\beta c \tau} \tag{18}$$

Figures 7 represents the ion lifetime in the laboratory frame, from Equation 16 for different magnetic fields. As we can see from Figure 7 the Keating and Jason parameters give consistent results with each other when used to calculate H^- lifetime for a beam kinetic energy of 8 GeV but are not consistent with the Stinson parameters. Figure 8 represents the fraction loss per meter from Equation 18 for different magnetic fields.

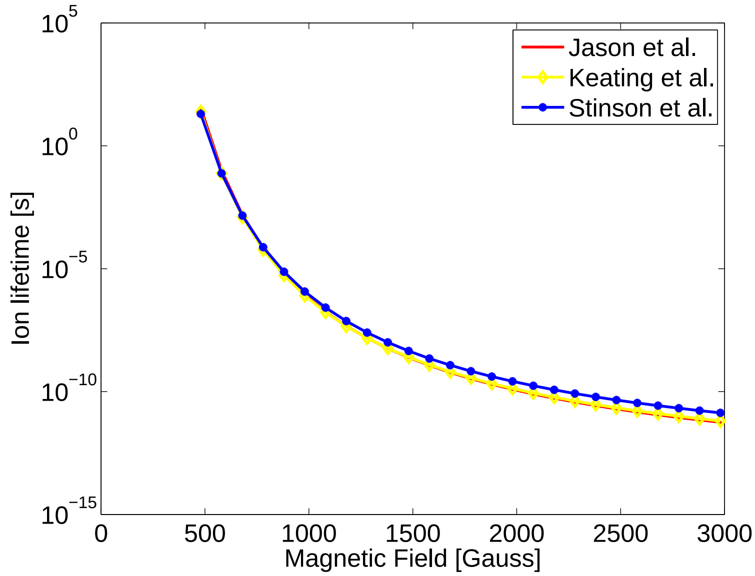


Figure 7: Ion lifetime in the laboratory frame due to magnetic field stripping for different magnetic fields and a beam kinetic energy of 8 GeV. From Equations 15 and 16.

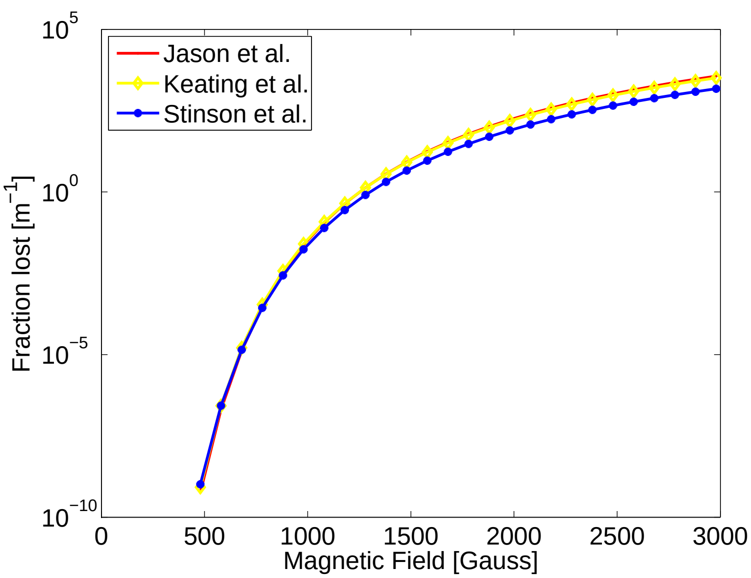


Figure 8: Fraction lost per meter due to magnetic field stripping for different magnetic fields and a beam kinetic energy of 8 GeV. From Equations 16 and 18.

3.2 Application to the FNAL HINS Transfer Line

The dipole magnets foreseen so far in the ~ 1 km transfer line are the refurbish Main Ring B2 dipoles with a magnetic length of ~ 6.071 meters and a magnetic field of ~ 480 G. From Equation 18 the corresponding fraction lost rate is $\sim 1.38 \times 10^{-10}$ m^{-1}. In terms of power, it corresponds to $\sim 1.77 \times 10^{-5}$ W·m^{-1} for a beam macropulse of 1.0×10^{14} particles, which is negligeable.

4 Residual Gas Stripping

4.1 Total loss per unit length by Residal Gas Stripping

During acceleration and transport, the H^- beam can be stripped by interation with the molecules of the residual gas. The lifetime τ_m of the ion is given by the relation [12] :

$$\tau_m = \frac{1}{d_m \sigma_m \beta c} \tag{19}$$

where d_m is the molecular density [m^{-3}], σ_m is the ionisation cross-section for molecule m [m^2]; βc is the velocity of the ion beam [m $\cdot$ s^{-1}]. If there are several types of molecules in the residual gas, then the total lifetime of the ion τ_i is given by the relation :

$$\frac{1}{\tau_i} = \sum_m \frac{1}{\tau_m} \tag{20}$$

and the total loss per unit length by the relation :

$$\frac{1}{L} = \frac{1}{\tau_i \beta c} \tag{21}$$

4.2 Energy dependence of electron cross-section

As presented in Figure 9, the electron cross-section for H^- ions on residual gas atoms decreases with increasing energy. The predicted cross-section scaling with respect to the energy is $1/\beta^2$, with β the relativitic factor (see Ref. [6] and [13]). There is no data available at 8 GeV but a scaling, as presented in Table 2, from measurements at lower energies.

Table 2: Electron cross-section for H^- (Units 10^{-18} cm^2). From Ref. [6] and [13].

Energy of H^-	**H**	**He**	**N**	**O**	**Ar**
400 MeV / 800 MeV	0.2 / -	0.2 / -	- / 1	- / 1	- / 3
8 GeV (scaled)	0.1	0.1	0.7	0.7	2.2

4.3 Application to the FNAL HINS Transfer Line

Figure 10 presents the residual gas spectrum measured on the Fermilab beam line A-150 with similar magnets and vaccum systems as foreseen for the 8 GeV transfer line.

To simplify the problem, we consider from Figure 10 the residual gas to be made of 50% $H2$, 25% $N2$ and 25% $O2$ assuming a pessimistic residual gas pressure of 10^{-7} Torr. The molecular density d_m is related at 20C to the pressure P_m by the relation [12] :

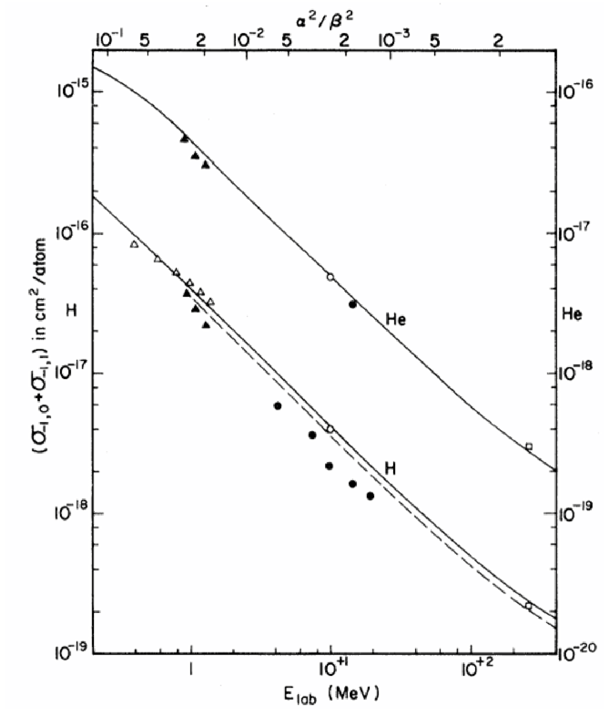
$$d_m[\mathrm{m}^{-3}] = 3.3 \times 10^{22} \cdot P_m[\mathrm{Torr}] \tag{22}$$

which implies a residual gas density in the order of $\sim 3.3 \times 10^{15}$ m^{-3} and from Equation 19, 20, 21 and Table 2 the fraction loss per unit length of :

$$\frac{1}{L} = 1.32 \times 10^{-7}[\mathrm{m}^{-1}] \tag{23}$$

Taking into account an H^- beam intensity of 1×10^{14} particles per macropulse at 8 GeV, the loss rate due to residual gas stripping in our example is in the order of ~ 0.016 W $\cdot$ m^{-1}. Appendix B presents a rigorous method of computing the molecular density d_m of any residual gas for any beam-line temperature and pressure, using the Van der Waals Equation.

(a)



(b)

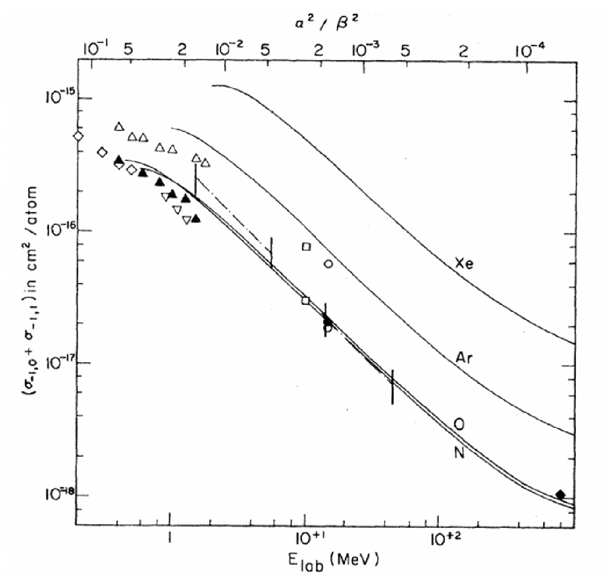


Figure 9: Electron loss cross-section for H^- on (a) He and H atoms and (b) Xe, Ar, O and N atoms as a function of energy. From reference [8].

Figure 10: Residual gas spectrum measured on Fermilab beam line A-150. Courtesy of T. Anderson.

5 Conclusion

Equations ruling the stripping of H^- ions from black-body radiation, magnetic field and residual gas has been presented in this document. Corresponding stripping loss rates computed for the transfer line (300 K and 150 K, 8 GeV) are summarized in Table 3, in agrement with results presented in [13] :

Table 3: Résumé of stripping loss rates from Black-body radiation, Magnetic Field and Residual Gas (300 K and 150 K, 8 GeV)

Stripping Effect	Loss Rate [m^{-1}]	
	[300K]	[150 K]
Black-body	7.86×10^{-7}	2.52×10^{-8}
Magnetic Field	1.38×10^{-10}	
Residual Gas	1.32×10^{-7}	
Total	9.18×10^{-7}	1.57×10^{-7}

All together, the predicted losses at 300 K and 8 GeV in the transfer line are in the order of $\sim\ 0.9 \times 10^{-6}$ [m^{-1}] which for the design beam intensity of 1×10^{14} particles per second at 8 GeV corresponds to beam losses in the order of $\sim\ 0.11$ W·m^{-1}. Reference [8] mentions that this continuous loss rate is not acceptable and previous simulations with the code MARS has shown that such a loss rate would be responsible for hot spots in the beam-pipe at 1000 mR/hr after 30 days of irradiation. As previously mentioned, cooling down of the transfer line at 150 K would reduce total beam losses to an acceptable ~ 0.02 W·m^{-1}.

A Frequently used Fundamental Physical Constants.

Table 4: Frequently used fundamental constants. (From [14]).

Quantity	Symbol	Value	Unit
Speed of light in vacuum	c	299792458	m·s^{-1}
Planck constant	h	$6.62606876(52) \times 10^{-34}$	J·s
$h/2\pi$	$\hbar$	$1.054571596(82) \times 10^{-34}$	J·s
Boltzmann constant	k	$1.3806503(24) \times 10^{-23}$	J·K^{-1}
Electron volt	eV	$1.602176462(63) \times 10^{-19}$	J
Atomic mass unit (a.m.u)	u	$931.494013(37) \times 10^{6}$	eV

$$\begin{aligned}\text{Rest mass of H}^- \text{ beam} : E_0 &\simeq 939.293976 \text{ MeV}\\ &\simeq 1.00838 \text{ a.m.u}\end{aligned}$$

$$\begin{aligned}\text{Lorentz factor} : \gamma &= \tfrac{W+E_0}{E_0} \text{ (with } W \text{ Kinetic Energy)}\\ &\simeq 9.517 \text{ (for } W = 8 \text{ GeV)}\end{aligned}$$

$$\begin{aligned}\text{Beta factor} : \beta &= \sqrt{1-\tfrac{1}{\gamma^2}}\\ &\simeq 0.9945 \text{ (for } W = 8 \text{ GeV)}\end{aligned}$$

Some relations concerning gas pressure :

$$\begin{aligned} 1 \text{ atm} &= 760 \text{ Torr} \\ &= 1013 \text{ mbar} \\ 1 \text{ bar} &= 750 \text{ Torr} \end{aligned}$$

B Molecular density from Van der Waals Equation

The molecular density of any residual gas can be determined from the Van der Waals Equation :

$$\left(P + a\frac{n^2}{V^2}\right)(V - nb) = nRT \tag{24}$$

where P is the pressure of the gas in [bar], V the volume in [L], T the temperature in [K], n the amount of gas in [moles], $R = 83.14472 \times 10^{-3}$ L · bar · K^{-1} · mol^{-1} the Universal Gas Constant. The Van der Waals constants a and b, characteristics of the substance and independant of the temperature, are given in Table 5 for selected gases :

Table 5: Van Der Waals constants for selected gases. From [14].

Substance	a bar L^2/mol^2	b bar L^2/mol^2
Argon	1.355	0.0320
Carbon Dioxide	3.658	0.0429
Hydrogen	0.2452	0.0265
Nitrogen	1.370	0.0387
Oxygen	1.382	0.0319
Water	5.537	0.0305
Xenon	4.192	0.0516

From Equation 24 and parameters of Table 5, the number of moles n per liter for each gas is determined, for any pressure P and temperature T, by the relation :

$$(-ab) \cdot n^3 + a \cdot n^2 - (Pb + RT) \cdot n + P = 0 \tag{25}$$

and the molecular density by :

$$d_m[\mathrm{m}^{-3}] = n \cdot N_A \cdot 10^3 \tag{26}$$

with $N_A = 6.0221367 \times 10^{23}$ the Avogadro's Number.

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