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ANALYTIC CROSS SECTIONS FOR COLLISIONS OF H^+, H_2^+, H_3^+, H, H_2, AND H^- WITH HYDROGEN MOLECULES

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Analytic expressions fitted to cross section data on collisions of H^+, H_2^+, H_3^+, H, H_2, and H^- with H_2 are given. The data used are those recommended by Phelps [J. Chem. Phys. Ref. Data **19**, 653 (1990)] and additional experimental data, when available, up to a projectile energy of about 100 keV, including new kinds of reactions not treated by Phelps. The analytic expressions are of the semiempirical functional forms proposed by Green and McNeal [J. Geophys. Res. **76**, 133 (1971)] and modifications of these, thus making it possible not only to interpolate but also to extrapolate the data to some extent. © 2000 Academic Press

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CONTENTS

INTRODUCTION

Data on cross sections of collision processes between hydrogen ions, atoms, and molecules with hydrogen molecules are important in thermonuclear fusion research in relation to edge plasma as well as in the study of hydrogen thyratrons, ion sources, and other devices. Comprehensive reviews of these processes were given by Barnett et al. [1], Tawara et al. [2], Janev et al. [3], and Phelps [4]. Consulting earlier reviews as well as original papers on experimental and theoretical works, Phelps [4] gave recommended data on cross sections for many different processes such as momentum transfer, excitations, ionization, etc., for H^+, H_2^+, H_3^+, H, H_2, and H^- in collisions with H_2. The purpose of the present paper is to give analytic expressions fitted to Phelps' recommended data [4] and additional experimental data [5] published later.

Forty-nine collision processes in the energy range from 0.1 to 10 keV were treated by Phelps [4]. Among these processes, 14 were considered also by Barnett et al. [1]. Little difference is seen between the recommended cross sections of Refs. [1] and [4] except for the charge transfer process in H_2^+ collisions with H_2 (Graph 12). For this latter process we have also adopted Phelps' recommendation, which is based on Ref. [2] and gives the trend of decreasing cross section with decreasing energy at energies below 5 keV. Reference [1] includes, in particular, the processes accompanied by electronic transitions at relatively high energies above 1 keV. Analytic cross sections fitted to the recommended data of processes included in Ref. [1] are available in a series of publications by Ito et al. [6–9].

Phelps' recommended data for dissociative collisions between H_3^+ and H_2 are considerably smaller than the recent experimental results of Peco and Champion [5] obtained by the crossed-beam technique in the energy range below 400 eV. We have adopted the latter results in the present compilation (Graphs 18–21), because the results of Peco and Champion for the H_3^+ destruction cross section join smoothly with those reported by Williams et al. [10] in the energy range above 2.5 keV, as shown in Graph 51. The data for the dissociative collisions at high energies in Graphs 18–21 have been taken from McClure [11] as in Ref. [4].

For fast H_2^+ production in H_3^+ collisions with H_2 (Graph 19), the cross section measured by Peco and Champion [5] increases with decreasing collision energy down to 55 eV. Considering that the threshold energy of the process is 15.5 eV, however, the extrapolated cross section has been made to fall off at energies below 55 eV. This also applies to the cross section for fast H production in H_3^+ collisions with H_2 (Graph 20), because fast H is produced together with fast H_2^+ simply through dissociation of H_3^+ at low energies below 125 eV. In the energy range above 125 eV, on the other hand, dissociative charge transfer becomes more dominant in this process. The corresponding cross sections, not available from the literature, have been estimated so as to be consistent with the results of McClure [11] at high energies.

Phelps' recommended data are confined to the energy region up to 10 keV. To extend the applicable energy region of the present recommendations by analytic expressions, we have also used additional data, when available, between 10 and 100 keV from the original experimental papers cited by Phelps. In doing so, we have found errors in his compilation of all inelastic data in H_3^+ collision with H_2 and have revised them by reanalyzing the data originally published (Graphs 22–24). We have also revised Phelps' recommended data for double electron loss collisions between H^- and H_2 according to the measurements of Williams [12] and Geddes et al. [13] (Graph 49). In the present compilation, fast H_2 production in the symmetrical $H_2^+ + H_2$ collisions (Graph 50), which shows a similar energy dependence for the cross section as for slow H_2^+ production (Graph 12), and four additional dissociative reactions in H_3^+ collisions with H_2 (Graphs 51–54) observed by Peco and Champion [5] have been included.

The list of the cross sections considered is given in Table A, where the number given with each reaction is the same as the corresponding Graph number.

Analytic Expressions

The functional forms used for the analytic expressions are those semiempirically developed by Green and McNeal [14] and modifications of them such as used in our previous work [6–9]. The relevant basic relations and definitions are as follows:

$$f_1(x; c_1, c_2) = \sigma_0 c_1 (x/E_R)^{c_2} \quad \text{(i)}$$

$$f_2(x; c_1, c_2, c_3, c_4) = f_1(x; c_1, c_2)/[1 + (x/c_3)^{c_2+c_4}] \quad \text{(ii)}$$

$$f_3(x; c_1, c_2, c_3, c_4, c_5, c_6) = f_1(x; c_1, c_2)/[1 + (x/c_3)^{c_2+c_4} + (x/c_5)^{c_2+c_6}] \quad \text{(iii)}$$

$$f_4(x; c_1, c_2, c_3, c_4, c_5, c_6, c_7, c_8) = f_1(x; c_1, c_2)[1 + (x/c_3)^{c_4-c_2}]/ [1 + (x/c_5)^{c_4+c_6} + (x/c_7)^{c_4+c_8}] \quad \text{(iv)}$$

$$\sigma_0 = 1 \times 10^{-16}\ \text{cm}^2 \quad \text{(v)}$$

$$E_R = 1.361 \times 10^{-2}\ \text{keV (Rydberg constant)} \quad \text{(vi)}$$

$$E_1 = E - E_{th} \quad \text{(vii)}$$

$$E = \text{incident projectile energy in keV} \quad \text{(viii)}$$

$$E_{th} = \text{threshold energy of reaction in keV.} \quad \text{(ix)}$$

The symbols x and c_i ($i = 1, 2, \ldots, 8$) denote dummy parameters. Depending on the formula to be chosen from Eqs. (1)–(14) below, the value of E_1 or E_1/a_i ($i = 6, 8, 10$ or 12) is put into x, and a_1, a_2, etc., are put into c_i.

The cross sections for the individual collisional processes are given by the following set of analytic expressions, according to the correlation between the "No." and "Eq." columns in Table I (read across two facing pages):

$$\sigma = f_2(E_1; a_1, a_2, a_3, a_4) \quad (1)$$

$$\sigma = f_2(E_1; a_1, a_2, a_3, a_4) + a_5 f_2(E_1/a_6; a_1, a_2, a_3, a_4) \quad (2)$$

$$\sigma = f_2(E_1; a_1, a_2, a_3, a_4) + f_2(E_1; a_5, a_6, a_7, a_8) \quad (3)$$

$$\sigma = f_2(E_1; a_1, a_2, a_3, a_4) + f_2(E_1; a_5, a_6, a_7, a_8) + a_9 f_2(E_1/a_{10}; a_5, a_6, a_7, a_8) \quad (4)$$

$$\sigma = f_2(E_1; a_1, a_2, a_3, a_4) + f_2(E_1; a_5, a_6, a_7, a_8) + f_2(E_1; a_9, a_{10}, a_{11}, a_{12}) \quad (5)$$

$$\sigma = f_3(E_1; a_1, a_2, a_3, a_4, a_5, a_6) \quad (6)$$

$$\sigma = f_2(E_1; a_1, a_2, a_3, a_4) + f_3(E_1; a_5, a_2, a_6, a_7, a_8, a_4) \quad (7)$$

$$\sigma = f_2(E_1; a_1, a_2, a_3, a_4) + f_3(E_1; a_5, a_2, a_6, a_7, a_8, a_9) \quad (8)$$

$$\sigma = f_2(E_1; a_1, a_2, a_3, a_4) + f_3(E_1; a_5, a_6, a_7, a_8, a_9, a_4) \quad (9)$$

$$\sigma = f_3(E_1; a_1, a_2, a_3, a_4, a_5, a_6) + a_7 f_3(E_1/a_8; a_1, a_2, a_3, a_4, a_5, a_6) \quad (10)$$

$$\sigma = f_3(E_1; a_1, a_2, a_3, a_4, a_5, a_6) + f_2(E_1; a_7, a_8, a_9, a_{10}) \quad (11)$$

$$\sigma = f_3(E_1; a_1, a_2, a_3, a_4, a_5, a_6) + f_2(E_1; a_7, a_8, a_9, a_{10}) + a_{11} f_2(E_1/a_{12}; a_7, a_8, a_9, a_{10}) \quad (12)$$

$$\sigma = f_3(E_1; a_1, a_2, a_3, a_4, a_5, a_6) + f_3(E_1; a_7, a_8, a_9, a_{10}, a_{11}, a_{12}) \quad (13)$$

$$\sigma = f_4(E_1; a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8). \quad (14)$$

The use of such expressions allows one not only to interpolate but also to extrapolate the data to some extent, in contrast to polynomial fits, which frequently show physically unreasonable behavior just outside the energy range of the available data.

The number of adjustable parameters used in the analytic expressions is between 4 and 12 according to the type of the function. The values of the adjustable parameters have been determined by least-squares fits to the data except for some values that were chosen to guarantee reasonable behavior outside the energy range of the available data. The values determined are given in the last six columns of Table I.

The analytic expressions obtained are compared with the recommended and experimental data in the set of Graphs, which show that agreement is quite good. The root-mean-square (rms) and the maximum deviations of the expressions

TABLE A
Collisional Cross Sections Represented by Analytic Expressions and Shown in Graphs 1-54

	H^+ collisions with H_2 ($H^+ + H_2$)	
1	Rotational excitation for $J = 0 \to 2$:	$H^+ + H_2\,(J=0) \to H_2\,(J=2)$
2	Rotational excitation for $J = 1 \to 3$:	$H^+ + H_2\,(J=1) \to H_2\,(J=3)$
3	Vibrational excitation for $v = 0 \to 1$:	$H^+ + H_2\,(v=0) \to H_2\,(v=1)$
4	Vibrational excitation for $v = 0 \to 2$:	$H^+ + H_2\,(v=0) \to H_2\,(v=2)$
5	Vibrational excitation for $v = 0 \to 3$:	$H^+ + H_2\,(v=0) \to H_2\,(v=3)$
6	Rearrangement or charge transfer to fast H:	$H^+ + H_2 \to$ fast H
7	Ly-α excitation:	$H^+ + H_2 \to$ Ly-α (Total)
8	Hα excitation:	$H^+ + H_2 \to$ Hα (Total)
9	Electron production:	$H^+ + H_2 \to$ e (Total)
10	Momentum transfer:	$H^+ + H_2 \to$ Momentum Transfer
	${H_2}^+$ collisions with H_2 (${H_2}^+ + H_2$)	
11	${H_3}^+$ formation:	${H_2}^+ + H_2 \to {H_3}^+ + H$
12	Slow H_2^+ formation:	${H_2}^+ + H_2 \to$ slow ${H_2}^+$ (Charge Transfer at $E \leqq 3$keV)
13	Vibrational excitation for $v = 0 \to 1$:	${H_2}^+ + H_2\,(v=0) \to H_2\,(v=1)$
14	Dissociation to fast H^+:	${H_2}^+ + H_2 \to$ fast H^+
15	Ly-α excitation:	${H_2}^+ + H_2 \to$ Ly-α (Total)
16	Hα excitation:	${H_2}^+ + H_2 \to$ Hα (Total)
17	Electron production:	${H_2}^+ + H_2 \to$ e (Total)
	${H_3}^+$ collisions with H_2 (${H_3}^+ + H_2$)	
18	Dissociation to fast H^+:	${H_3}^+ + H_2 \to$ fast H^+
19	Dissociation to fast ${H_2}^+$:	${H_3}^+ + H_2 \to$ fast ${H_2}^+$
20	Charge transfer to form fast H:	${H_3}^+ + H_2 \to$ fast H
21	Charge transfer to form fast H_2:	${H_3}^+ + H_2 \to$ fast H_2
22	Ly-α excitation:	${H_3}^+ + H_2 \to$ Ly-α (Total)
23	Hα excitation:	${H_3}^+ + H_2 \to$ Hα (Total)
24	Electron production:	${H_3}^+ + H_2 \to$ e (Total)
25	Momentum transfer:	${H_3}^+ + H_2 \to$ Momentum Transfer
	H collisions with H_2 ($H + H_2$)	
26	Rotational excitation for $J = 0 \to 2$:	$H + H_2\,(J=0) \to H_2\,(J=2)$
27	Rotational excitation for $J = 1 \to 3$:	$H + H_2\,(J=1) \to H_2\,(J=3)$
28	Vibrational excitation for $v = 0 \to 1$:	$H + H_2\,(v=0) \to H_2\,(v=1)$
29	Production of fast H^-:	$H + H_2 \to$ fast H^-
30	Production of slow ${H_2}^+$:	$H + H_2 \to$ slow ${H_2}^+$
31	Production of fast H^+:	$H + H_2 \to$ fast H^+
32	Ly-α excitation:	$H + H_2 \to$ Ly-α (Total)
33	Hα excitation:	$H + H_2 \to$ Hα (Total)
34	Hβ excitation:	$H + H_2 \to$ Hβ (Total)
35	Electron production:	$H + H_2 \to$ e (Total)
36	Momentum transfer:	$H + H_2 \to$ Momentum Transfer
	H_2 collisions with H_2 ($H_2 + H_2$)	
37	Fast H_2 destruction:	$H_2 + H_2 \to$ fast H_2 Destruction
38	Production of fast ${H_2}^+$:	$H_2 + H_2 \to$ fast ${H_2}^+$
39	Rotational excitation for $J_1,\, J_2 = 0, 0 \to 0, 2$:	$H_2(J=0) + H_2\,(J=0) \to H_2(J=0) + H_2\,(J=2)$
40	Rotational excitation for $J_1,\, J_2 = 0, 0 \to 2, 2$:	$H_2(J=0) + H_2\,(J=0) \to H_2(J=2) + H_2\,(J=2)$
41	Rotational excitation for $J_1,\, J_2 = 0, 0 \to 0, 4$:	$H_2(J=0) + H_2\,(J=0) \to H_2(J=0) + H_2\,(J=4)$
42	Vibrational excitation for $v = 0 \to 1$:	$H_2 + H_2\,(v=0) \to H_2\,(v=1)$
43	Production of fast H^+:	$H_2 + H_2 \to$ fast H^+
44	Hα excitation:	$H_2 + H_2 \to$ Hα (Total)
45	Ionization:	$H_2 + H_2 \to$ fast ${H_2}^+$ or fast H^+
46	Momentum transfer:	$H_2 + H_2 \to$ Momentum Transfer
	H^- collisions with H_2 ($H^- + H_2$)	
47	Detachment to form an electron and fast H:	$H^- + H_2 \to$ e + fast H
48	Momentum transfer:	$H^- + H_2 \to$ Momentum Transfer
49	Production of fast H^+:	$H^- + H_2 \to$ fast H^+
	Additional data	
50	Fast H_2 formation in collisions of ${H_2}^+$ with H_2:	${H_2}^+ + H_2 \to$ fast H_2 (Charge Transfer at $E \leqq 3$keV)
51	H_3^+ destruction in collisions with H_2:	${H_3}^+ + H_2 \to {H_3}^+$ Destruction
52	Production of slow ${H_3}^+$ in collisions of ${H_3}^+$ with H_2:	${H_3}^+ + H_2 \to$ slow ${H_3}^+$
53	Production of slow H^+ in collisions of ${H_3}^+$ with H_2:	${H_3}^+ + H_2 \to$ slow ${H_3}^{+*} \to$ slow H^+
54	Production of slow ${H_2}^+$ in collisions of ${H_3}^+$ with H_2:	${H_3}^+ + H_2 \to$ slow ${H_3}^{+*} \to$ slow ${H_2}^+$ (Dissociative Charge Transfer at $E > 125$eV)

Note. "Total" means total emission from the projectile and the target. In the last two reaction formulas, the symbol $*$ means the excited state.

from the data are given in the fifth and the sixth column of Table I. For the processes for which the data have been taken only or mostly from Phelps' recommendation, the rms deviation is generally smaller than about 5%. Large deviations are due to experimental uncertainties rather than the errors of fitting as can typically be seen, for example, from Graph 18.

Example of Use of Table I

As an illustration, we calculate the cross section for Ly-α excitation in collisions of H^+ with H_2 at incident energy $E = 0.500$ keV (Reaction No. 7). From Table I, we find that the cross section is given by Eq. (2) as

$$\sigma = f_2(E_1; a_1, a_2, a_3, a_4) + a_5\, f_2(E_1/a_6; a_1, a_2, a_3, a_4).$$

The coefficients a_1 to a_6 are listed in Table I together with the threshold energy E_{th}. Using basic relation (vii), we have $E_1 = E - E_{th} = 0.475$ keV. Substituting basic relation (i) into (ii) and using (v) and (vi), we find for the terms in Eq. (2)

$$\begin{aligned} & f_2(E_1; a_1, a_2, a_3, a_4) \\ & = 10^{-16} \times 3.2 \times 10^{-5} \left(\frac{0.475}{0.01361}\right)^{2.228} \Big/ \\ & \quad \left[1 + \left(\frac{0.475}{0.83}\right)^{2.228+0.428}\right] \\ & = 8.762 \times 10^{-18}/1.227 \\ & = 7.14 \times 10^{-18} \end{aligned}$$

and

$$\begin{aligned} & f_2(E_1/a_6; a_1, a_2, a_3, a_4) \\ & = 10^{-16} \times 3.2 \times 10^{-5} \left(\frac{0.475}{9.44 \times 0.01361}\right)^{2.228} \Big/ \\ & \quad \left[1 + \left(\frac{0.475}{9.44 \times 0.83}\right)^{2.228+0.428}\right] \\ & = 5.893 \times 10^{-20}/1.001 \\ & = 5.89 \times 10^{-20}. \end{aligned}$$

Hence, the fitted cross section is

$$\begin{aligned} \sigma & = 7.14 \times 10^{-18} + 3.12 \times 5.89 \times 10^{-20} \\ & = 7.32 \times 10^{-18}\,(\text{cm}^2), \end{aligned}$$

which agrees with Graph 7.

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Notes added after completion of this work. It has come to our attention that Krstić and Schultz [15] have recently performed fully quantum-mechanical calculations in infinite order sudden approximation on the cross sections for vibrational excitation and momentum transfer in H^+ collisions with H_2 (Graphs 3–5 and 10) and H collisions with H_2 (Graphs 28 and 36) in the energy range from 0.15 to 150 eV. Their results show an oscillatory structure or a narrow peak of the cross section at energies near the threshold on the respective cross section curve for vibrational excitation, while Phelps' recommended cross section decreases monotonically with decreasing energy. This discrepancy stems from the fact that Phelps did not have actual data in this energy region and had based his recommended values on extrapolation. As far as the cross sections for vibrational excitation in H^+ collisions with H_2 (Graphs 3–5) are concerned, the sets of results agree well with each other at energies above 6 eV, except for excitation to the third excited vibrational state, for which the cross section of Krstić and Schultz is larger than that of Phelps by about 40%. For excitation to the first excited vibrational state in H collisions with H_2 (Graph 28), on the other hand, we see considerable discrepancies between these two data sets; that is, the new data are lower by about a factor of four than Phelps', which had been simply taken from the corresponding data set for H^+ collisions with H_2 at high energies. For momentum transfer, the results of Krstić and Schultz are in approximate agreement with Phelps' recommended data at low collision energies, though the former are a little smaller than Phelps' in H^+ collisions with H_2, and larger than Phelps' in H collisions with H_2. However, their results become smaller by a factor up to about 10 than Phelps' with increasing energy. A similar situation had been noted by Phelps [4] during his data assessment, where he found that the cross sections inferred from his procedure were about an order of magnitude larger than a calculation at 38 eV and suggested that more work in the energy range from 1 eV to 1 keV would be important.

In the present work we have aimed at providing users with analytic fits to Phelps' recommended data sets and new experimental data. Therefore, we do not include the analytic fits to the single theoretical data sets of Krstić and Schultz, but the results of fits to these data sets are available upon request to one of the present authors (T.S.).

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EXPLANATION OF TABLE

TABLE I. **Energy Ranges of Data, Fitting Errors, and Parameters of the Analytic Expressions**

No.	Number label identifying a particular reaction process in the same sequence as in Table A and in the Graphs.
Process	The relevant reaction process.
E_{min}	Minimum energy (in keV) of the recommended data.
E_{max}	Maximum energy (in keV) of the recommended data.
δ_{rms}	Root-mean-square relative deviation (in %) of the analytic expression from the data.
δ_{max}	Maximum relative deviation (in %) of the analytic expression from the data.
$E_{\delta_{max}}$	Energy (in keV) at which the relative deviation takes on the value δ_{max}.
Eq.	The identifying number of the equation to be used for deriving the cross sections.
n	Number of applicable fit parameters.
E_{th}	Threshold energy of the reaction (in keV); values are often the ones adjusted to give good fits to data rather than the actual threshold energy.
a_i ($i = 1, 2, \ldots, 12$)	Fit parameters.

The notation 1.23–1 means 1.23×10^{-1}.

EXPLANATION OF GRAPHS

GRAPHS. **Cross Section vs Projectile Energy**

Graphs are numbered in the same sequence as in Table A and Table I.

Ordinate	Cross section in cm^2.
Abscissa	Laboratory energy in eV of the projectile for H_2 at rest.
Solid line	Recommended data from the analytic formula of the present work.
Circles	Phelps’ recommended data (Ref. [4]).
Pentagons	Experimental data in the energy range above 10 keV from the same sources as used by Phelps.
Other symbols	Experimental or theoretical data from other sources as explained in the legend.

TABLE I. Energy Ranges of Data, Fitting Errors, and Parameters of the Analytic Expressions

See page 7 for Explanation of Tables

No.	Process	E_{min}	E_{max}	δ_{rms}	δ_{max}	$E_{\delta_{max}}$
1	$H^+ + H_2(J=0) \to H_2(J=2)$ (Rotational Excitation)	1.00−4	3.16−1	2.3	4.7	3.16−4
2	$H^+ + H_2(J=1) \to H_2(J=3)$	1.33−4	2.37−1	2.4	6.1	1.33−1
3	$H^+ + H_2(v=0) \to H_2(v=1)$ (Vibrational Excitation)	1.00−3	1.00+1	1.7	4.2	7.50−3
4	$H^+ + H_2(v=0) \to H_2(v=2)$	1.78−3	1.00+1	2.7	7.3	3.16−3
5	$H^+ + H_2(v=0) \to H_2(v=3)$	2.37−3	3.16	2.1	4.5	1.00−1
6	$H^+ + H_2 \to$ fast H	3.16−3	1.00+2	1.3	2.8	3.16−1
7	$H^+ + H_2 \to$ Ly α (Total)	7.50−2	2.46+1	6.6	2.3+1	7.50−2
8	$H^+ + H_2 \to$ Hα (Total)	1.00−1	1.00+2	5.0	1.6+1	2.00+1
9	$H^+ + H_2 \to e$ (Total)	7.50−2	1.00+2	1.9	5.5	1.00−1
10	$H^+ + H_2 \to$ Momentum Transfer	1.00−4	1.00+1	4.9	1.1+1	1.00−1
11	${H_2}^+ + H_2 \to {H_3}^+ + H$	1.00−4	1.78−2	1.2	2.1	1.00−4
12	${H_2}^+ + H_2 \to$ slow ${H_2}^+$ (Charge Transfer at $E \leq 3$keV)	1.00−4	4.67+1	2.0	5.2	1.78−3
13	${H_2}^+(v=0) + H_2 \to {H_2}^+(v=1)$	1.78−3	1.00+1	1.5	4.9	1.33−2
14	${H_2}^+ + H_2 \to$ fast H^+	5.62−3	9.90+1	2.2	4.1	1.33−1
15	${H_2}^+ + H_2 \to$ Ly α (Total)	1.78−1	2.49+1	2.6	7.7	7.50
16	${H_2}^+ + H_2 \to$ Hα (Total)	7.50−1	1.00+2	1.1	2.2	1.33
17	${H_2}^+ + H_2 \to e$ (Total)	3.16−2	1.00+2	6.8	1.9+1	4.22−2
18	${H_3}^+ + H_2 \to$ fast H^+	1.48−2	1.15+2	1.8+1	6.0+1	1.48−2
19	${H_3}^+ + H_2 \to$ fast ${H_2}^+$	5.40−2	1.17+2	1.0+1	2.2+1	2.46−1
20	${H_3}^+ + H_2 \to$ fast H	5.40−2	1.19+2	5.9	1.4+1	5.4−2
21	${H_3}^+ + H_2 \to$ fast H_2	5.40−2	1.18+2	3.9	1.3+1	1.98−1
22	${H_3}^+ + H_2 \to$ Ly α (Total)	7.50−2	1.00+1	2.3	7.2	7.07−1
23	${H_3}^+ + H_2 \to$ Hα (Total)	7.50−2	1.00+2	1.7	3.4	2.00+1
24	${H_3}^+ + H_2 \to e$ (Total)	7.50−2	1.00+2	3.9	9.8	6.00
25	${H_3}^+ + H_2 \to$ Momentum Transfer	1.00−4	1.00+1	5.6	3.1+1	5.62
26	$H + H_2(J=0) \to H_2(J=2)$	1.00−4	1.00+1	4.7	1.3+1	7.50−4
27	$H + H_2(J=1) \to H_2(J=3)$	1.00−4	1.00+1	4.4	8.6	1.78−3

TABLE I. Energy Ranges of Data, Fitting Errors, and Parameters of the Analytic Expressions

See page 7 for Explanation of Tables

No.	Eq.	n	E_{th}	a_1	a_2	a_3	a_4	a_5	a_6
				a_7	a_8	a_9	a_{10}	a_{11}	a_{12}
1	9	9	9.0−5	1.13+2	5.4−1	7.0−5	3.459	7.0+1	2.6−1
				2.96−3	1.113	1.307−2			
2	9	9	1.2−4	1.14+3	1.0	9.15−5	3.588	8.1+1	4.4−1
				2.2−3	1.058	1.189−2			
3	6	6	5.0−4	6.18	1.523	1.786−2	1.48−1	7.27−2	1.164
4	6	6	1.3−3	1.375	1.21	3.8−2	6.59−1	8.39−2	1.482
5	6	6	2.1−3	5.258−1	9.673−1	3.965−2	9.83−1	1.36−1	1.88
6	8	9	2.5−3	2.12+2	1.721	6.7−4	3.239−1	4.34−3	1.296
				1.42−1	9.34	2.997			
7	2	6	2.5−2	3.2−5	2.228	8.3−1	4.28−1	3.12	9.44
8	2	6	3.0−2	9.8−5	1.272	1.88	1.747	7.58	1.326+1
9	1	4	2.0−2	1.864−4	1.216	5.31+1	8.97−1		
10	1	4	0.0	5.74	−5.765−1	2.79−2	1.737		
11	6	6	0.0	6.05	−5.247−1	4.088−3	2.872	7.3−3	6.99
12	5	12	0.0	8.07	6.85−1	6.43−3	3.95	2.58+2	2.85
				4.65−3	1.201−1	4.85−5	1.67	1.49+1	1.09
13	6	6	1.0−3	1.838	1.875	1.459−2	1.89−1	6.87−2	9.51−1
14	4	10	5.0−3	6.34+1	1.78	1.38−3	4.06−1	1.63−1	3.27−1
				1.554+1	3.903	1.735	1.02+1		
15	6	6	9.0−2	3.22−4	2.007	4.35−1	−2.55−1	4.0+1	1.0
16	6	6	4.0−1	3.26−4	1.195	2.09	−2.53−1	2.88+1	1.62
17	6	6	3.0−2	1.086−3	1.153	1.24+1	−4.44−1	5.96+1	1.0
18	3	8	1.1−2	6.67−1	1.35	4.42−2	7.1−1	6.7−5	1.54
				1.1+1	−1.0−1				
19	3	8	1.55−2	5.03−1	1.0	2.5−2	2.0	1.17−1	3.18−1
				9.4+1	1.35				
20	3	8	1.55−2	5.89−1	1.0	2.5−2	1.5	4.05−2	7.59−1
				4.64+1	1.1				
21	3	8	0.0	3.78+1	1.0	2.0−3	2.5−1	4.14−2	6.25−1
				4.89+1	1.69				
22	2	6	6.2−2	1.098−2	6.644−1	1.519	1.89	1.17+2	3.06+3
23	2	6	6.2−2	5.33−3	5.27−1	7.46−1	9.66−1	1.312+1	2.48+2
24	2	6	3.6−2	2.63−3	9.31−1	4.05−1	1.0	1.26+2	2.13+2
25	14	8	0.0	1.16	−8.12−1	4.29−4	−1.38−1	1.28−2	1.33
				8.67−2	2.18				
26	14	8	0.0	7.54−1	9.57−1	3.98−4	3.69	1.235−3	−2.97−1
				3.74−3	1.005				
27	14	8	0.0	7.81−1	1.03	5.41−4	3.98	1.342−3	−3.46−1
				3.72−3	9.91−1				

TABLE I. Energy Ranges of Data, Fitting Errors, and Parameters of the Analytic Expressions

See page 7 for Explanation of Tables

No.	Process	E_{min}	E_{max}	δ_{rms}	δ_{max}	$E_{\delta_{max}}$
28	$H + H_2(v = 0) \to H_2(v = 1)$	1.00−3	1.00+1	2.2	5.3	3.16
29	$H + H_2 \to$ fast H^-	2.37−2	9.11+1	3.6	8.8	1.00
30	$H + H_2 \to$ slow $H_2{}^+$	2.37−2	2.53+1	2.9	7.1	2.53+1
31	$H + H_2 \to$ fast H^+	5.62−2	1.12+2	4.7	2.0+1	7.50−2
32	$H + H_2 \to$ Ly α (Total)	2.37−2	2.50+1	3.9	1.1+1	3.16−2
33	$H + H_2 \to H\alpha$ (Total)	3.16−2	1.00+2	6.5	2.3+1	4.22−2
34	$H + H_2 \to H\beta$ (Total)	1.00−1	1.00+1	2.9	7.1	1.33−1
35	$H + H_2 \to e$ (Total)	4.22−2	2.84+1	4.7	1.1+1	1.00+1
36	$H + H_2 \to$ Momentum Transfer	1.00−4	1.00+1	1.5	5.0	3.16
37	$H_2 + H_2 \to$ fast H_2 Destruction	1.78−2	1.00+2	2.8	9.2	3.16−2
38	$H_2 + H_2 \to$ fast $H_2{}^+$	4.21−2	9.90+1	3.4	8.1	7.50−2
39	$H_2(J = 0) + H_2(J = 0) \to H_2(J = 0) + H_2(J = 2)$	1.00−4	7.50−1	3.2	1.3+1	7.50−1
40	$H_2(J = 0) + H_2(J = 0) \to H_2(J = 2) + H_2(J = 2)$	2.37−4	1.78	1.9	4.2	7.50−4
41	$H_2(J = 0) + H_2(J = 0) \to H_2(J = 0) + H_2(J = 4)$	7.50−4	4.22−1	1.6	5.9	4.21−2
42	$H_2 + H_2(v = 0) \to H_2(v = 1)$	2.37−3	1.00+1	2.2	6.6	3.16−3
43	$H_2 + H_2 \to$ fast H^+	1.78−1	9.95+1	4.8	9.8	7.50
44	$H_2 + H_2 \to H\alpha$ (Total)	1.78−2	1.00+2	2.1	7.1	6.00+1
45	$H_2 + H_2 \to$ fast $H_2{}^+$ or fast H^+ (Ionization)	4.21−2	9.90+1	2.9	6.8	7.50−2
46	$H_2 + H_2 \to$ Momentum Transfer	1.00−4	1.00+1	1.2	3.6	1.78
47	$H^- + H_2 \to e +$ fast H	2.37−3	5.00+1	6.4	2.1+1	1.0 + 1
48	$H^- + H_2 \to$ Momentum Transfer	1.00−4	1.00+1	2.6	6.5	4.21−1
49	$H^- + H_2 \to$ fast H^+	1.00	3.00+2	9.0	2.1+1	3.00
50	$H_2{}^+ + H_2 \to$ fast H_2 (Charge Transfer at $E \leq 3$keV)	1.00−4	1.2+2	2.1	7.2	3.32
51	$H_3{}^+ + H_2 \to H_3{}^+$ Destruction	5.40−2	1.00+2	4.1	1.4+1	1.98−1
52	$H_3{}^+ + H_2 \to$ slow $H_3{}^+$	8.41−3	5.13−2	1.2+1	2.8+1	4.16−2
53	$H_3{}^+ + H_2 \to$ slow $H_3{}^{+*} \to$ slow H^+	8.41−3	3.13−1	3.4	8.9	1.88−2
54	$H_3{}^+ + H_2 \to$ slow $H_3{}^{+*} \to$ slow $H_2{}^+$	1.67−2	3.13−1	9.4	2.7+1	1.88−2

TABLE I. Energy Ranges of Data, Fitting Errors, and Parameters of the Analytic Expressions

See page 7 for Explanation of Tables

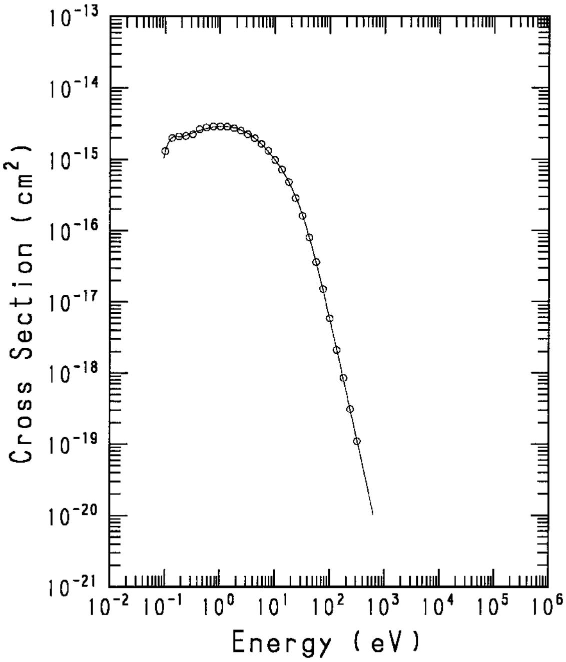
No.	Eq.	n	E_{th}	a_1 a_7	a_2 a_8	a_3 a_9	a_4 a_{10}	a_5 a_{11}	a_6 a_{12}
28	6	6	9.0−4	1.602+1	1.082	5.96−3	2.7−2	4.37−2	1.005
29	13	12	2.1−2	9.73−3	2.38	1.39−2	−5.51−1	7.7−2	2.12
				1.97−6	2.051	5.5	6.62−1	2.02+1	3.62
30	11	10	2.1−2	9.69−3	2.4	1.36−2	−5.52−1	7.91−2	2.27
				1.05−4	1.538	8.35	2.6−2		
31	2	6	2.0−2	2.53−4	1.728	2.164	7.74−1	1.639	1.43+1
32	2	6	2.0−2	7.64−2	1.097	1.77−1	3.66−1	5.49−1	4.0
33	12	12	2.3−2	4.06−4	2.214	2.23−1	−1.1−1	6.41−1	1.34
				2.67−11	3.33	1.09+1	4.97−1	1.79	2.0+1
34	1	4	7.0−2	1.211−3	1.012	8.47−1	6.74−1		
35	2	6	3.2−2	4.21−4	1.64	2.35	4.68−1	2.79	1.25+1
36	6	6	0.0	2.97+1	4.095−3	1.11−4	5.55−1	6.0−3	1.607
37	10	8	1.1−2	4.69−3	2.946	4.36−2	−9.01−1	1.52−1	5.01−1
				2.6	3.229+1				
38	10	8	3.2−2	1.879−3	2.497	6.62−2	−4.67−1	3.58−1	5.0−1
				7.67	2.01+2				
39	6	6	6.0−5	3.25+1	1.073	3.65−3	6.24−1	7.68−3	1.678
40	6	6	9.0−5	2.47+1	1.746	7.94−3	8.42−1	2.106−2	1.98
41	6	6	9.0−5	2.54+1	2.81	3.76−3	−2.48−1	8.63−3	1.73
42	6	6	1.2−3	1.041+1	3.214	8.1−3	−3.22−1	1.44−2	6.2−1
43	2	6	2.0−2	1.307−5	1.586	1.066+1	2.03	2.73	4.71
44	7	8	1.1−2	1.07−8	2.211	2.31+1	8.38−1	4.79−3	2.044−2
				−3.58−1	2.4−1				
45	7	8	2.9−2	1.06−10	2.914	2.97+1	2.79	1.023−3	6.15−2
				−5.035−1	4.95				
46	6	6	0.0	1.018+1	−1.413−1	6.53−3	1.083	5.88−2	2.234
47	11	10	2.25−3	4.19−2	1.89	1.78−1	−2.3−1	1.04	8.7−1
				1.65+1	1.088	5.33−3	1.66−1		
48	6	6	0.0	6.36	−3.37−1	5.5−3	8.3−1	2.6−2	1.766
49	6	6	0.0	1.75−8	3.88	9.06−1	−2.74−1	3.19	1.19
50	12	12	0.0	2.29+2	2.78	4.75−3	1.248−1	2.14−1	2.33
				7.96	6.82−1	6.59−3	4.51	1.67−1	1.164+4
51	3	8	1.55−2	3.2+2	1.0	3.7−4	2.5−1	1.17−1	5.37−1
				7.68+1	1.59				
52	1	4	0.0	6.0	−5.0−1	1.322−2	4.26		
53	3	8	1.1−2	2.1	3.91−1	1.535−2	2.84	1.48−1	3.0−1
				1.4	2.5−1				
54	3	8	1.55−2	2.8−1	4.4	1.61−2	1.8	2.11−1	5.24−1
				8.4−1	2.5−1				

GRAPHS. Cross Section vs Projectile Energy

See page 7 for Explanation of Graphs

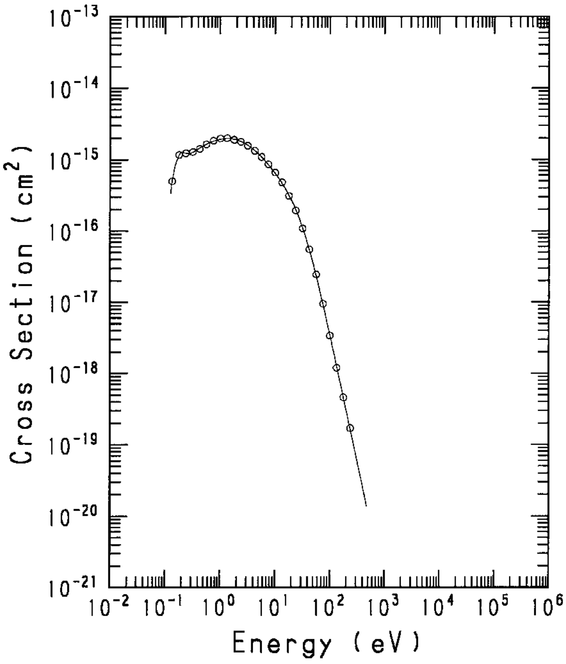
GRAPH 1

$H^+ + H_2(J=0) \rightarrow H_2(J=2)$



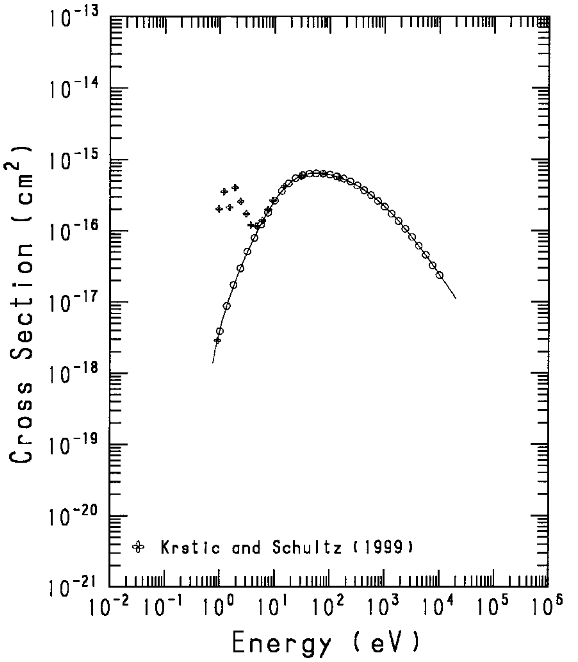
GRAPH 2

$H^+ + H_2(J=1) \rightarrow H_2(J=3)$



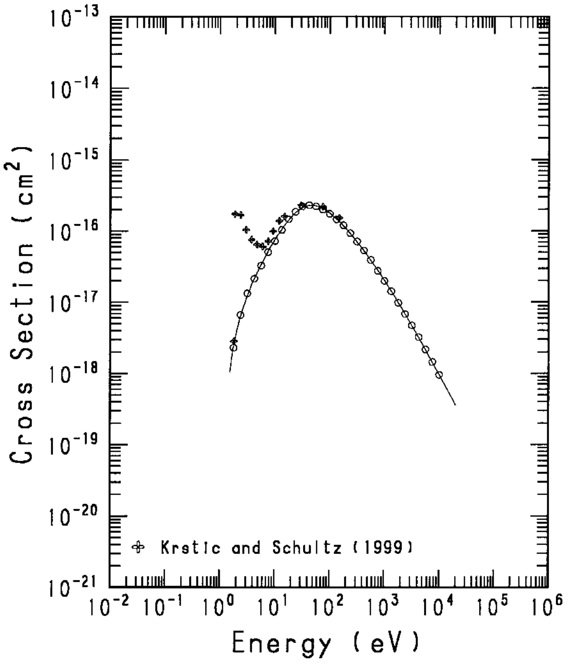
GRAPH 3

$H^+ + H_2(v=0) \rightarrow H_2(v=1)$



GRAPH 4

$H^+ + H_2(v=0) \rightarrow H_2(v=2)$

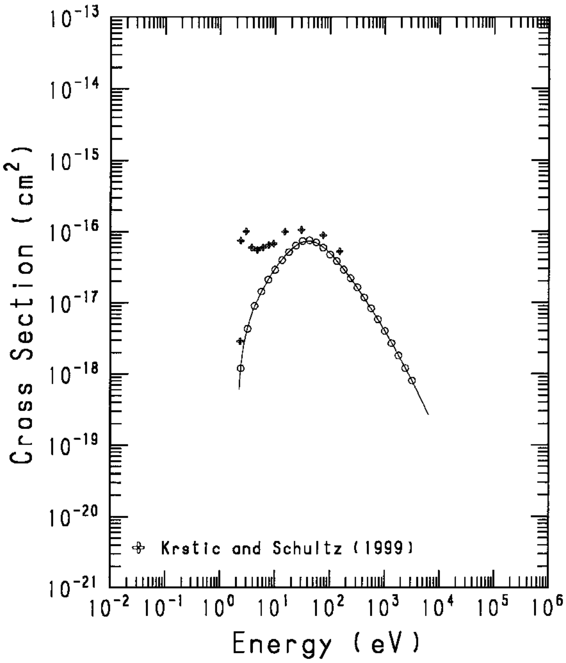


GRAPHS. Cross Section vs Projectile Energy

See page 7 for Explanation of Graphs

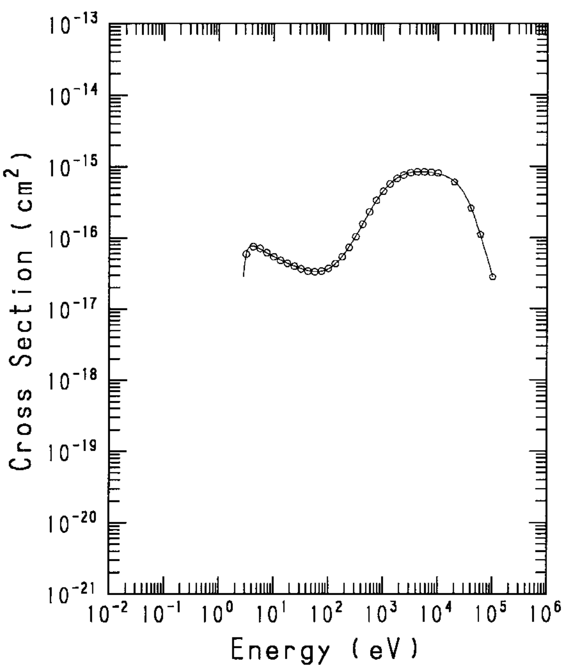
GRAPH 5

$H^+ + H_2(v=0) \rightarrow H_2(v=3)$



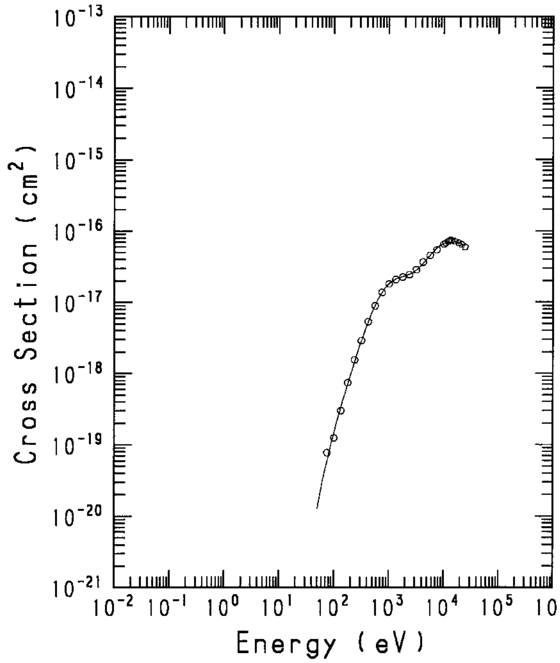
GRAPH 6

$H^+ + H_2 \rightarrow$ fast H



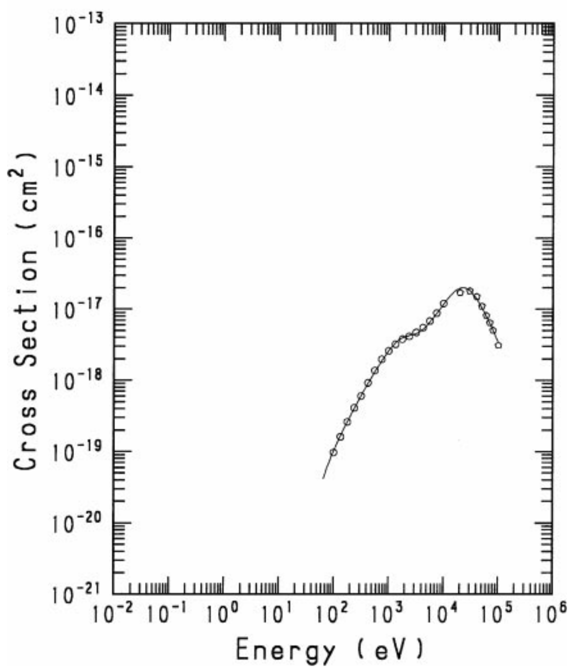
GRAPH 7

$H^+ + H_2 \rightarrow$ Ly α (Total)



GRAPH 8

$H^+ + H_2 \rightarrow$ Hα (Total)

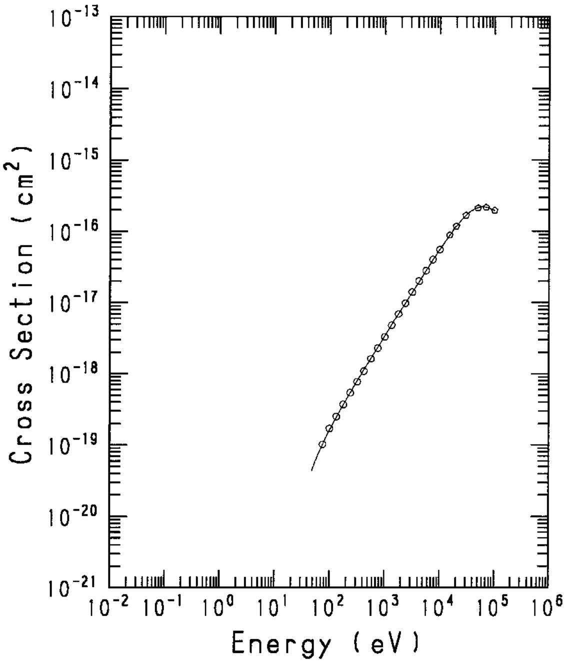


GRAPHS. Cross Section vs Projectile Energy

See page 7 for Explanation of Graphs

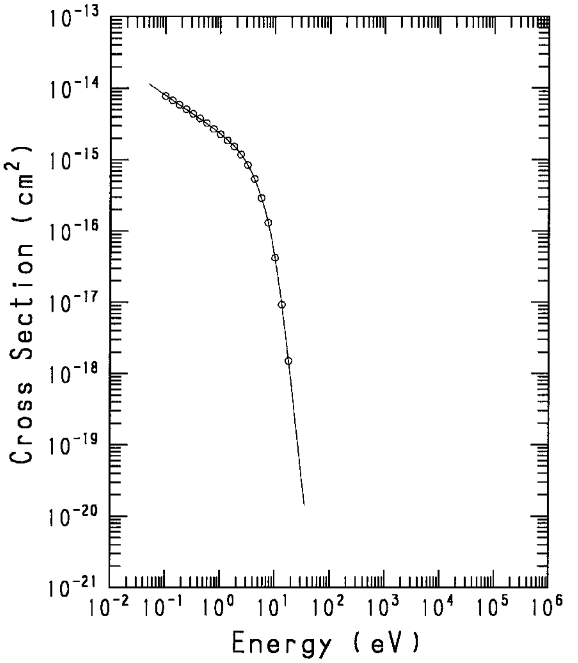
GRAPH 9

$H^+ + H_2 \rightarrow e$ (Total)



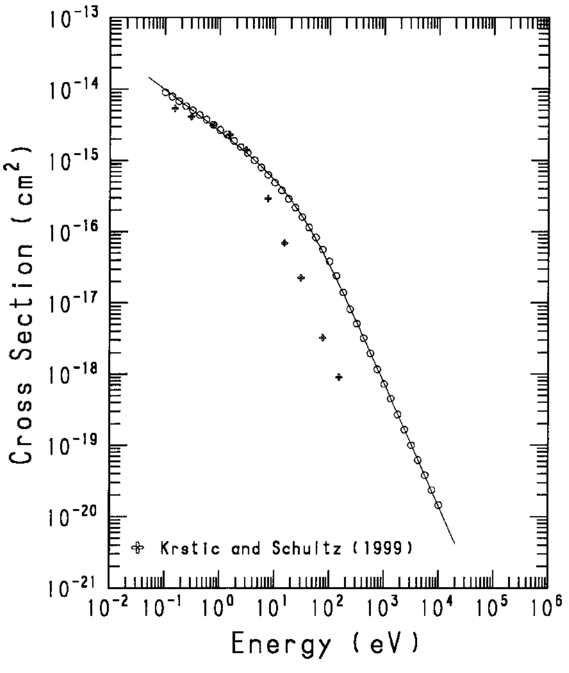
GRAPH 11

$H_2^+ + H_2 \rightarrow H_3^+ + H$



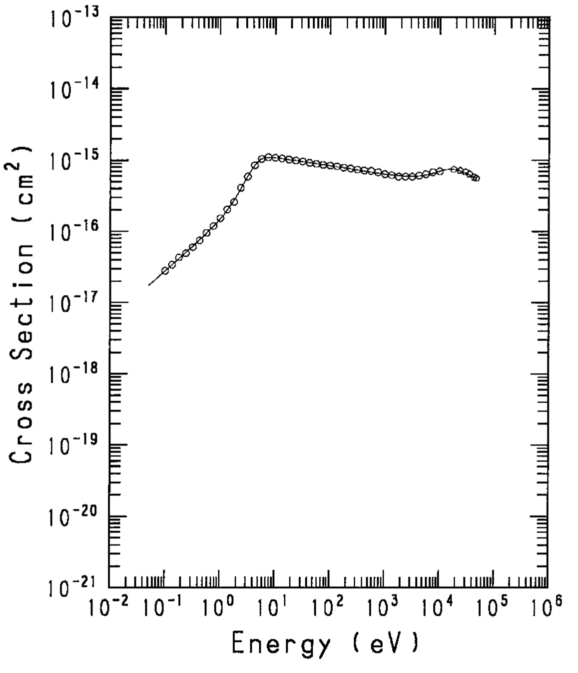
GRAPH 10

$H^+ + H_2 \rightarrow$ Momentum Transfer



GRAPH 12

$H_2^+ + H_2 \rightarrow$ slow H_2^+

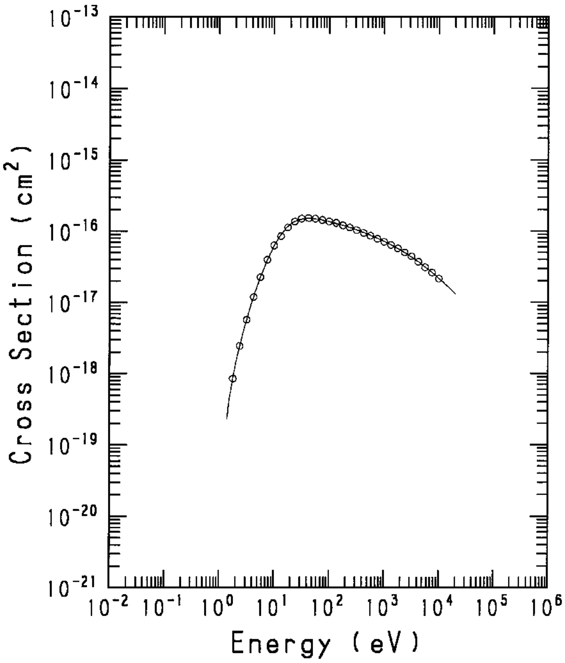


GRAPHS. Cross Section vs Projectile Energy

See page 7 for Explanation of Graphs

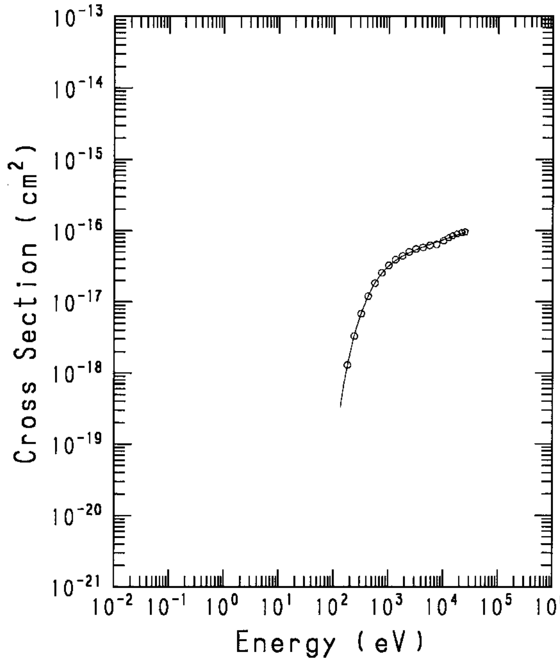
GRAPH 13

$H_2^+(v=0) + H_2 \rightarrow H_2^+(v=1)$



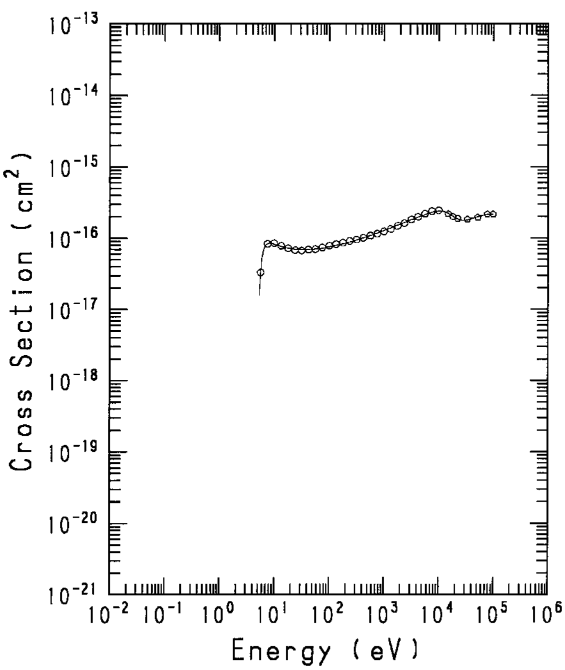
GRAPH 15

$H_2^+ + H_2 \rightarrow Ly\ \alpha$ (Total)



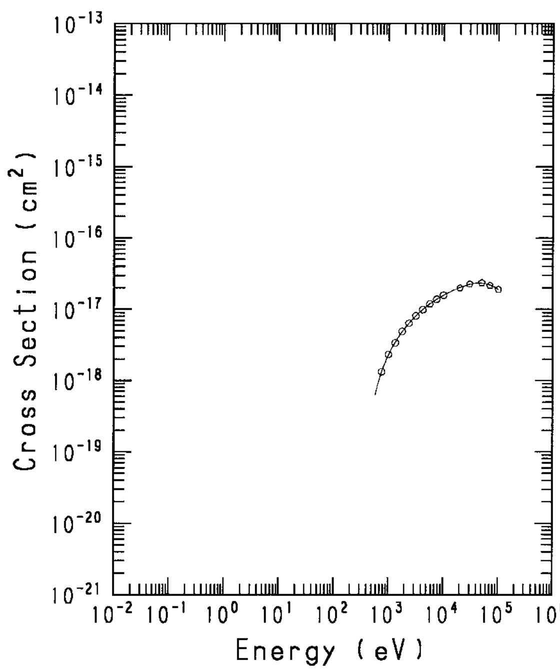
GRAPH 14

$H_2^+ + H_2 \rightarrow$ fast H^+



GRAPH 16

$H_2^+ + H_2 \rightarrow H\alpha$ (Total)

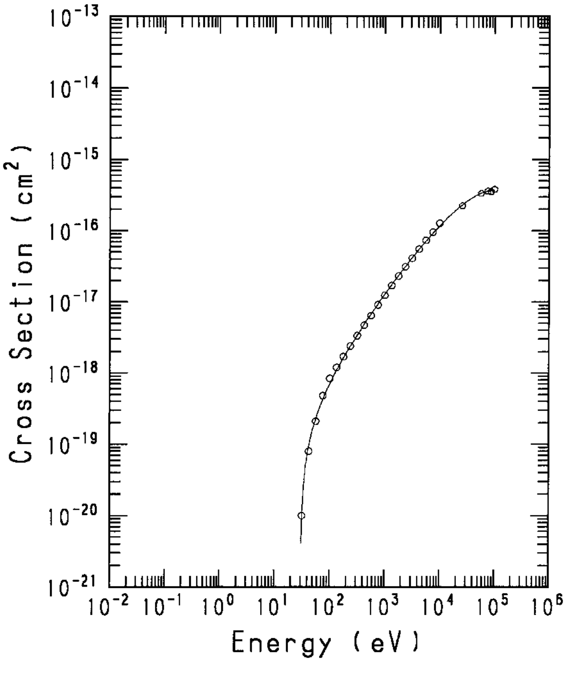


GRAPHS. Cross Section vs Projectile Energy

See page 7 for Explanation of Graphs

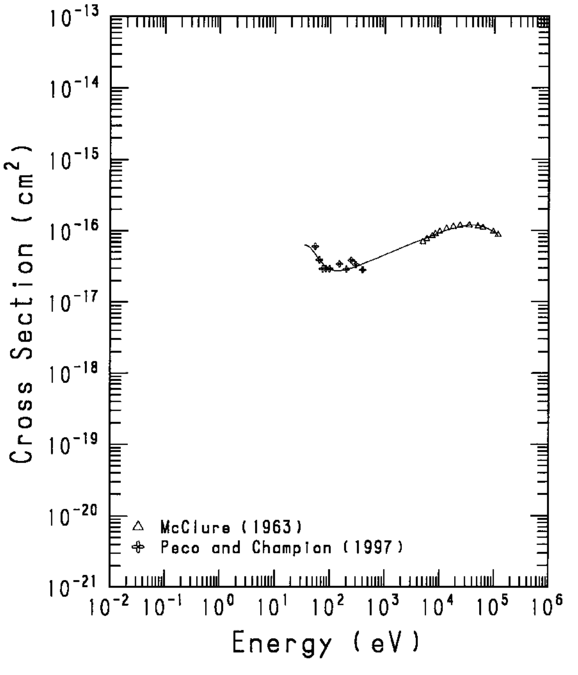
GRAPH 17

$H_2^+ + H_2 \rightarrow e$ (Total)



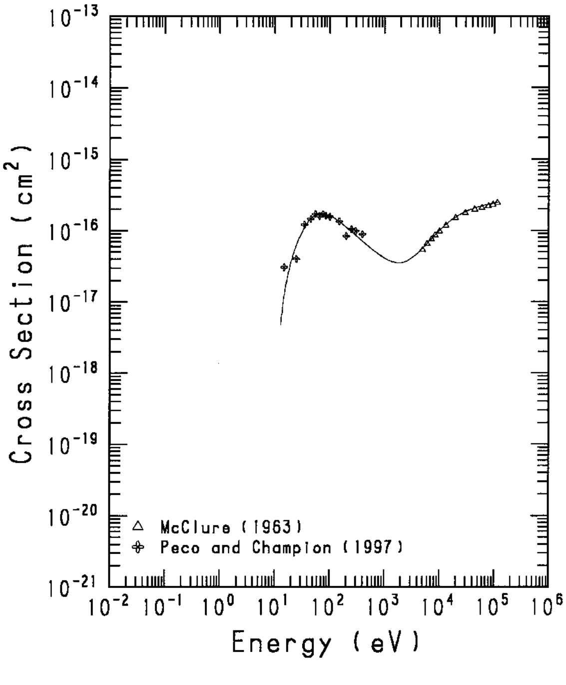
GRAPH 19

$H_3^+ + H_2 \rightarrow$ fast H_2^+



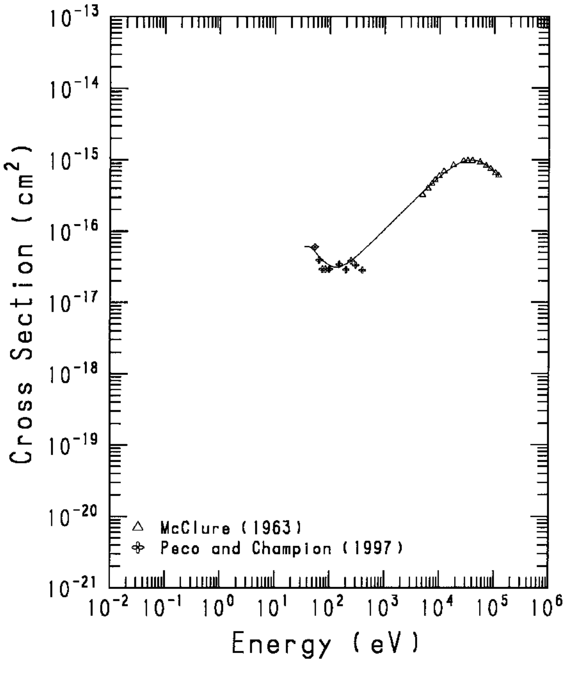
GRAPH 18

$H_3^+ + H_2 \rightarrow$ fast H^+



GRAPH 20

$H_3^+ + H_2 \rightarrow$ fast H

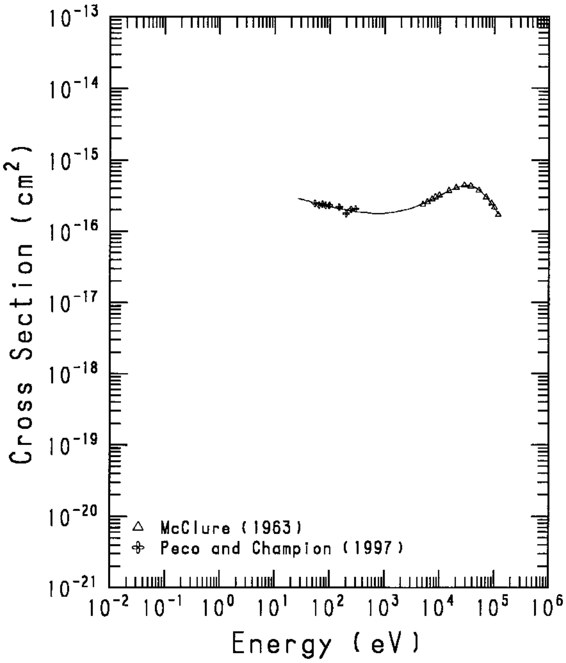


GRAPHS. Cross Section vs Projectile Energy

See page 7 for Explanation of Graphs

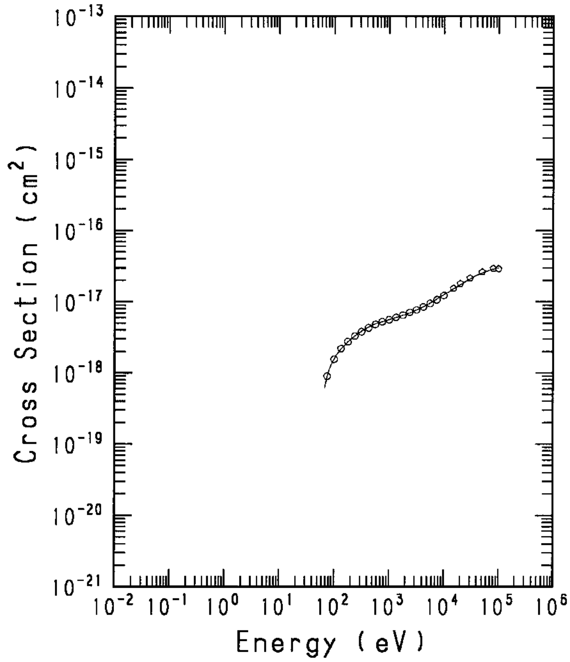
GRAPH 21

$H_3^+ + H_2 \rightarrow$ fast H_2



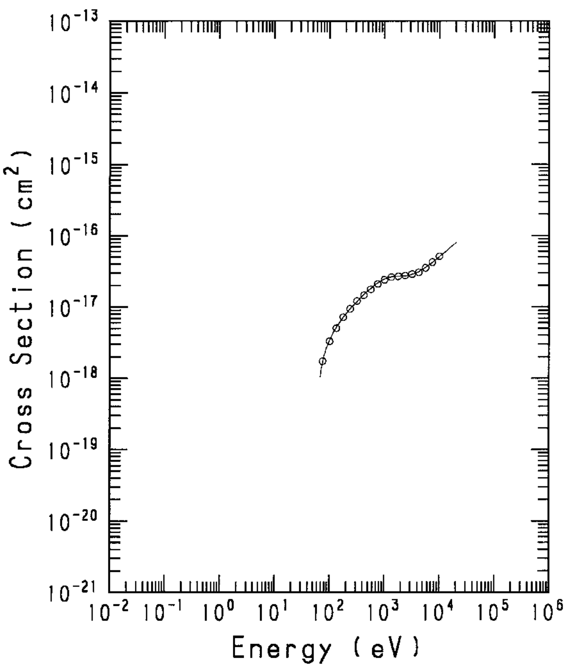
GRAPH 23

$H_3^+ + H_2 \rightarrow H\alpha$ (Total)



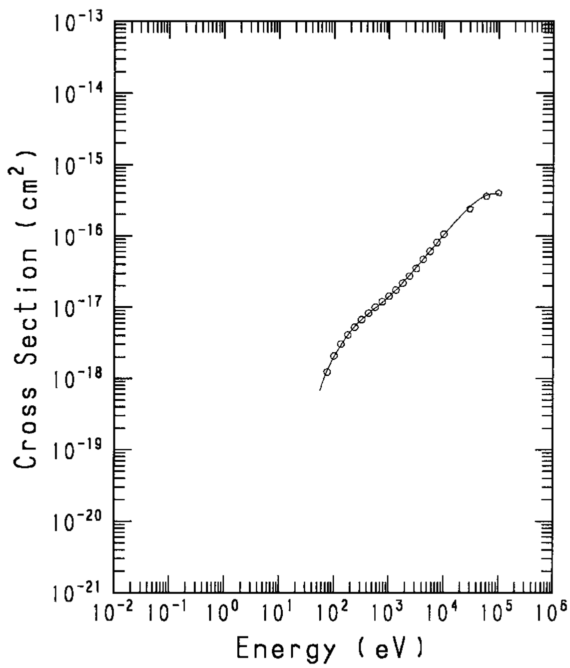
GRAPH 22

$H_3^+ + H_2 \rightarrow$ Ly α (Total)



GRAPH 24

$H_3^+ + H_2 \rightarrow$ e (Total)



GRAPHS. Cross Section vs Projectile Energy

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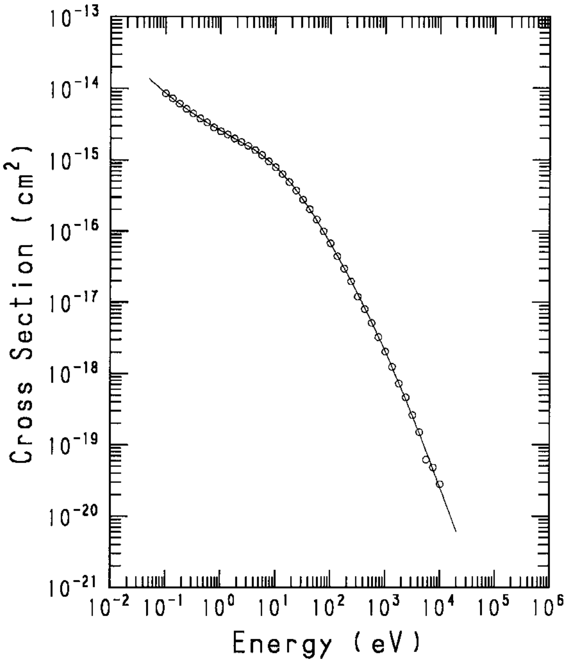
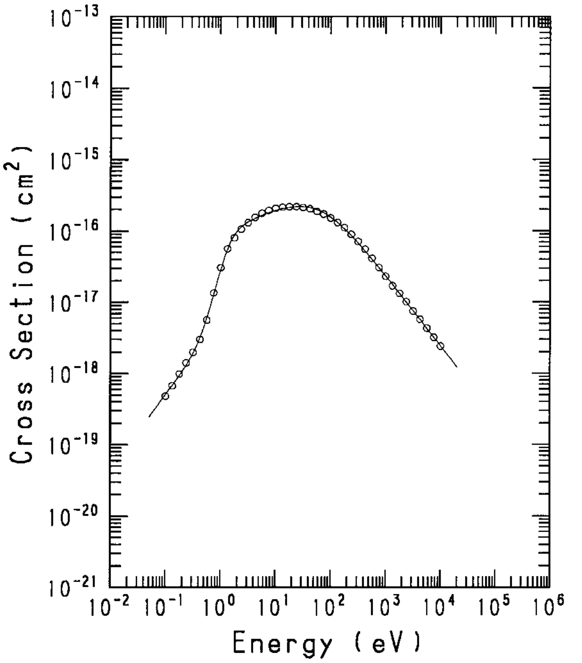
GRAPH 25

$H_3^+ + H_2 \rightarrow$ Momentum Transfer



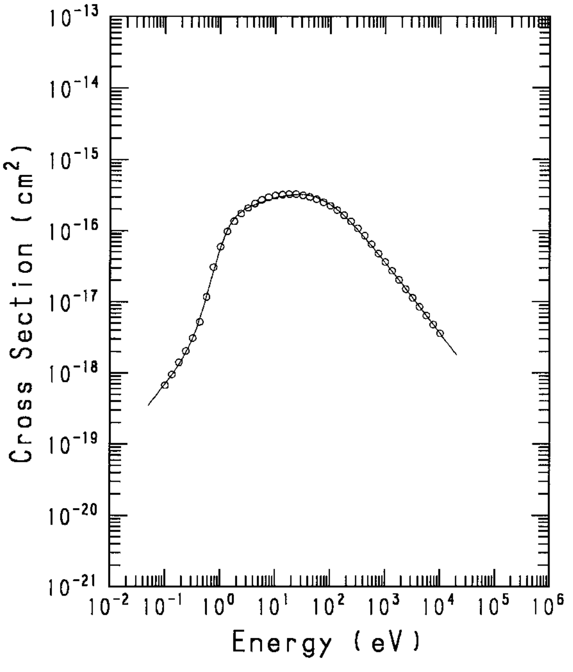
GRAPH 27

$H + H_2(J=1) \rightarrow H_2(J=3)$



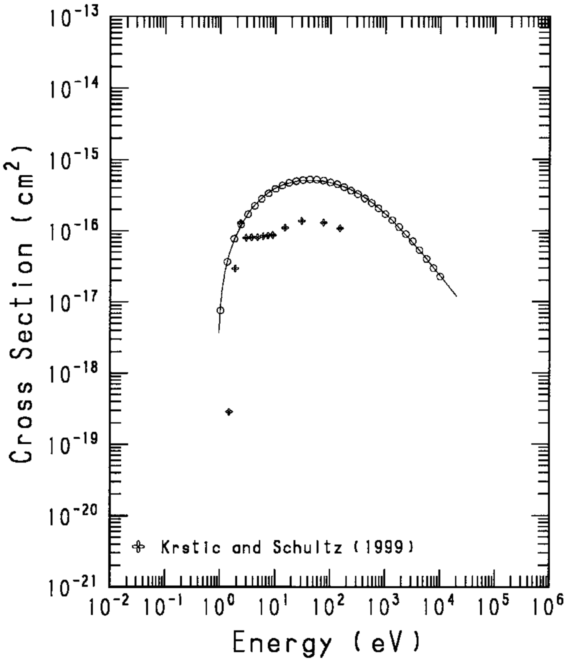
GRAPH 26

$H + H_2(J=0) \rightarrow H_2(J=2)$



GRAPH 28

$H + H_2(v=0) \rightarrow H_2(v=1)$



GRAPHS. Cross Section vs Projectile Energy

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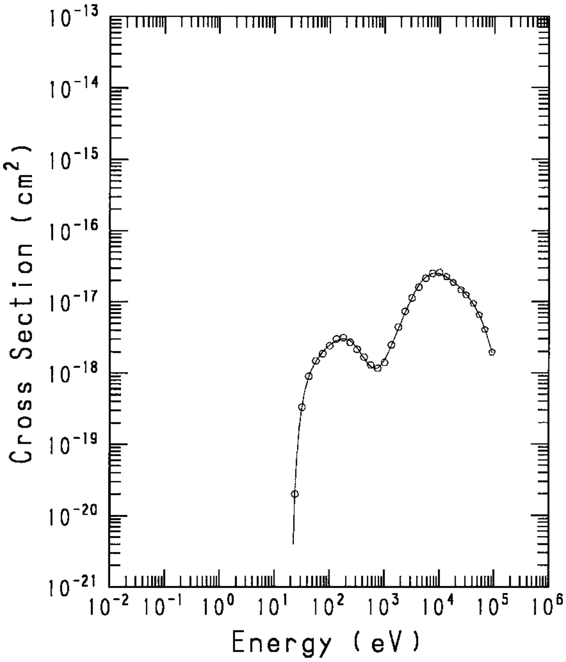
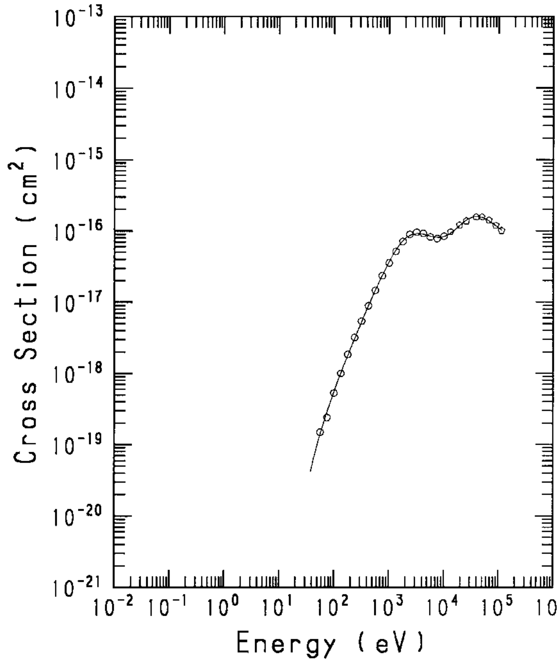
GRAPH 29

H + H_2 → fast H^-



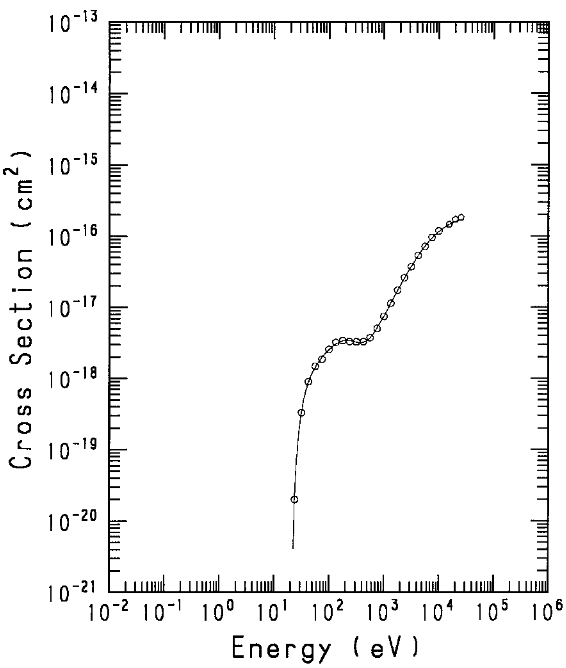
GRAPH 31

H + H_2 → fast H^+



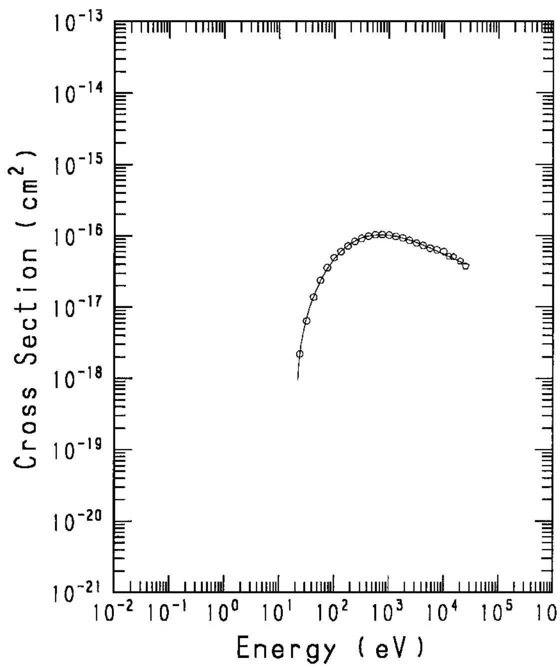
GRAPH 30

H + H_2 → slow H_2^+



GRAPH 32

H + H_2 → Ly α (Total)



GRAPHS. Cross Section vs Projectile Energy

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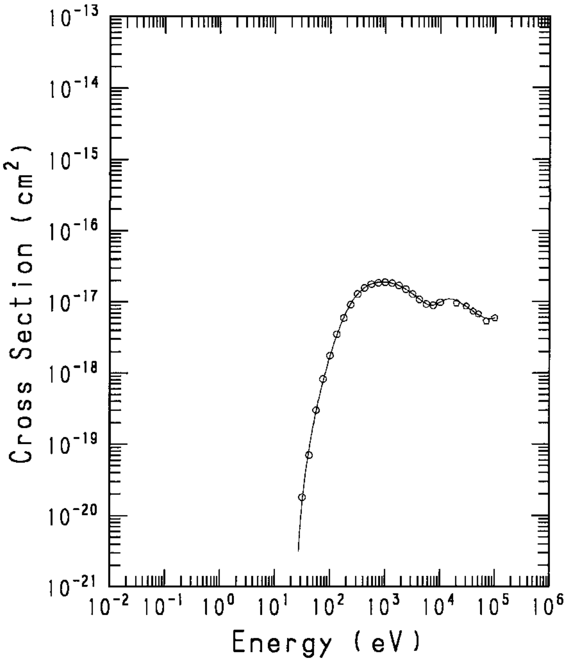
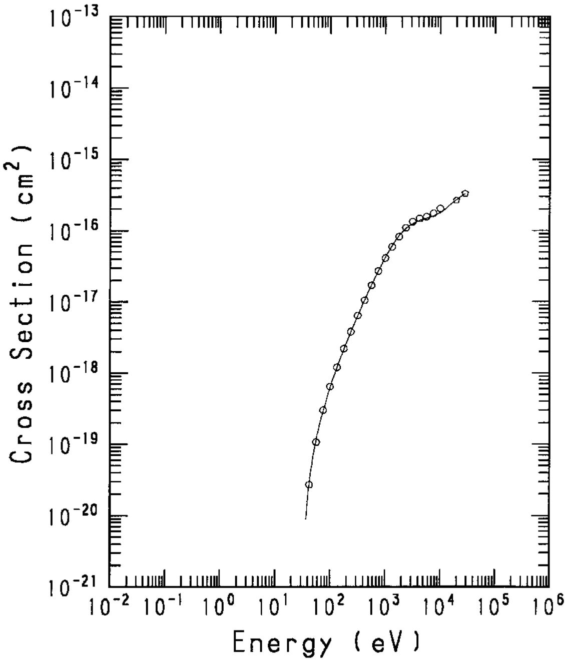
GRAPH 33

$H + H_2 \rightarrow H\alpha$ (Total)



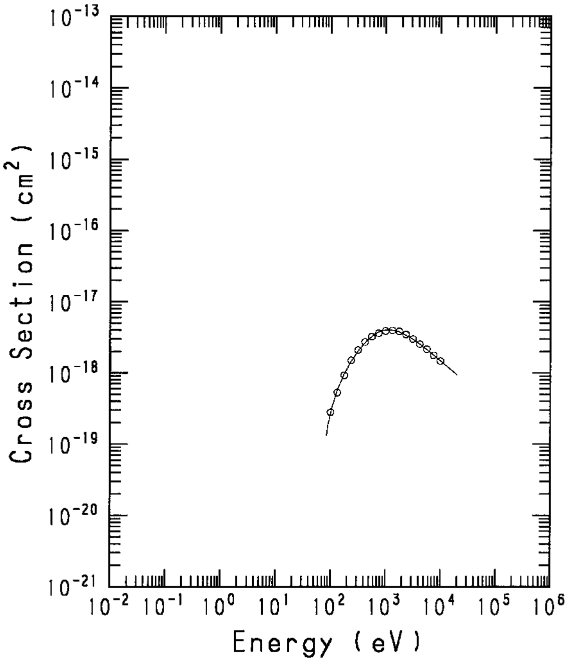
GRAPH 35

$H + H_2 \rightarrow e$ (Total)



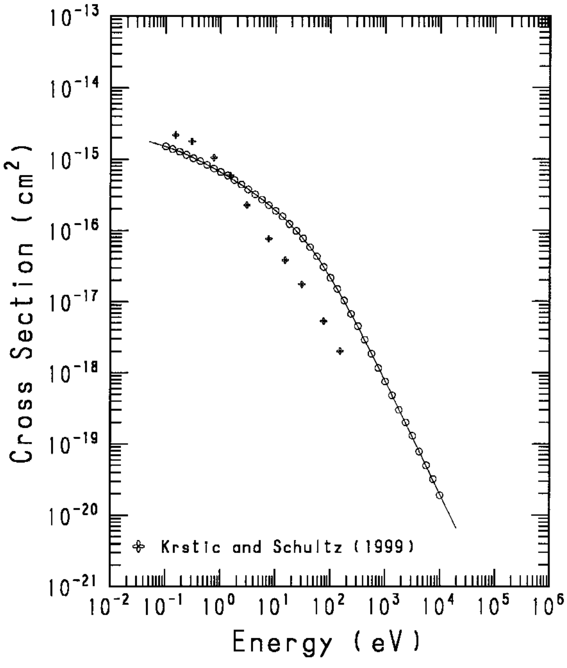
GRAPH 34

$H + H_2 \rightarrow H\beta$ (Total)



GRAPH 36

$H + H_2 \rightarrow$ Momentum Transfer



GRAPHS. Cross Section vs Projectile Energy

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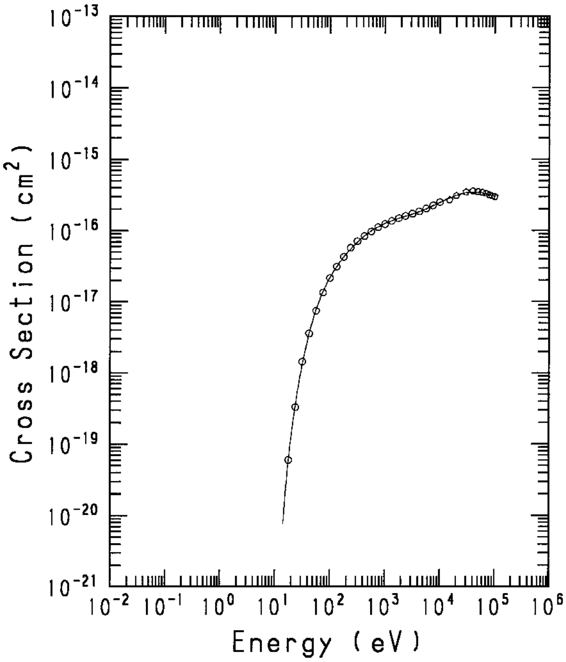
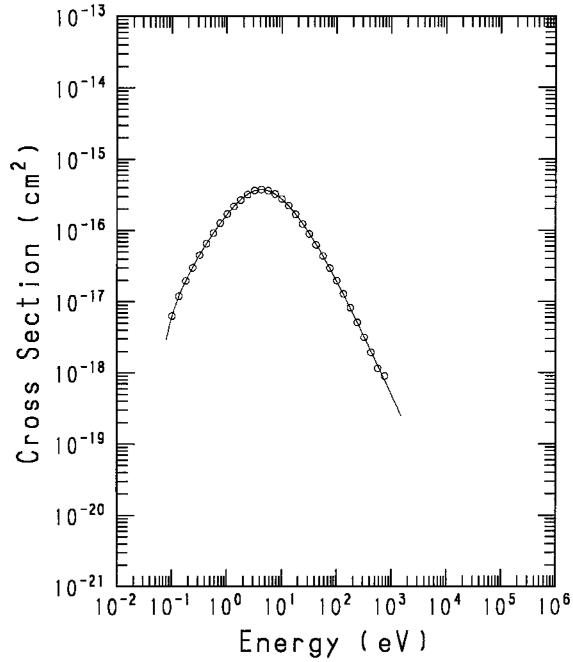
GRAPH 37

$H_2 + H_2 \rightarrow$ fast H_2 Destruction



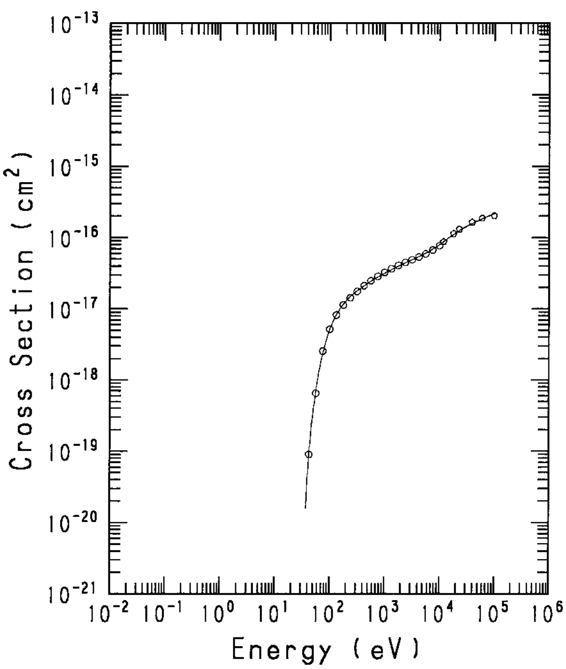
GRAPH 39

$H_2(J=0) + H_2(J=0) \rightarrow H_2(J=0) + H_2(J=2)$



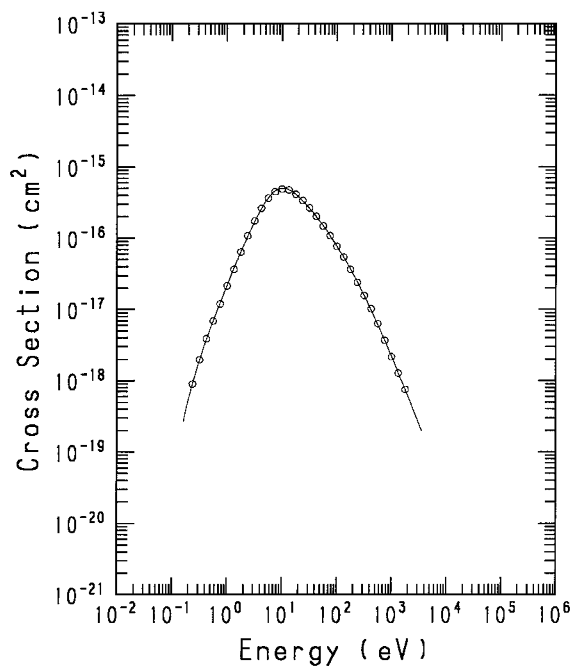
GRAPH 38

$H_2 + H_2 \rightarrow$ fast H_2^+



GRAPH 40

$H_2(J=0) + H_2(J=0) \rightarrow H_2(J=2) + H_2(J=2)$



GRAPHS. Cross Section vs Projectile Energy

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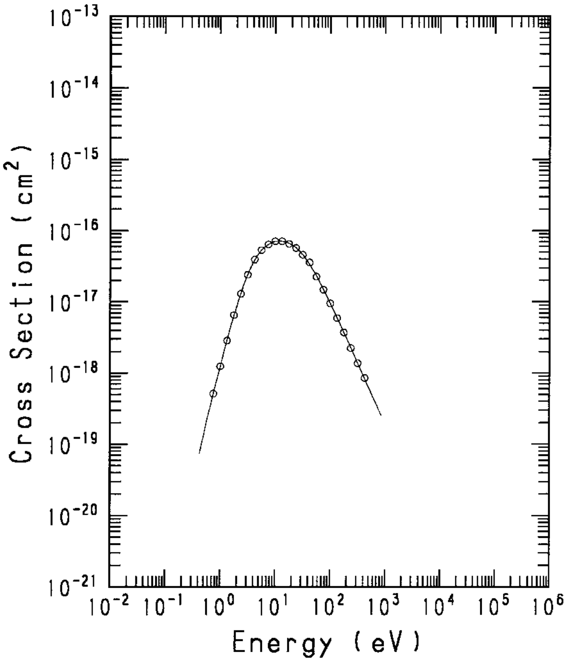
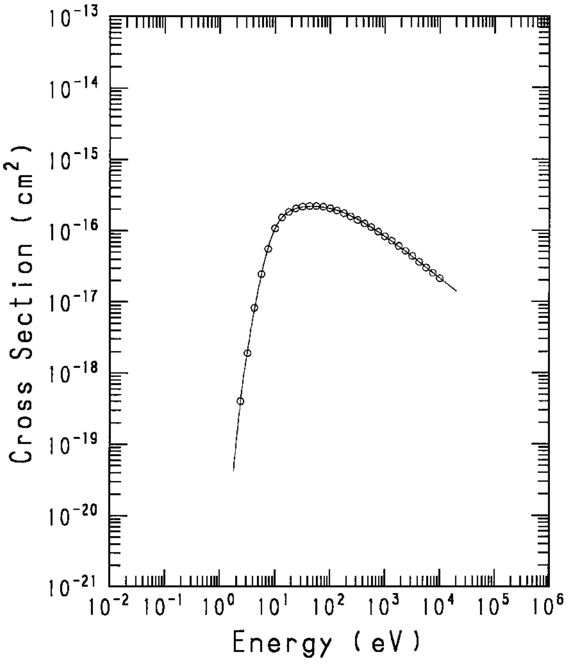
GRAPH 41

$H_2(J=0) + H_2(J=0) \rightarrow H_2(J=0) + H_2(J=4)$



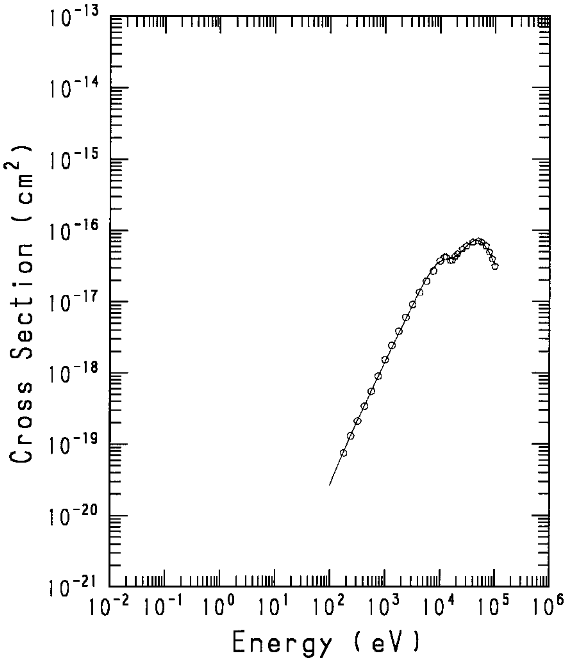
GRAPH 42

$H_2 + H_2(v=0) \rightarrow H_2(v=1)$



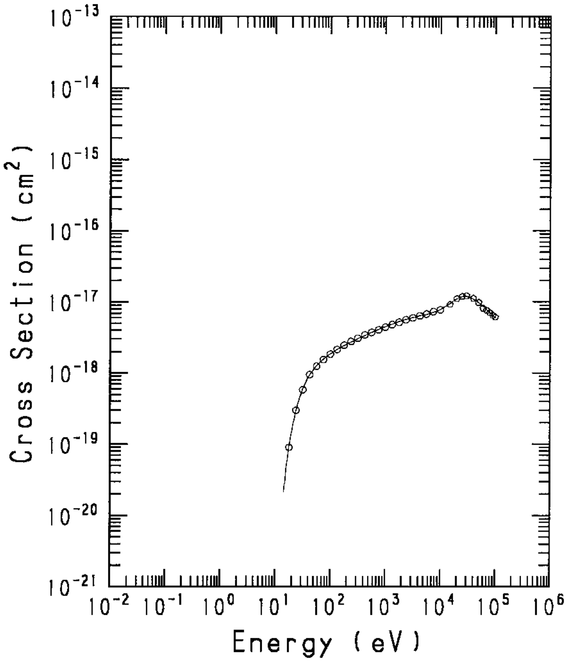
GRAPH 43

$H_2 + H_2 \rightarrow$ fast H^+



GRAPH 44

$H_2 + H_2 \rightarrow H\alpha$ (Total)



GRAPHS. Cross Section vs Projectile Energy

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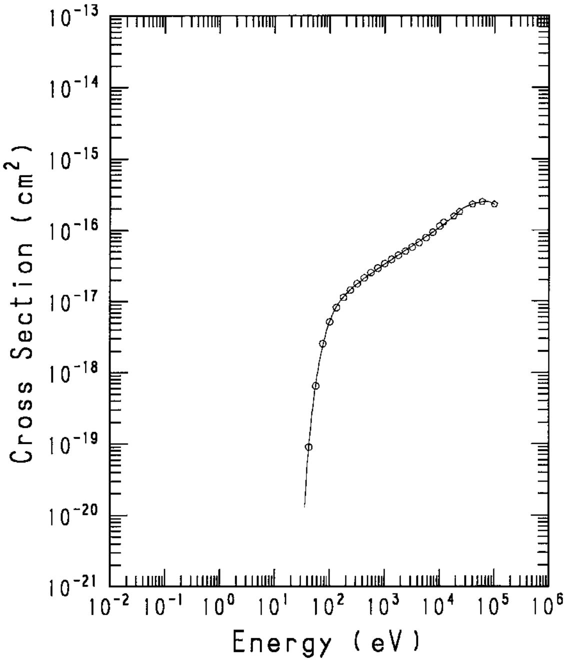
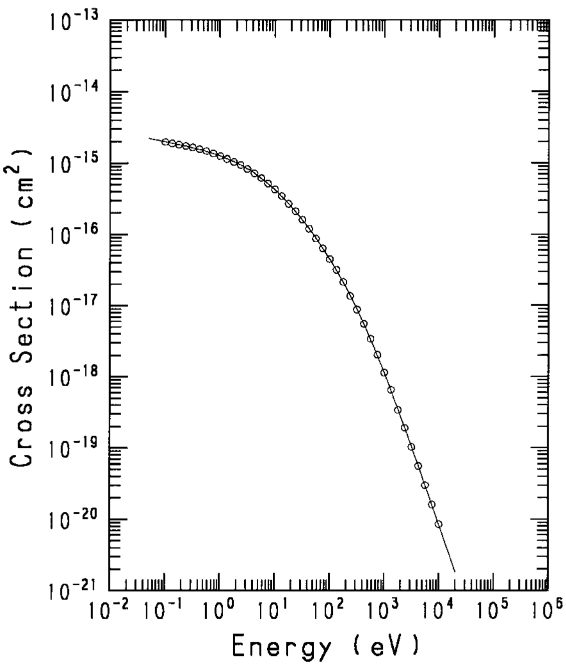
GRAPH 45

$H_2 + H_2 \rightarrow$ fast H_2^+ or fast H^+



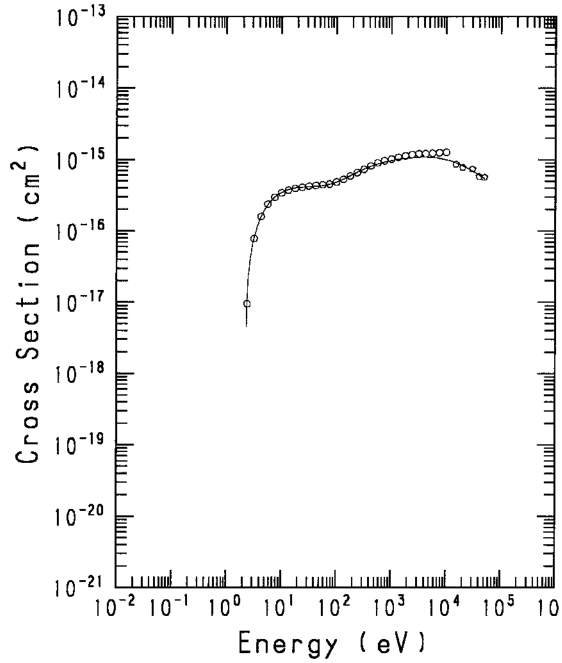
GRAPH 46

$H_2 + H_2 \rightarrow$ Momentum Transfer



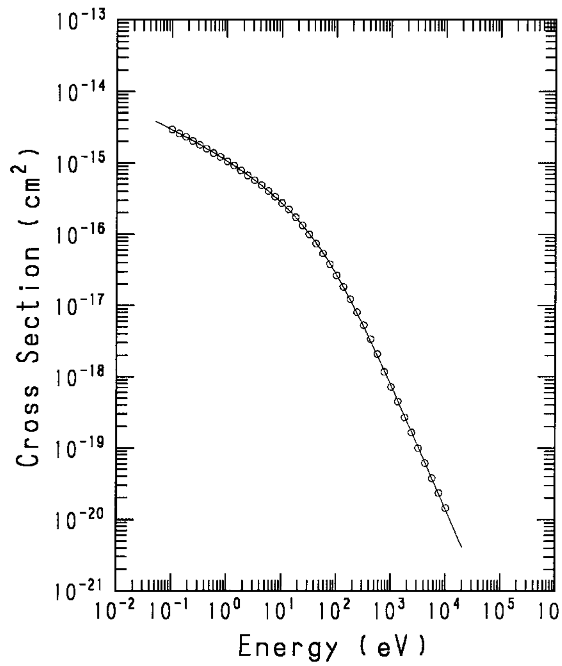
GRAPH 47

$H^- + H_2 \rightarrow e +$ fast H



GRAPH 48

$H^- + H_2 \rightarrow$ Momentum Transfer



GRAPHS. Cross Section vs Projectile Energy

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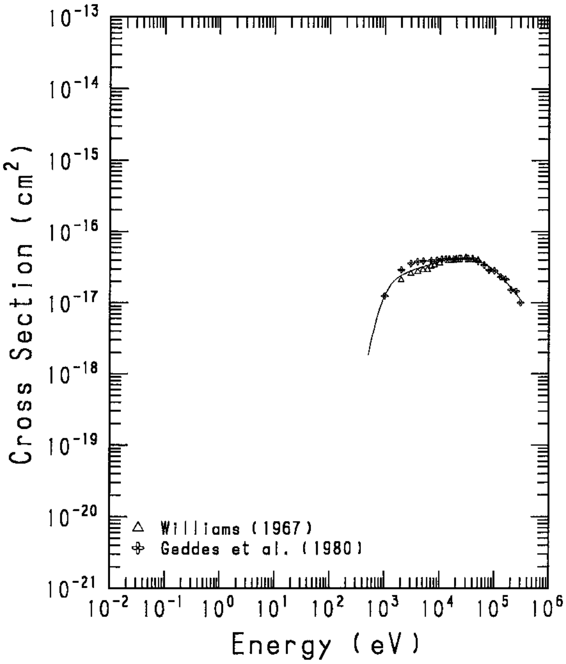
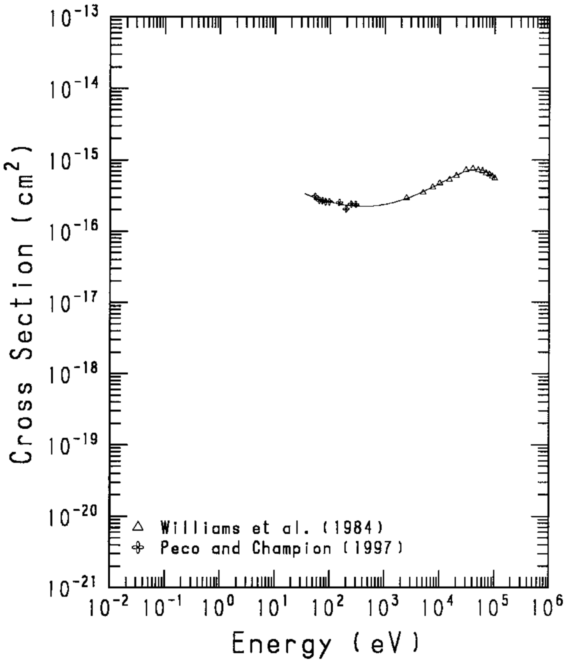
GRAPH 49

$H^- + H_2 \rightarrow$ fast H^+



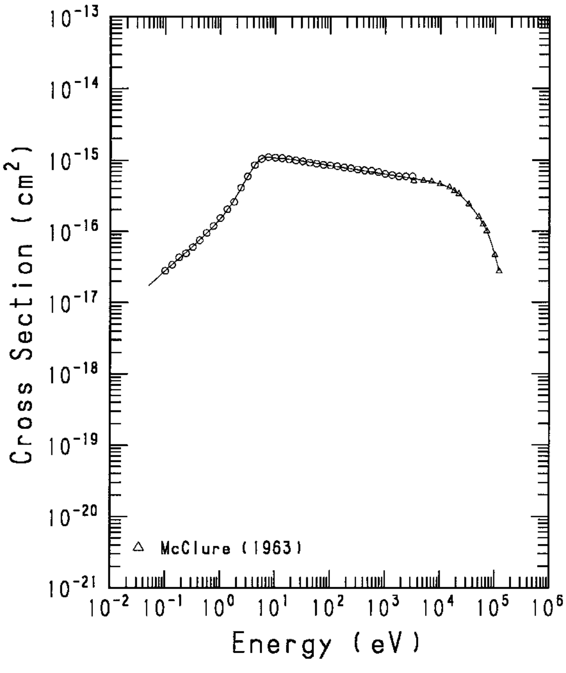
GRAPH 51

$H_3^+ + H_2 \rightarrow H_3^+$ Destruction



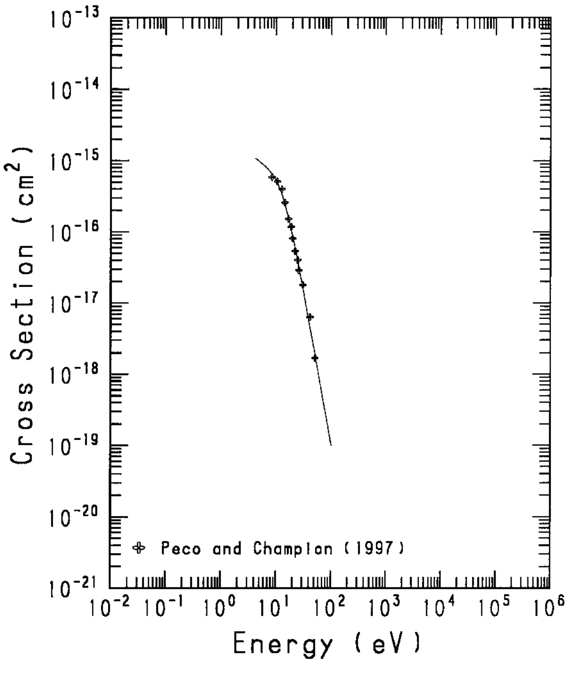
GRAPH 50

$H_2^+ + H_2 \rightarrow$ fast H_2



GRAPH 52

$H_3^+ + H_2 \rightarrow$ slow H_3^+



GRAPHS. Cross Section vs Projectile Energy

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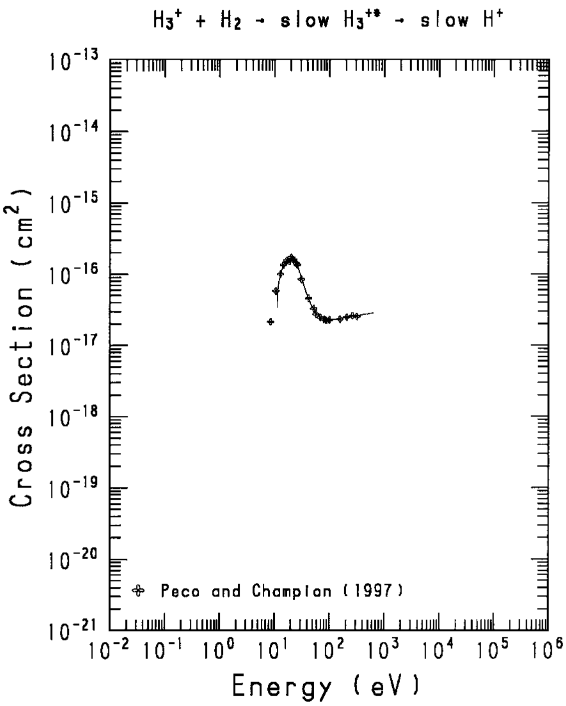
GRAPH 53



GRAPH 54