GEM TN-93-382

Conductance Calculation - Molecular Flow, Long Tube of Circular Cross Section

Sanyi Zheng

April 6, 1993

Abstract:

A formular for the conductance calculation in molecular flow is presented. The formular gives the conductance of pipes which are constant cross section, as well as in which the circumference and cross section of the pipes are increasing or decreasing function of the pipe length.

Vacuum Group

Technical Note # GEM TN-93-382	Date: 4 - 6 - 93
Title: Conductance Calculation - Molecular Flow, Long Tube of Circular Cross Section	Pages: 4
Requested By: Gerry Chapman	Checked By: Jiaxiang Zhou
Originator: Sanyi Zheng	Approved By: Gerry Chapman

Abstracts:

A formular for the conductance calculation in molecular flow is presented. The formular gives the conductance of pipes which are constant cross section, as well as in which the circumference and cross section of the pipes are increasing or decreasing function of the pipe length.

References:

1) Knudsen, M., 1910, Ann. Phys. 31, 205; 32, 890; 33, 1435
2) O'Hanlon, J.F., 1989, A User's Guide to Vacuum Technology, 2nd edition
3) Pinson, J.D., and Peck, A.W., Trans 9th Nat. Vac. Symp., MacMillan, N.Y., 1962

Distributions:

CONDUCTANCE CALCULATION
MOLECULAR FLOW, LONG TUBE OF CIRCULAR CROSS SECTION

Sanyi Zheng
Physics Research Division
EFD Vacuum Group
February 6, 1993

1. MOLECULAR VELOCITIES

$$V_{av} = 1.45x10^4 (T/M)^{\frac{1}{2}} \text{ cm/sec.} \qquad (1)$$

at air molecule $M = 29$, temperature $T = 300°K$

The average air molecule has a velocity of about $V_{av} = 4.6x10^4 \text{cm/sec.}$

2. MOLECULAR FLOW - LONG TUBE OF CONSTANT CROSS SECTION

$$q = \Phi BL \qquad \text{(Knudsen)} \qquad (2)$$

q is the number of molecules striking the wall each second
Φ is the number of molecules impinging on the unit surface per unit time($\Phi = n\ V_{av} / 4$ n is the molecular density)
B is the periphery of the cross section
L is the length of the tube

$$q^{'} = qmv = BL\, nV_{av}mv / 4 \qquad (3)$$

q' is the momentum transferred by all the molecules to the wall

Assume the number N of molecules crossing the cross section A of the pipe per unit time is $N = Avn$ and the pressure difference ΔP achieved corresponds to a force(4)

$$\Delta F = A\Delta P = AkT\Delta n \qquad (5)$$

k is Boltzmann's constant

For equilibrium condition $q' = \Delta F$, then

$$4AkT\Delta n = BLnV_{av}mv \qquad (6)$$

m is the mass (of molecule)

From eqs. (4) & (6), we have

$$N / \Delta n = \left[4A^2 / (BL)\right]\left[kT / \left(mV_{av}\right)\right] \qquad (7)$$

By using definition of the conductance $N / \Delta n = C$, also

$$V_{av} = \left(2 / \sqrt{\pi}\right)\left(2kT / m^{\frac{1}{2}}\right)$$

(8)

Therefore, we can obtain

$$C = \left[2A^2 / (BL)\right]\left[\pi kT / (2m)\right]^{\frac{1}{2}} = \left[2A^2 / (BL)\right]\left[\pi R_0 T / (2M)\right]^{\frac{1}{2}} \tag{9}$$

R_0 is the gas constant (per mole)
M is molecular weight

The equation (9) contains the assumption that a uniform drift velocity v is superimposed on the random Maxwell - Boltzmann distribution of the molecules. Knudsen modifies the above equation by multiplying a factor $8/(3\pi)$ and assume the superimposed drift velocity of a molecule is proportional to its random velocity. The conductance will become

$$C = \frac{8}{3\sqrt{\pi}}\left(\frac{2kT}{m}\right)^{\frac{1}{2}}\left(\frac{A^2}{BL}\right) = \frac{3.44x10^4}{\sqrt{\pi}}\left(\frac{T}{M}\right)^{\frac{1}{2}}\left(\frac{A^2}{BL}\right) \tag{10}$$

in CGS units.

For the uniform circular cross section, $A = \pi D^2/4$, and $B = \pi D$. The conductance of a tube is

$$C = 3.81\left(T/M\right)^{\frac{1}{2}}\left(D^3/L\right) \tag{11}$$

where D (cm), L (cm) and C (liter/sec.)

For air at 20°C, $\left(T/M\right)^{\frac{1}{2}} = 3.18$, therefore

$$C_{air} = 12.1D^3 / L \tag{12}$$

It can be seen that conductance (molecular flow) is independent of the pressure.

3. MOLECULAR FLOW - TAPERED TUBES (CONICAL SHAPE TYPICALLY)

We can rewrite the equation (10) by using equation (1)

$$C = \frac{4}{3} V_{av} K / \left(BL / A^2\right) \tag{13}$$

K is the shape factor

If the conductance results from a series connection of conductance of length dL, then

$$\frac{1}{C} = \left[\frac{3}{4} / \left(V_{av} K\right)\right]\int_0^L \left(B/A^2\right) dL \tag{14}$$

$$\text{or } C = \frac{4}{3} V_{av} K \Big/ \int_0^L \left(B/A^2 \right) dL \tag{15}$$

For the constant cross section, B and A are not functions of L, $\int_0^L dL = L$ and equation (15) results in equation (13) or (10).

It is the short conclusion that equation (15) is a general formula which gives the conductance of pipes which are constance cross section, as well as in which B and A are continuously increasing or decreasing function of L.

For a conical pipe, B_1 and A_1, represent the parameter of cross section at the small end while B_2 and A_2 are the parameter of cross section at the large end. At a distance x, B_x and A_x will be

$$B_x = B_1 + (B_2 - B_1)(x/L) = K_B\left[a_1 + (a_2 - a_1)(x/L)\right]$$

$$A_x = K_A\left[a_1 + (a_2 - a_1)(x/L)\right]^2 \tag{16}$$

a is the radius

K_B is the constant ratio between the circumference and radius $(K_B = 2\pi a / a = 2\pi)$

K_A is the constant ratio between the cross section area and the square of the radius $(K_A = \pi a^2 / a^2 = \pi)$

Substitute eq. (16) into eq. (15)

$$\int_0^L \frac{B_x}{A_x^2} dL = \frac{K_B}{K_A^2} \int_0^L \frac{dx}{\left[a_1 + 1(a_2 - a_1)(x/L)\right]^3} = \left(\frac{K_B}{K_A^2}\right)\left(\frac{L}{2}\right)\left(\frac{a_1 + a_2}{a_1^2 \cdot a_2^2}\right) \tag{17}$$

Therefore the conductance (eq 15) will be given by

$$C = \frac{8}{3} V_{av} \left(K_A^2 / K_B\right)\left[a_1^2 \cdot a_2^2 / (a_1 + a_2)\right](K/L) \tag{18}$$

For a circular cross section

$$B = 2\pi r;\ A = \pi r^2;\ K_B = 2\pi;\ K_A = \pi;\ K_A^2 / K_B = \pi^2/(2\pi) = \pi/2; \text{ and } K = 1$$

Then the conductance of a tapered pipe of circular cross section will become

$$C = (4\pi/3)\left[r_1^2 r_2^2 / (r_1 + r_2)\right] V_{av} \tag{19}$$

or for $D = 2r$

$$C = 7.62\left(T/M\right)^{1/2} D_1^2\, D_2^2 / \left[(D_1 + D_2)\, L\right] \qquad (20)$$

Where D, L (cm), and C (liter/sec.), D_1 and D_2 being the diameters of the tapered pipe at its end.

By comparing eq (20) with eq (11), it results that the equivalent diameter for a tapered tube is

$$D_e = \left[2D_1^2\, D_2^2 / (D_1 + D_2)\right]^{1/2} \qquad (21)$$

4. THE CONCLUSION

The equation of the conductance of long tubes for molecular flow with the circular cross section is

$$C = 3.81\left(T/M\right)^{1/2}\left(D_e^3 / L\right) \text{ or } C_{air} = 12.1\, D_e^3 / L$$

where D_e (cm), L (cm), C (liter/sec.), $\left(T/M\right)^{1/2} = 3.18$ at 20°C for air

in case of constant crass section:

$D_e = D = 2r$

in case of a tapered tube with a circular crass section

$$D_e = \left[2\, D_1^2\, D_2^2 / (D_1 + D_2)\right]^{1/2}$$

5. REFERENCE

1) Knudsen, M., 1910, Ann. Phys. 31, 205; 32, 890; 33, 1435
2) O'Hanlon, J. F., 1989, A User's Guide to Vacuum Technology, 2nd edition
3) Pinson, J. D., and Peck, A.W., Trans 9th Nat. Vac. Symp., MacMillan, N.Y.,. 1962