THE PLASMA SHEATH

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1. INTRODUCTION AND OUTLINE

Outline: Section 2 simply describes, without attempt at explanation, what happens electrically when a plasma is in contact with a solid surface. The practical implications of the interaction are briefly described. Section 3 provides a physical explanation of why these effects occur and deduces initial estimates of the plasma-solid voltage difference and the spatial extent of this voltage drop. Section 4 deduces the Bohm Criterion (ion drift velocity out of the plasma must equal the ion acoustic speed) using ion fluid models. The Criterion is obtained from both the plasma and the sheath equations separately. Section 5 deduces simple formulae for the particle and energy flux which is transmitted by a sheath, both for electrically floating and biased objects. Section 6 gives a brief indication of how the sheath particle and energy transmission characteristics influence the modeling of the edge plasma in magnetically confined plasma devices, while Section 7 gives a similar brief introduction to their use in the interpretation of plasma probe data. Section 8 indicates various refinements on sheath theory and concludes with recommended formulations for floating potential, particle and energy transmission coefficients for the sheath.

Some caveats: The plasma/sheath analysis presented in this Chapter does not explicitly include the presence of a magnetic field. It is therefore suitable for describing the flow of plasma along magnetic field lines to a surface, since such flow is not impeded by the magnetic field. Such parallel-field plasma flow is generally a good first approximation to the interaction of plasma with probes and other solid objects, such as limiters, inserted into a plasma. When the plasma flow must cross magnetic field lines in order to reach the object, then some corrections can be made to the simple theory; see Chapter by Chodura. Also, if the ion or electron Larmor radii are large compared with the object size, then the simple theory presented in this Chapter is not applicable.

Some omissions: Because of space limitations, only the steady-state, wall sheath will be considered here, i.e., the case of a solid on one side of the sheath, with plasma on the other side. There is thus no treatment of other important types of sheath: (a) dynamic sheaths which occur when, for example, a very rapidly varying voltage is applied to an object immersed in a plasma (monotonic time variation or oscillatory), or when a plasma expands into a vacuum, etc., (b) double sheaths, or free-standing sheaths, which are bounded on each side by plasmas and which may occur, for example, at magnetic mirrors in fusion devices.

2. THE BASIC FACTS

A plasma is generally highly conducting, and we may often consider it to be an equipotential at the plasma potential Φ_p. An object placed in the plasma will generally not assume this potential. Rather, when a plasma is in contact with any solid object, such as a limiter, divertor plate or diagnostic probe, a voltage difference spontaneously develops between the plasma and the object called the floating potential, Φ_f (often, for convenience we take plasma potential as reference potential thus making $\Phi_p = 0$). The object will generally float electrically at a potential which is negative relative to the plasma with $|\Phi_f| =$ a few kT_e/e where T_e is the electron temperature of the plasma actually contacting the object. Of course, one may bias the object relative to the plasma at any desired potential using an external circuit provided that one has a means of applying this extra potential difference. This latter situation is generally only achievable for small objects, typically probes, where one can use other (large) objects contacting the plasma, e.g., limiters or divertor plates, as the bias reference. Taken in aggregate, the solid surfaces contacting the plasma will float negatively relative to the plasma, or alternatively, one may say that the plasma will adjust to assume a positive potential relative to the aggregate of surfaces.

The potential drop, Φ_f, occurs in a thin sheath which is established between the plasma and the solid. The sheath thickness is of order the Debye length

$$\lambda_D = (\varepsilon_o kT_e/ne^2)^{1/2} \simeq 743\ T^{1/2}\ (\text{eV})\ n^{-1/2}\ (\text{cm}^{-3})\ \text{cm}. \quad (1)$$

Example: Typical tokamak edge plasma with $n = 10^{12}$ cm^{-3}, T_e = 15 eV, then $\lambda_D = 0.003$ cm.

In the plasma itself, $n_e = n_i$ to a high order, i.e., the plasma is quasi-neutral. The sheath, by contrast, has a net positive charge per unit volume since the plasma electrons are repelled by the negative potential on the solid. The areal negative charge density on the solid surface approximately equals the areal positive charge density in the sheath. The sheath thus acts to shield the plasma from the potential on the solid surface. This shielding effect also occurs if the object is biased more negatively than Φ_f; the sheath thickness increases in this case, but is still usually very small compared with plasma dimensions. If the object is biased positively relative to the plasma, then the sheath disappears and the random, Maxwellian flux of electrons strikes the surface, unattenuated (in the simplest cases).

The shielding effect of the sheath is imperfect and a small residual field, the pre-sheath, penetrates deep into the plasma. In the simplest case this pre-sheath field extends all the way to the symmetry point between two opposite-facing solid surfaces. The potential drop in the pre-sheath is small, ~1/2 kT_e/e and acts to draw ions from the plasma into the sheath. This accelerating field is just such as to cause the ion drift velocity, at the sheath/plasma interface, to equal the ion acoustic speed

$$C_s = [k(T_e + T_i)/m_i]^{1/2}. \quad (2)$$

This is the (generalized) Bohm Criterion. A (collisionless) plasma itself cannot support potential differences greater than ~1/2 kT_e/e without breaking down into a space-charge zone and thus ions cannot be accelerated to supersonic velocities (in terms of C_s) by plasma fields.

The potential variations in sheath and pre-sheath, ion drift speed and n_e, n_i variations are shown in Fig. 1.

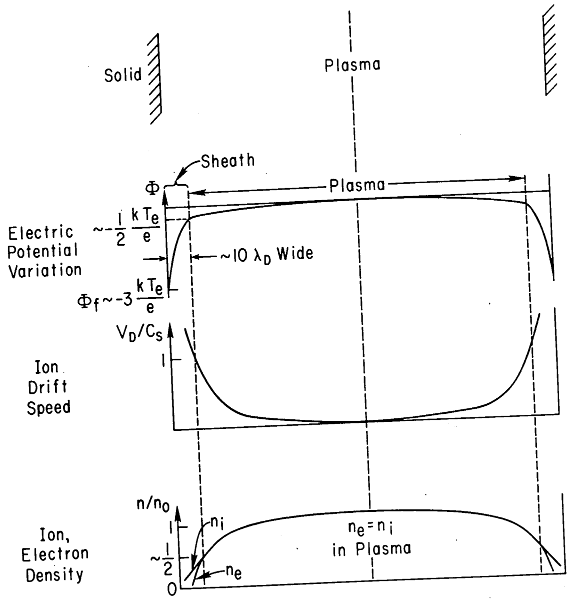

Fig. 1. Schematic of the variation of electric potential, ion drift speed and ion/electron densities in the plasma between two semi-infinite planes. So long as (i) a magnetic field is perpendicular to the plane surfaces and (ii) it is strong enough that Larmor radii are small compared with object dimension then the magnetic field will not effect the analysis. The thickness of the sheath is exaggerated for clarity.

The existence of the potential drop Φ_f has a number of consequences: (a) Ions are accelerated through the sheath and thus impact the solid surface with an energy which is greater than that associated with T_i. This generally increases sputtering. It also influences backscattering/retention/release and thus the ability of the solid and plasma to come into equilibrium with regard to the recycle of particles (hydrogen in the case of fusion devices). (b) The sheath controls the rates at which particles and energy are removed from the plasma by the solid surface. One wishes then to know the sheath transmission factors as a boundary conditions for modeling of the edge plasma. (c) For probe analysis one also requires knowledge of these transmission factors for all potentials, in addition to Φ_f.

The fact that the potential drop generally occurs in a very thin layer simplifies sheath analysis; for example, one can usually neglect ionization and many other effects within the sheath itself.

3. A PRELIMINARY ANALYSIS OF THE SHEATH

Why can't the plasma simply contact the solid without generating a potential difference?

Consider the solid suddenly introduced into the plasma at t = 0, with the solid initially biased to be at the plasma potential (thereafter the solid potential is allowed to adjust itself). The random flux of electrons strikes the solid at flux density 1/4 $n_e\bar{c}_e$ while the ions strike it at flux density 1/4 $n_i\bar{c}_i$ where $\bar{c}_{e,i} = (8\ kT_{e,i}/\pi m_{e,i})^{1/2}$.[1] We can easily show that in the plasma $n_e = n_i$, to a high order. Consider a plasma of density 10^{20} ions/m^3 between two flat metal plates across which is suddenly applied a voltage which pulls all of the electrons out of the plasma before the heavier ions have a chance to move. What voltage is required? We use Maxwell's equation,[2] $dE/dx = en/\varepsilon_o$ to obtain a required field of about 10^{12}V/m! Clearly it is not practical to generate such voltages; the potential energy represented by such charge separation is also unphysically large. Hence, unless $T_i >> T_e$, the electron flux to the solid will greatly exceed the ion flux and within a very short time the solid will gain a negative charge. This is the origin of the potential difference which is generated between plasma and solid.

How large is the potential difference? If the solid is floating relative to the plasma it must, in the steady state, receive equal currents of positive and negative charge, i.e., equal ion and electron particle fluxes if z = 1, where z = charge on each ion. The negative potential on the solid will then adjust itself so as to reduce the electron flux to the surface until it is equal to the ion flux.

We may now obtain a first estimate of the magnitude of Φ_f. The electrons from the plasma find themselves in a retarding electrostatic field. From elementary statistical mechanics we know that in a repelling, conservative force-field, a Maxwellian distribution remains Maxwellian but the number density is simply reduced.[3] Example: the vertical variation of air density in the earth's atmosphere. In the present case the flux of electrons reaching the solid is thus reduced to 1/4 $n\bar{c}_e$ exp $(e\Phi_f/kT_e)$; note $\Phi_f < 0$. (Strictly, this simple relation for the electrons only holds if all electrons are reflected and the electron distribution is therefore fully Maxwellian. The effect of the distortion to the distribution caused by the escaping electrons on the calculation of floating potential has been shown by Self[4] to be negligible. For objects which are biased more positively the effect can be more substantial; Andrews and Varey[5] have provided the corrections).

Considering now the ion flux to the surface: the situation is more complex since the ions are attracted and their distribution is distorted from Maxwellian. As indicated in Section 2, the ions actually enter the principal potential drop region with a net drift velocity C_s and are then accelerated freely to potential Φ_f. The ion flux to the surface is therefore simply nC_s. We mustn't jump ahead, however, and we want to actually prove this result for the ion flux. A crude estimate would be to take the ion flux into the accelerating region (hence into the solid surface) as the free Maxwellian value, $1/4\ n\bar{c}_i$, neglecting pre-sheath accelerations. (This approximation will be seriously invalid when $T_i < T_e$, but otherwise will turn out not to be too gross.)

Thus in steady state we have

$$1/4\ n\bar{c}_e \exp(e\Phi_f/kT_e) \simeq 1/4\ n\bar{c}_i \tag{3}$$

thus

$$\Phi_f \simeq \frac{kT_e}{e} \ln\left(\frac{T_i m_e}{T_e m_i}\right) . \tag{4}$$

Example: H^+ ions, $T_e = T_i$ then $\Phi_f \simeq -3.8\ kT_e/e$.

For this example, one can see that the impact energy of the ion on the solid is substantially increased over the average ion energy in the plasma.

From where does the ion obtain the increased impact energy? This energy comes from the electrons as one may now see. Consider those electrons at the plasma/sheath interface which have enough energy to escape over the potential barrier Φ_f to reach the solid surface. For a floating surface, the wall flux of such electrons equals the wall flux of ions. In traversing the sheath, these electrons will lose an amount of energy $e\Phi_f$ which is, of course, precisely the energy gained by the ions in the same transversal. The sheath thus acts as a mechanism for transferring energy from the energetic tail of the electron distribution, to the ions.

Although the ion impact energy exceeds the electron impact energy, one should note that the power removal rate from the

population of electrons in the plasma exceeds that from the ions. Thus a surface in contact with a plasma exerts a preferential cooling effect on the electrons.[6]

We turn next to estimating the sheath thickness. We will approximate the sheath as a planar diode in which only one charge species is present (the ions) emitted at one surface (the plasma/sheath interface) and collected at the other (the solid surface). How can we justify neglecting the electrons in the sheath? Because the electron density falls off exponentially as $\Phi(x)$ decreases through the sheath $\{n_e(x) \propto \exp[e\Phi(x)/kT_e]\}$ while the ions in being accelerated by $\Phi(x)$ fall off more slowly in density, namely $n_i(x) \propto [-\Phi(x)]^{1/2}$.[7] The latter result is seen as follows: Neglecting local ionization the ion flux, $n_i(x)V_i(x)$, is constant through the sheath, where $V_i(x)$ is the local ion velocity. Neglecting any initial ion velocity we then have $V_i(x) = [-2e\Phi(x)/m_i]^{1/2}$. Thus, within a short distance of the sheath/plasma interface, the electron density becomes negligible compared with the ion density.

Next we employ Poisson's equation[8] which relates potential variation to space charge density

$$\frac{d^2\Phi}{dx^2} = -\frac{e}{\varepsilon_o}(n_i - n_e) \tag{5}$$

$$\approx -\frac{e}{\varepsilon_o} n_i(x) \tag{6}$$

The ion current density to the wall

$$j^+ = e n_i(x)\ V_i(x) \tag{7}$$

is constant throughout the sheath hence giving

$$\frac{d^2\Phi}{dx^2} = \frac{j^+}{\varepsilon_o}(2e/m_i)^{-1/2}(-\Phi)^{-1/2} . \tag{8}$$

The solution to this equation is the famous Child-Langmuir[9] relation for current in a space-charge limited diode

$$j^+ = 4\varepsilon_o (2e/m_i)^{1/2} (-\Phi_d)^{3/2}/(9d^2) , \tag{9}$$

where d = thickness of acceleration zone, Φ_d is the total voltage drop and the boundary conditions $\Phi = d\Phi/dx = 0$ at the emitting surface are assumed. This relation thus gives the current which can be drawn between two plates, when the space charge between the plates (rather than the current emitted by one of them) limits the achievable current density. Only one charged particle species was assumed to be present in the original Child-Langmuir derivation, and since we neglected the presence of electrons in the sheath we have simply regained this classical result. It is however, only a first approximation for the wall sheath; see next Section.

We now insert our estimates of j^+ and Φ_d for the sheath to calculate the sheath thickness. We use for the present our rough estimate

$$j^+ = 1/4\, n\bar{c}_i e \tag{10}$$

where n represents the (constant) plasma density and $\Phi_d = -\eta kT_e/e$, where for floating conditions in a hydrogen plasma with $T_e = T_i$ we found $\eta \simeq 4$. Inserting these values into Eq. (9) gives

$$d = 1.3\, \eta^{3/4}\, \lambda_D$$

$$\simeq 3.6\, \lambda_D \text{ for } \eta = 4\,. \tag{11}$$

We have thus deduced that the sheath is of order a few Debye lengths[10] in thickness. Even for very large applied (negative) potentials, $\eta \gg 1$, the sheath is still generally very thin compared to typical plasma scale lengths.

The foregoing derivation is clearly an approximate one since (a) electrons have been neglected in the sheath, (b) the initial velocity of the ions entering the sheath has been taken as zero [which would require that $n_i(0) = \infty$ if flux continuity is to hold] instead of $V_i(0) = C_s$. A complete treatment, however, only changes the numerical factor in Eq. (11) somewhat. Since for most practical purposes the precise thickness of the sheath is not important we will not attempt any further refinement of this first estimate of sheath thickness.

4. THE BOHM CRITERION

In this section we deduce two results: (a) that the voltage drop between the plasma and object cannot be accommodated within the plasma itself and thus that a charge layer or sheath must occur between the two to accommodate the drop, (b) that a small pre-sheath electric field extends into the plasma and is sufficient to accelerate the ions such that by their point of entering the sheath they have attained a drift velocity equal to the ion acoustic speed. The latter result is known as the Bohm Criterion.[11]

We deduced in Section 3 that a substantial voltage drop must exist between a plasma and an electrically floating object with which it is in contact. Can this potential drop occur in the plasma itself? That is, in a region where $n_e = n_i$? The answer is no. We will show, in fact, that a collisionless plasma cannot contain a potential drop greater than ~1/2 kT_e/e. (This also assumes either no magnetic field or at least we restrict our attention to motion along the B-field.)

We will apply the fluid equations to the plasma, e.g., we will assume that all of the ions at point x in the plasma have the same drift velocity V(x). We assume one dimensional, steady-state flow. This analysis could therefore be applied to the case of plasma flow along magnetic field lines to solid surfaces such as limiters, divertor plates, probes, etc. We must allow for a source of ion pairs since particles are removed by the solid surface and steady-state is assumed. The equation of continuity[12] is thus

$$\frac{d}{dx}(nV) = S\,, \tag{12}$$

where S = volume source rate of ion pairs (ion pairs/m^3/s)

$$n = n_e = n_i\,.$$

S might, for example, represent local ionization of neutral atoms by impact with plasma electrons--the situation in low pressure, partially-ionized discharges such as neon lights, etc.[13] S can also represent the appearance of ion pairs in a magnetic flux tube via cross-field diffusion. This latter case would characterize the scrape-off region of magnetically-confined plasma devices where ion pairs may enter the edge flux tube primarily from the main or core plasma,[6,14-22] rather than being created locally by neutral ionization. Note also that S may vary, S(x).

The momentum equation[12] for the ions is

$$m_i V \frac{dV}{dx} = -\frac{dp_i}{dx} + enE - m_i VS \tag{13}$$

where $p_i = nkT_i$, ion pressure $E = -d\Phi/dx$ electric field in the plasma.

The first term in Eq. (13) gives the convective rate of change of momentum. The last term allows for the drag on the ion flow when a new ion is created at point x with zero velocity and has to be brought up to velocity V(x). For a derivation of Eqs. (12) and (13) from basic principles, see Ref. 23.

We can write similar equations for the electrons; however, since they are in a retarding field, it is convenient to employ the result mentioned earlier, i.e., one has the <u>Boltzmann Relation</u>[3] for the electrons in the plasma:

$$n_e(x) = n_o \exp\,[e\Phi(x)/kT_e]\ , \tag{14}$$

where n_o is the reference density, conveniently taken to be that at the symmetry point (Fig. 1), i.e., at a great distance from the object. This relation permits a replacement of the third term in Eq. (13)

$$enE = -en\frac{d\Phi}{dx} = -kT_e \frac{dn}{dx}\ . \tag{15}$$

It is convenient to consider the ion flow to be either <u>isothermal</u> (T_i = constant) or <u>adiabatic</u> (no energy addition) thus one has that

$$\frac{dp_i}{dx} = \gamma_s kT_i \frac{dn}{dx}\ , \tag{16}$$

where γ_s = 1, 5/3 for isothermal, adiabatic flow. One may therefore rewrite the ion momentum conservation equation as

$$V\frac{dV}{dx} = -\frac{C_s^2}{n}\frac{dn}{dx} - \frac{SV}{n} \tag{17}$$

where

$$C_s = [(\gamma_s\, kT_i + kT_e)/m_i]^{1/2}, \tag{18}$$

the ion acoustic speed at the sheath/plasma interface.[24] For isothermal flow $C_s = C$ = constant.

One thus sees that in the plasma the natural ion velocity for normalization is the ion acoustic speed and one may define a local ion <u>Mach number</u> $M(x) \equiv V(x)/C_s$. We will see that something rather spectacular occurs when the ions <u>"break the sound barrier"</u>, namely the plasma is terminated and forms a <u>sheath</u> (Ref. 25). We combine these equations to give

$$\frac{dM}{dx} = \frac{S}{nC_s}\,\frac{1+M^2}{1-M^2}\ . \tag{19}$$

We assign V(0) = M(0) at the symmetry point. This last equation is a combination of the fundamental expressions of the conservation of mass and momentum and it provides us with the basic picture of what happens to the ion flow in the plasma. Thus from Eq. (19) we have the obvious result that $dM/dx > 0$, i.e., the ion velocity increases as one approaches the surface. (Note that we used the fact that S/nC_s is inherently positive and that the flow starts subsonically at the stagnation point.) Most importantly we have from Eq. (19) that as $M \to 1$, $dM/dx \to \infty$, i.e., the ions are abruptly accelerated to very high velocities and the plasma solution fails, <u>"blows up"</u>. This corresponds physically to the termination of the plasma and the start of the sheath with its large electric fields and high ion acceleration.[4,11,25-30]

The foregoing equations have simple solutions[31] for relating the plasma density and potential to the local Mach number

$$\frac{n(M)}{n_o} = \frac{1}{(1+M^2)} \tag{20}$$

$$\Phi(M) = -\frac{kT_e}{e}\,\ln\,(1+M^2)\ . \tag{21}$$

[To calculate n(x), Φ(x), V(x), etc. requires knowledge about the spatial variation of S(x)]. We thus have the useful fluid model pre-sheath results that at the plasma/sheath edge where M = 1, then $n \equiv n_{se} = 1/2\ n_o$ and $\Phi = -0.69\ kT_e/e$.[32] That is, the plasma density at the sheath edge is half that far from the surface and the voltage drop in the plasma cannot exceed ~0.7 kT_e/e. We thus have obtained our result that the potential drop that we know must exist between the plasma and the solid cannot be accommodated in the plasma itself.

Instead of employing the fluid equations to describe the ion flow to the surface one can use equations which allow for individual particle motion (as is most appropriate when the ions are collisionless). This type of analysis was first carried out (for the case of $T_i = 0$) by Tonks and Langmuir[26] in their classic 1929 paper on plasma flow to a surface. They showed that the plasma solution "blows up" when the potential reaches a value $(0.854\text{-}1.418)kT_e/e$ (depending on geometry and other details) giving an average ion velocity at the plasma/sheath interface of $(1.144\text{-}1.213)C_s$.

The result that $V_i = C_s$ at the plasma/sheath interface is the all important Bohm Criterion and was first derived explicitly by David Bohm (Ref. 11) in the 1940's by considering the sheath equations rather than the plasma equations as we have done above. Because of the historical importance of this result, we now deduce the Bohm Criterion from an analysis of the sheath. Our analysis will in fact be a more refined version of the one employed in Section 3 to estimate the sheath thickness. We now allow for (a) electrons in the sheath, (b) a finite ion velocity for the ions entering the sheath. We follow Bohm[11] in assessing the case of $T_i = 0$ and all ions have the same (drift) velocity at any given point in the sheath. The ion momentum and particle conservation equations therefore give the particularly simple result that

$$n_i(x) = n_{se}\,[\Phi_o/\Phi(x)]^{1/2}, \qquad (22)$$

where n_{se} = ion (and electron) number density at the plasma/sheath interface, $|\Phi_o| = 1/2\ m_i V_o^2/e$, the pre-sheath potential drop (assumed to be experienced by all ions) and which we are attempting to calculate, V_o = the monoenergetic ion velocity at the plasma/sheath interface. As noted earlier we can neglect the creation of new ions in the sheath since it is so thin. The electrons obey, to a good approximation,[3,4]

$$n_e(x) = n_{se}\exp\,[e(\Phi-\Phi_o)/kT_e]\ . \qquad (23)$$

We now insert Eqs. (22) and (23) into Poisson's Eq. (5) and integrate once to obtain the electric field in the sheath:

$$E^2(x) = \frac{2}{\varepsilon_o} n_{se}\, e\{2(\Phi_o\Phi)^{1/2} + \frac{kT_e}{e}\exp\,[e(\frac{\Phi-\Phi_o}{kT_e})]\} + C' , \qquad (24)$$

where C' is the constant of integration. To find C', Bohm reasoned that $E(0) \simeq 0$ by noting that in the plasma itself we know that Φ only changes over scale lengths which are (generally) extremely large compared with the sheath thickness, i.e., in the plasma $E \simeq 0$ at least compared with typical values in the sheath where $E \sim kT_e/e\lambda_D$. Thus to obtain the physically satisfactory situation of a continuous electric field at the plasma/sheath interface we take in our sheath analysis that $E(0) = 0$. This gives

$$E^2(x) = \frac{2}{\varepsilon_o} n_{se}\, e\{-2\Phi_o[(\Phi/\Phi_o)^{1/2}-1] + \frac{kT_e}{e}[\exp(\frac{e(\Phi-\Phi_o)}{kT_e})-1]\}. \qquad (25)$$

We now consider $E(x)$ for small $\Delta\Phi \equiv \Phi-\Phi_o$ and expanding Eq. (25) find

$$E^2(x) \simeq \frac{n_{se}}{\varepsilon_o}\, e\,(\frac{e}{kT_e} - \frac{1}{2\Phi_o})(\Delta\Phi)^2\ . \qquad (26)$$

Thus the electric field is only real provided

$$|\Phi_o| \geq \frac{kT_e}{2e}\ , \qquad (27)$$

that is the ion velocity at the plasma/sheath interface, V_o, must satisfy

$$V_o \geq (kT_e/m_i)^{1/2}$$
$$= C_s \text{ for } T_i = 0\ . \qquad (28)$$

(Aside: The small difference between the pre-sheath potential drop calculated using the plasma fluid equations, $-0.69\ kT_e/e$ and using the sheath equations, $-0.5\ kT_e/e$, is due to the simplifying assumption used in the latter case that the ions originated from a point source in the plasma.)

We thus find that a physically realistic, steady-state sheath solution requires that the ions enter the sheath with a speed of at least C_s, while from the plasma analysis, we found that no steady-state solution is possible for ion drift velocity exceeding C_s. Thus we may conclude that the plasma terminates and the sheath commences when the ion velocity equals the ion acoustic speed, precisely. The sheath is thus seen to be analogous to the shock which can form at supersonic velocities in conventional fluid flow.[25]

5. SIMPLE EXPRESSIONS FOR THE FLOATING POTENTIAL, PARTICLE AND HEAT FLUX DENSITIES THROUGH THE SHEATH[33]

In this section we seek to establish expressions--if possible, simple and convenient ones--for nine key sheath quantities, namely: Φ_f and the electron (and ion) particle (and energy) flux densities to floating (and biased) surfaces.

We seek to relate these nine quantities to the plasma density and temperatures (ion, electron), and in the case of the biased surface to the bias voltage.

The Bohm Criterion is the starting point in this undertaking since it provides a value for the ion particle flux density from a plasma to a surface (thus also the electron particle flux density if the surface is floating). We can now replace our first estimate of j^+, $1/4\ n\bar{c}_i e$, with

$$j^+ = en_{se}C_s \ . \tag{29}$$

If in flowing through the plasma the ions do not suffer momentum loss due to collisions with other particles, and if T_e and T_i are spatially constant, then we can also relate j^+ to conditions far from the surface,[31,32] namely,

$$j^+ \simeq 1/2\ en_o C_s(T_e,\ T_i) \ . \tag{30}$$

In more complex plasmas the relation between n_o and n_{se} can be quite different and temperature variations (Refs. 19, 20, 22, 34, 35) in the flow direction can also occur. In such cases detailed modeling of the plasma[19,20,35-37] (pre-sheath) is required in order to relate n, T_e, and T_i at the sheath edge to values far away. Such modeling is the subject of other chapters and will not be dealt with further here. Equation (29) is generally true, however, that is one can express the particle

flux to the surface in terms of the local plasma density and temperature.

We are now in a position to deduce a more accurate value for the floating voltage Φ_f. We wish to allow for secondary electron emission from the surface, arising from electron impact, since this is significant even at rather low energies, $T_e \gtrsim 30$ eV (by contrast, ion-induced secondary electron emission is usually only important for ion impact energies of $\gtrsim 1$ keV). We thus have that the secondary electron current density is

$$j^-_{SEC} = \gamma_e j^-_{TOT} = \gamma_e(j^-_{NET} + j^-_{SEC}) \tag{31}$$

where

γ_e = secondary electron emission coefficient (includes both true s.e.e. and electron backscatter),

j^-_{TOT} = total electron current density striking the surface,

$$j^-_{NET} \equiv j^-_{TOT} - j^-_{SEC} = (1-\gamma_e)j^-_{TOT} \ .$$

We thus have that

$$j^-_{NET} = 1/4\ n_{se}\ \bar{c}_e(1-\gamma_e)\ \exp\ (e\Phi_f/kT_e) \ . \tag{32}$$

For floating conditions we have that

$$j^-_{NET} = j^+ \ , \tag{33}$$

hence equating Eqs. (29) and (32) one has

$$\frac{e\Phi_f}{kT_e} = 0.5\ \ell n\ \left[\left(2\pi\ \frac{m_e}{m_i}\right)\left(1 + \frac{T_i}{T_e}\right)(1-\gamma_e)^{-2}\right] \ . \tag{34}$$

One may note that, if we put $\gamma_e = 0$, our first estimate of Φ_f, Eq. (4), is not greatly different from our more refined value, provided also $T_i \gtrsim T_e$. Equation (34) does not, of course, include the pre-sheath voltage drop and one may add ~1/2 kT_e/e to find the potential difference between the surface and the plasma far from the surface. The value of Φ_f, Eq. (34), is shown in Fig. 2. As can be seen, Φ_f is reduced by increasing

T_i/T_e or by increasing secondary electron electron emission. The often-quoted statement that "the floating voltage is about $3kT_e/e$" can thus be in error.

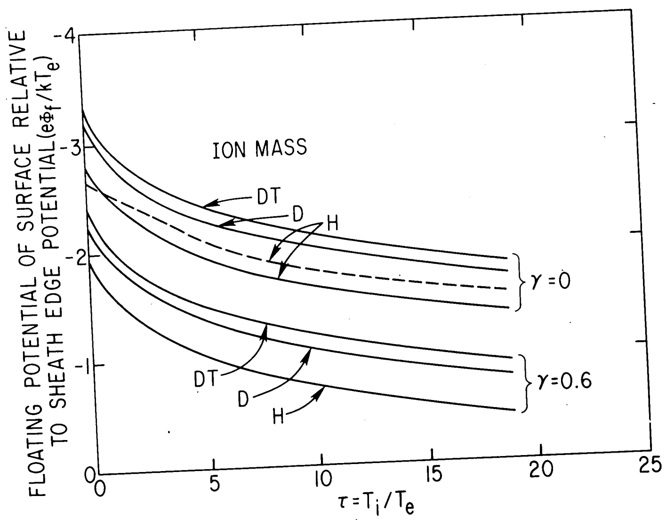

Fig. 2. The voltage difference between a floating surface and the potential at the plasma/sheath interface (normalized), i.e., excluding the pre-sheath voltage, Eq. (34). Dotted line from Emmert et al.[86]

We consider next the <u>energy transmission</u> of the sheath. As indicated earlier, the electron distribution at the sheath solid interface is still Maxwellian, at least in the forward direction. We may therefore use the fact that for a Maxwellian distribution the energy flux in the x-direction is just 2 kT times the particle flux in the x-direction:

$$\iiint v_x 1/2m(v_x^2+v_y^2+v_z^2) f_{max} dv_x dv_y dv_z = 2kT \iiint v_x f_{max} dv_x dv_y dv_z$$

$$= 2kT\ (1/4\ n\bar{c}) \qquad (35)$$

If we wish to calculate the power removal rate from the plasma electron distribution, then we must note that these escaping electrons actually possessed a higher kinetic energy as they were removed from the plasma, namely, one higher by the amount $e\Phi_f$. Thus the electron power flux density <u>removed from the plasma</u> is

$$Q_e = (2kT_e - e\Phi_f)\,\frac{j^-_{TOT}}{e} + e\Phi_f\,\frac{j^-_{SEC}}{e}\,. \qquad (36)$$

(Note that since Φ_f is -ve, the $-e\Phi_f$ term is +ve.)

The last term in Eq. (36) represents the energy reinjected <u>into</u> the plasma by secondary electrons accelerated to energy $-e\Phi_f$. We neglect the thermal energy of the secondary electrons since it is only a few electron volts. Thus we can write

$$Q_e = \left(\frac{2kT_e}{1-\gamma_e} - e\Phi_f\right)\frac{j^+}{e}\,. \qquad (37)$$

Consider now the ion energy flow. To calculate this we need to know the actual ion velocity distribution at the sheath edge--for more on this see Section 8. As a first approximation we can neglect the pre-sheath in which case the relation between particle and energy flow is given by the Maxwellian distribution result:

$$Q_i = 2kT_i\,\frac{j^+}{e}\,. \qquad (38)$$

One may add pre-sheath contributions to Q_e and Q_i, however, these contributions are comparatively small and their values depend on assumptions about pre-sheath conditions[33]; see Section 8 for further discussion on this point.

It is generally found to be useful to define the <u>sheath energy transmission factor</u>, which is the ratio of the power flux to $(kT_e) \times$ (particle flux).

Thus the electron energy transmission factor

$$\delta_e \equiv \frac{Q_e}{kT_e(j^+/e)} = \frac{2}{1-\gamma_e} - \frac{e\Phi_f}{kT_e} \qquad (39)$$

and so

$$\delta_e = \frac{2}{1-\gamma_e} - 0.5\ \ell n\ \left[\left(2\pi \frac{m_e}{m_i}\right)\left(1+\frac{T_i}{T_e}\right)\left(1-\gamma_e\right)^{-2}\right] \qquad (40)$$

while for the ions

$$\frac{Q_i}{kT_e(j^+/e)} \equiv \frac{2T_i}{T_e} . \qquad (41)$$

The total energy transmission factor $\delta = \delta_e + \delta_i$ is thus

$$\delta = \frac{2T_i}{T_e} + \frac{2}{1-\gamma_e} - 0.5\ \ell n\ \left[\left(2\pi \frac{m_e}{m_i}\right)\left(1 + \frac{T_i}{T_e}\right)\left(1-\gamma_e\right)^{-2}\right] . \qquad (42)$$

Relation (42) is shown in Fig. 3.

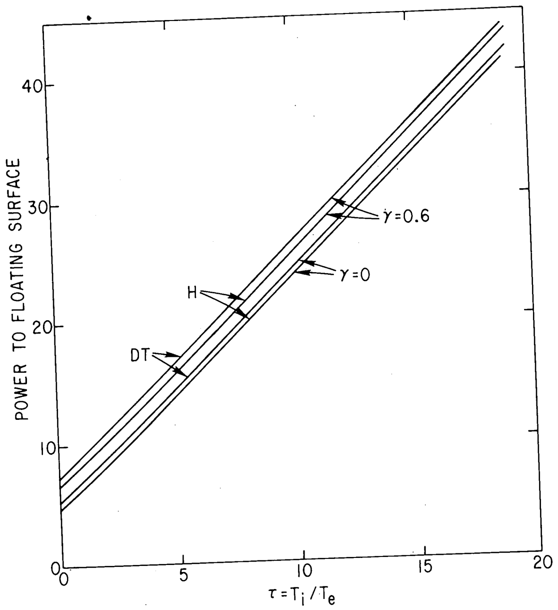

Fig. 3. Power flux density to a floating surface (normalized), Eq. (42). Note that the refinements of Eq. (50) are not included here.

Note: When $\gamma_e = 1 - [(2\pi m_e/m_i)(1+T_i/T_e)]^{1/2}$ then $\Phi_f = 0$; for example for $T_i = 10T_e$ and H^+ ions this occurs for γ_e = 0.8. Then the electrons reach the surface unimpeded by any sheath, so $Q_e = 1/4\ n\bar{c}_e\ 2kT_e$ [equivalent to Eq. (39)] and the surface heating rate is extremely high.

The inclusion of the effect of secondary electron emission in the calculation of $e\Phi_f$ (thus also of δ_e and δ) warrants further discussion. In their pioneering work on this problem, Hobbs and Wesson[34] fully accounted for the existence of secondary electrons by allowing for the presence of injected secondary electrons in the plasma far from the sheath (thus reducing the density of primary electrons there below the ion density). It turns out that this has a negligible influence on Φ_f, provided $\gamma_e \neq 1$, and the Hobbs and Wesson value for Φ_f is the same as that given in Eq. (34), (taking $T_i = 0$, their assumed value). These authors also found the (electron) heat flux density through the sheath to be

$$Q_e = 1/4\ n_o\bar{c}_e\ 2kT_e\ F(\gamma_e) \qquad (43)$$

where they define

$$F(\gamma_e) \equiv \left(\frac{\pi m_e}{8m_i}\right)^{1/2} \left[\ell n\ \left(\frac{(1-\gamma_e^2)}{2\pi m_e/m_i}\right) + \frac{5-\gamma_e}{1-\gamma_e}\right] . \qquad (44)$$

For $T_i = 0$ Eq. (37) gives the same value for Q_e as obtained by Hobbs and Wesson except that:

(a) These authors (incorrectly) take $j^+ = en_oC_s$, neglecting the pre-sheath density drop; thus the n_o in Eq. (43) should read n_{se}.

(b) Hobbs and Wesson include a pre-sheath energy contribution, $1/2\ kT_e$ in Eq. (43).

Clearly neither Eqs. (42) nor (43) can be applied for strong secondary electron emission, $\gamma_e \geq 1$, since a singularity occurs at $\gamma_e = 1$. Hobbs and Wesson, in fact, showed that for $\gamma_e \gtrsim 0.8$, an electron space charge layer will occur at the surface inhibiting any further secondary emission. Thus these equations do not apply for strong secondary electron emission which can set in at $T_e \gtrsim 100$ eV.

We should, in principal, also include secondary electron emission due to ion, photon, metastable atom impact etc.[34] Usually ions will not create significant amounts of secondary electrons for fusion edge conditions since the required energy, $\gtrsim$1 keV, normally implies intolerable levels of sputtering and surface heating. Photon fluxes are only ~1 W/cm^2 on average in the edge region (contrasted with the 10's - 1000's W/cm^2 of charged particle power flux along magnetic field lines to surfaces) and so usually one can neglect this process compared with electron impact s.e.e.; however, near probes, limiters etc. high gas levels are often present (due to the release of hydrogen initially deposited in the solid as ions), and so strong local sources of radiation can exist arising from electron impact with this gas. These photons may increase s.e.e. locally to substantial levels.

It is left as an exercise to show that Eqs. (31)-(34) can be generalized to allow for these additional s.e.e. processes by simply replacing γ_e with γ where[34]

$$\gamma = (\gamma_e + \gamma_i + j)/(1 + \gamma_i + j)$$

and

$\gamma_i \equiv$ ion s.e.e. coefficient

$j \equiv j_{ph}/n_{se} C_s$

$j_{ph} \equiv$ s.e.e. flux density due to photons, metastable atoms, etc.

Now that we have dealt with the particle and energy flow to an electrically floating surface, let us examine the case of an electrically biased surface.[33,38,39] We will restrict our attention to the case of surfaces which are still biased negatively with respect to the plasma. As we shall see, as the surface potential approaches the plasma potential, the heat flux can reach extremely high values.

So long as the surface is negative with respect to the plasma, a sheath still exists, although its thickness varies. We thus have that the ion current is still given by $j^+ = en_{se}C_s$. The net electron current is

$$j^-_{NET} = 1/4\ n_{se}\bar{c}_e e\ (1-\gamma)\ \exp\ (e\Phi/kT_e), \tag{45}$$

where Φ is the (negative) potential applied to the surface relative to plasma potential. The net total current density can therefore be written

$$\frac{j_{TOT}(\Phi)}{1/4\ en_{se}\bar{c}_e} = \left[\left(1+\frac{T_i}{T_e}\right)\left(\frac{2\pi m_e}{m_i}\right)\right]^{1/2} - (1-\gamma)\ \exp\ \left(\frac{e\Phi}{kT_e}\right). \tag{46}$$

Relation (46) is shown in Fig. 4. The net total current reaches the "saturation ion current" j^+_{SAT}, which is just the j^+ of Eq. (29), at sufficiently negative potentials. Increasing the potential above the floating potential causes an exponential increase in the electron current. In the simplest model, the electron current attains the saturation value $j^-_{SAT} = 1/4\ n_{se}\bar{c}_e e$ at $\Phi = 0$ and this then remains constant for all $\Phi > 0$. Often a true saturation electron current is not observed but the current continues to rise with Φ; however, the increase is slower than exponential. The solid-plasma interaction is difficult to model for $\Phi > 0$ since no sheath is present and the applied electric field penetrates far into the plasma causing a significant perturbation. In some circumstances, for example, operation in a strong magnetic field parallel to the direction of plasma collection, there can be a reduction[40] of the maximum electron current from the value of $1/4\ n_{se}\bar{c}_e e$. (See Section 7.4.)

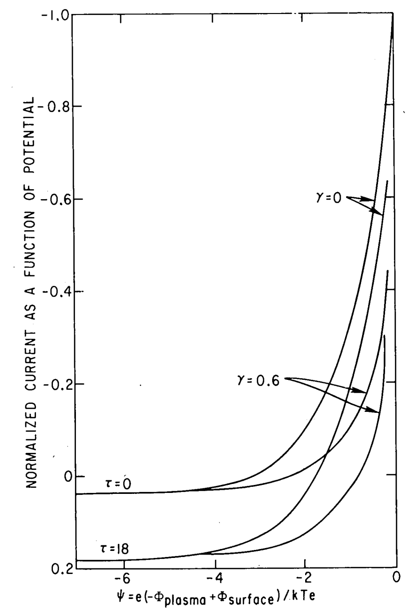

Fig. 4. The normalized current as a function of the applied potential to a surface, i.e., the Langmuir J-Φ characteristic, Eq. (46).

Next consider the power flux density to an electrically biased surface. The ion power is

$$Q_i(\Phi) = j^+(2kT_i - e\Phi) \tag{47}$$

while for the electrons

$$Q_e(\Phi) = 2kT_e \, j^-_{TOT} \, . \tag{48}$$

Note that we use j^-_{TOT}, in the last expression rather than j^-_{NET}, since every electron striking the surface deposits, on average, $2kT_e$ there. Again, we neglect the thermal energy of the emitted secondaries. Thus we may write the total power flux density in the form of a transmission coefficient

$$\frac{Q(\Phi)}{kT_e(j^+/e)} \equiv \delta(\Phi) = -\frac{e\Phi}{kT_e} + \frac{2T_i}{T_e} + 2\left[\left(1 + \frac{T_i}{T_e}\right)\left(\frac{2\pi m_e}{m_i}\right)\right]^{-1/2} \exp\left(\frac{e\Phi}{kT_e}\right). \tag{49}$$

Note that γ does not appear in this last expression. Relation (49) is shown in Fig. 5.

Note that the heat flux is nearly a minimum at the floating potential.[38] At potentials below floating, the power flux increases slightly due to the increasing impact energy of the ions. For potentials above floating, the electron particle, and thus energy, flux increases exponentially fast. Operation at potentials near or above the plasma potential is often hazardous due to this strong heating.

The foregoing analysis applies to the case of no magnetic field. Provided ion motion is along B-field lines, this theory should also apply when B is finite. For ion motion oblique to magnetic field lines, as occurs when the plane of the collecting surface is not parallel to the B-field, further analysis[41-43] is required. This has been carried by Chodura[42,43] and will be discussed in the next lecture.

Further refinements that can be added to the δ-values already calculated include:

(a) Ion backscatter. The impacting ions tend to be

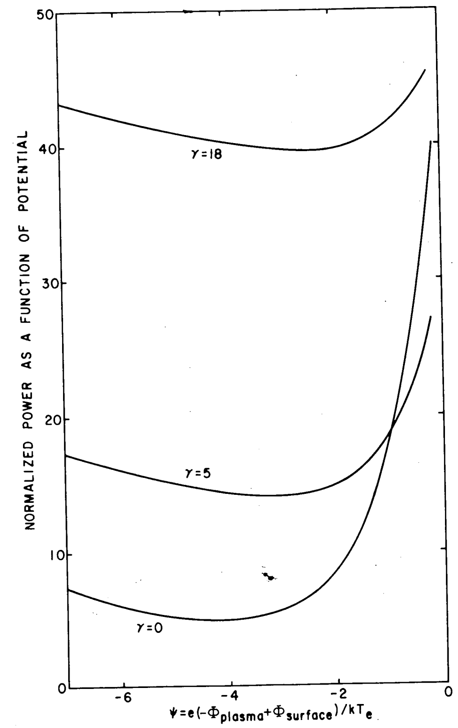

Fig. 5. The normalized power flux density as a function of the potential applied to a surface, Eq. (49). $\tau = T_i/T_e$. Note that refinements are neglected here: $R_{iE} = R_{iN} = R_{eN} = \chi_i = \chi_r = 0$.

reflected from the surface (usually as neutrals), depositing only a fraction R_{iE} of their impact energy there. Values of R_{iE} are dependent on the energy, mass, and incidence angle of the impacting ion and the material of the solid; values of R_{iE} are given in various references.[44-46]

(b) Ionization. Recombination energy χ_i should be added.

(c) Atom. Atom recombination energy χ_r should be added (if molecular formation ensues). Here also an ion particle reflection coefficient R_{iN}, should be allowed for.

(d) Electron backscatter.[47] For purposes of computing *particle* fluxes (hence floating potential) it is not necessary to distinguish between electron-induced secondary electron emission and electron backscatter. For purposes of calculating *heat* flux and δ_e, however, the distinction can be worth making since a secondary electron only removes a few eV from the solid, while the backscattered electron can return a sizeable fraction of $2kT_e$ to the plasma.

(e) Pre-sheath contributions. As indicated above, pre-sheath contributions can also be added. In the simplest case this introduces $1/2\ kT_e$ to δ_e, but for more complex pre-sheaths, other values may be appropriate, see Section 8.

Including these latter refinements one may rewrite Eq. (42) as

$$\delta kT_e = \left[2kT_i - e\Phi_f\left(\frac{m_e}{m_i}, \frac{T_i}{T_e}, \gamma\right)\right](1-R_{iE})$$

$$+ \frac{2kT_e}{1-\gamma}(1-R_{eE})$$

$$+ \varepsilon_{pre.sh.}$$

$$+ \chi_i$$

$$+ \chi_r(1-R_{iN}) \ . \qquad (50)$$

Example: H^+ on W, $T_i = 20$ eV, $T_e = 10$ eV. Then $\chi_i = 13.6$ eV, $\chi_r = 2.2$ eV, $\gamma \cong 0.3$, $R_{iN} \cong 0.5$, $R_{iE} \cong 0.3$, $R_{eE} \cong 0.15$, $e\Phi_f = -2.1\ kT_e$, and thus

$\delta kT_e = 42.7 + 24.3 + (\sim 5) + 13.5 + 1.1,$

hence $\delta \simeq 8.7$.

Ideally one should employ values of R_{iE}, R_{eE}, R_{iN}, and γ which are appropriate for the actual incident angle of the particles. In practice, surface roughness implies that one should probably use average values. The effect on heat flux and δ of terms which are independent of T_e and T_i, such as χ_i, is only significant when T_e and T_i are rather small, i.e., generally smaller than χ_i. For many edge plasma conditions, e.g., the foregoing example, the T-independent contributions to δ are not great and can be ignored, to first order. In some divertor devices, however, very low temperature (and high density) plasmas have been achieved near the surface, with $T < 5$ eV. In such cases one should include further T-independent contributions to δ, for example, the energy removed from the solid for each secondary electron emitted, a few eV. For a non-floating surface the solid will also again (lose) the electron work function energy corresponding to the net gain (loss) of an electron from the plasma. For the case of high density edge plasmas, neutrals emitted from the solid into the plasma will be dissociated (into Frank-Condon atoms of a few eV energy), electronically and vibrationally excited and ionized all quite close to the surface. One should then include in the calculation of heat flux to the surface contributions due to Frank-Condon atoms, photons, excited neutrals etc., impacting on the surface. Clearly the calculation of heat flux to a surface for high density, low temperature edge plasmas requires special treatment, and this will not be dealt with further here.

6. APPLICATIONS: EDGE MODELING

For many magnetic confinement configurations one can consider the plasma as consisting of two regions the *core* plasma and the *edge* or *scrape-off* plasma, see Fig. 6. The behavior of the scrape-off layer, *SOL*, is strongly influenced by the presence of solid surfaces inserted into this plasma, since plasma motion along magnetic field lines to these surfaces is scarcely impeded. In the simplest magnetic topology these inserted solids are called *limiters*, while if the magnetic field lines near the edge are distorted and drawn away from the main vessel in the *divertor* arrangement, then the solid surfaces are called *divertor target plates*. In either case we can model the SOL as unimpeded, one-dimensional flow to a solid surface and thus use the results we have obtained in Section 5. This topic of *plasma edge modeling*[14-22,35-38] is dealt with more extensively in other lectures and here we merely wish to gain some basic insight into the influence of the sheath on SOL properties. We will consider a few basic results.