

# Stretching the time distribution of ion pulses ejected from the ReA EBIT: Theoretical concepts for the ramp

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## I. THEORY

So long as the instantaneous rate delivered to the experimental stations is not a constraint, the simplest extraction method to release ions is to lower the collector-side potential barrier with (discontinuous) step time function. With this scheme, the extracted ion time distribution depends in most part on the physical length of the trapping region and the energy distribution of the trapped ions. For a trap length of 635 mm like the ReA EBIT's, the full width of the extracted pulse is in the range of fifty microseconds. Hence, for stretching extracted pulses up to tens of milliseconds, the effect of the trap length on the extracted ion time distribution can be neglected, and the time distribution can be assumed to only depends on the energy distribution. If not affected by undesirable ion losses, the total number of ions ( $N_i^{tot}$ ) of a species  $i$  is conserved during ion extraction and is the sum of the number of trapped ( $N_i^{tr}$ ) and extracted ( $N_i^{ex}$ ) ions:

$$N_i^{tot} = N_i^{tr} + N_i^{ex}. \quad (1)$$

The variation in time of the number of extracted ions (*i.e.*, the ion extraction rate) can be obtained from the time derivative of Eq. 1, which, because  $N_i^{tot}$  is a conserved quantity and  $dN_i^{tot}/dt = 0$ , yields:

$$\frac{dN_i^{ex}}{dt} = -\frac{dN_i^{tr}}{dt}. \quad (2)$$

For a given ion energy distribution  $f(E_i)$ , the number of trapped ions of energy  $E_i$  within an interval  $dE_i$  can be expressed as:

$$dN_i^{tr} = f(E_i)dE_i, \quad (3)$$

so that the number of trapped ions can be obtained by integrating the energy distribution over  $E_i$ , where  $E_i^{max}$  is the maximum energy of the ions confined in the trap:

$$N_i^{tr} = \int_0^{E_i^{max}} f(E_i) dE_i. \quad (4)$$

The variation in time of the number of trapped ions can then be expressed as:

$$\frac{dN_i^{tr}}{dt} = \frac{d}{dt} \left[ \int_0^{E_i^{max}} f(E_i) dE_i \right]. \quad (5)$$

Substituting Eq. 6 into Eq. 5, one can obtain a general expression describing the extraction rate as a function of the ion energy distribution:

$$\frac{dN_i^{ex}}{dt} = -\frac{d}{dt} \left[ \int_0^{E_i^{max}} f(E_i) dE_i \right]. \quad (6)$$

$dN_i^{ex}/dt$  expresses the (instantaneous) extracted rate as a function of time, and for simplicity can be renamed as  $R(t)$ . Eq. 6 can then be re-written as:

$$R_i(t) = -\frac{d}{dt} \left[ \int_0^{E_i^{max}} f(E_i) dE_i \right]. \quad (7)$$

In several cases, it may be more desirable to spread uniformly in time the release of ions in order to extract a constant rate over the entire extraction time period. Hence, replacing  $R_i(t)$  with  $R_i$  in Eq. 7 to eliminate the time dependence and integrating, one can obtain a special-case expression:

$$R_i t = - \int_0^{E_i^{max}} f(E_i) dE_i + C, \quad (8)$$

where  $C$  is a constant that depends on the initial conditions. Note that it is assumed here that the ion energy distribution of the trapped ions do not change during the extraction time. The ion extraction rate depends on the lowering voltage rate of the trapping potential, which for a constant extraction rate has to vary with time to match the initial ion energy distribution. In the next subsection, we present different ramp voltage functions obtained assuming different ion energy distribution to release a constant extraction rate.

### A. Boltzmann distribution

The energy distribution of the trapped ions can be assumed to be a Boltzmann distribution, such as:

$$f(E_i) = A e^{-E_i/(k_B T_i)}, \quad (9)$$

where  $k_B$  is the Boltzmann constant,  $T_i$  is the ion temperature, and where  $A$  is a constant equal to  $N_i^{tot}/(k_B T_i)$

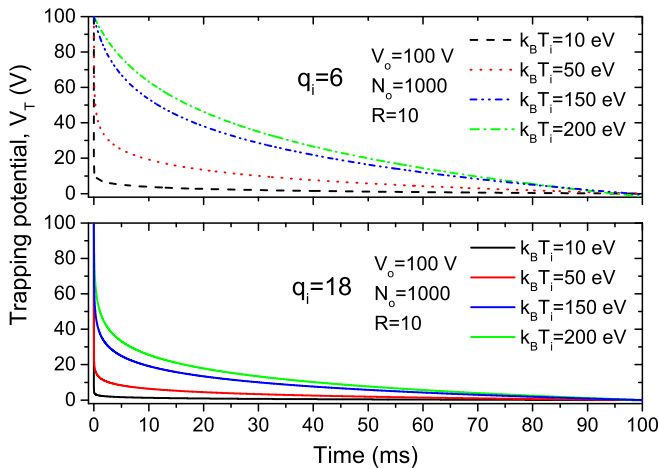


FIG. 1. Lowering voltage time function assuming a Boltzmann ion energy distribution.

after integrating from 0 to Infinity to normalize by the total number of trapped ions,  $N_i^{tot}$ . Substituting 9 into 8, this latter equation can be re-written as:

$$\frac{R_i}{N_i^{tot}} t = \left[ e^{-E_i^{max}/(k_B T_i)} - 1 \right] + C, \quad (10)$$

Assuming that the electron-beam space charge potential is significantly larger than the axial trapping potential well and hence that no ions are lost radially, the maximum energy of ions of charge  $q_i$  confined in the trapping potential  $V_T$  is defined as  $E_i^{max} = q_i V_T$ . Replacing  $E_i^{max}$  in Eq. 10 with this expression and solving  $V_T$  for the initial condition  $V_T(t = 0) = V_o$  where  $V_o$  is the initial trapping potential, one obtains:

$$V_T(t) = -\frac{k_B T_i}{q_i} \ln \left[ \frac{R_i}{N_i^{tot}} t + e^{-q_i V_o / (k_B T_i)} \right], \quad (11)$$

Fig. 1 shows voltage waveforms (or functions) expressing how the trapping potential has to be lowered with time to release the entire initial ion population with a constant rate over the whole extraction time for different ion species and initial ion temperatures. In this figure,  $R$  was calculated from the ratio of the total number of trapped ions to the desired extraction time period, which can be adjusted for the instantaneous rate to remain lower than the maximum rate the detection and acquisition systems can handle at the experimental stations. The overall shape and steepness of the curves is governed by the  $k_B T_i / q_i$  ratio. For instance, the trapping potential lowering time function of  $C^{6+}$  with an initial ion temperature of 50 eV is equivalent to that of  $Ar^{18+}$  with a temperature of 150 eV. The fast steep voltage decrease during the first milliseconds is to accommodate for the long exponential tail of the Boltzmann distribution which contains few ions compared with the peak of the distribution around  $E_i = k_B T_i$ .

## B. Maxwell-Boltzmann distribution

The energy distribution of the trapped ions can be assumed to have a Maxwell-Boltzmann distribution for all degrees of freedoms (normalized to  $N_i^{tot}$ ) as:

$$f(E_i) = 2N_i^{tot} \sqrt{\frac{E_i}{\pi}} \left( \frac{1}{k_B T_i} \right)^{3/2} e^{-E_i / (k_B T_i)} \quad (12)$$

Substituting Eq. 12 into Eq. 8, integrating, and replacing  $E_i^{max}$  with  $q_i V_T$ , one can obtain an expression describing the lowering voltage time function of the trapping potential to release all trapped ions  $N_i^{tot}$  within a given time period with a constant rate  $R_i$ :

$$\frac{R_i}{N_i^{tot}} t = 2 \sqrt{\frac{q_i V_T}{\pi k_B T_i}} e^{-q_i V_T / (k_B T_i)} + erf \left( -\sqrt{\frac{q_i V_T}{k_B T_i}} \right) + K \quad (13)$$

where  $erf$  is the error function and  $K$  is constant depending on the initial condition at  $t = 0$ :

$$K = -2 \sqrt{\frac{q_i V_o}{\pi k_B T_i}} e^{-q_i V_o / (k_B T_i)} - erf \left( -\sqrt{\frac{q_i V_o}{k_B T_i}} \right) \quad (14)$$

Solving  $V_T$  numerically, Fig. 2 shows lowering voltage time functions for different ion species and ion temperatures. Overall, the lowering time function of the trapping potential for a Maxwell-Boltzmann distribution is nearly identical to that for the Boltzmann distribution. In both cases, the fast decrease of the trapping potential at the very beginning of the extraction period (negative slope) is to accommodate for the long high-energy tail of the energy distributions driven by the exponential function. However, unlike the Boltzmann distribution, the Maxwell-Boltzmann distribution shows a fast potential decrease towards the end of the extraction period which is caused by its  $\sqrt{q_i V_T}$  factor.

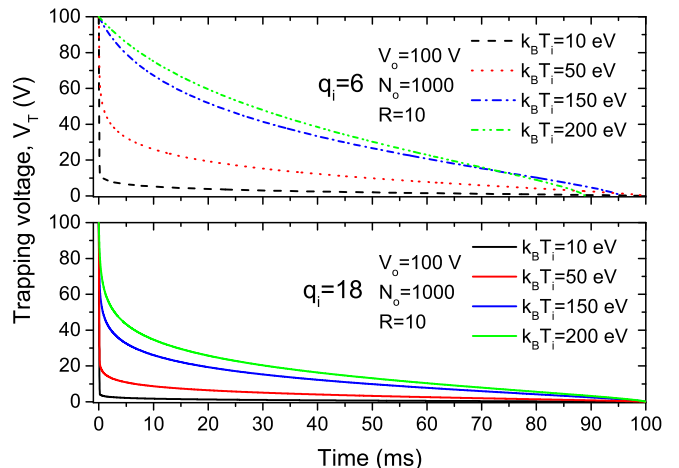


FIG. 2. Lowering voltage time function assuming a Maxwell-Boltzmann ion energy distribution.

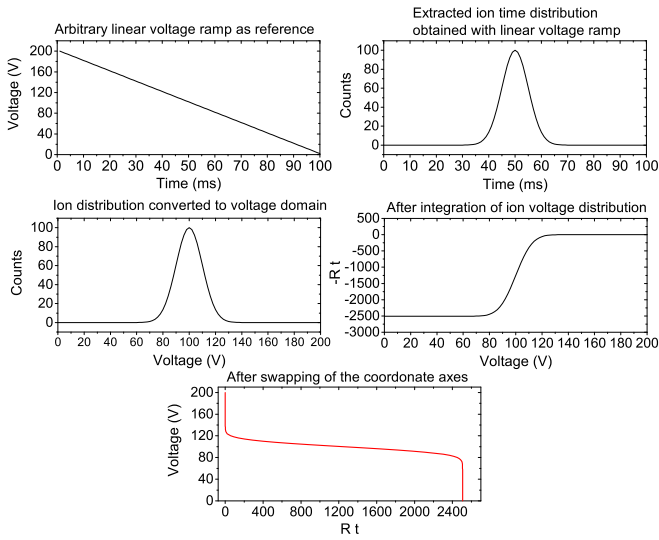


FIG. 3. Procedure to obtain the optimum ramp voltage time function from a measured ion time distribution.

### C. Experimental energy distribution

A more pragmatic approach is to infer from a measured ion time distribution the optimum ramp voltage time function that would uniformly distribute ions over the entire extraction period. The optimum ramp voltage function can be calculated from the ion energy distribution in the trap using Eq. 8. A way to obtain this distribution is to eject ions with well-known (arbitrary) ramp voltage function (such as a linear ramp) and measure the resulting ion time distribution. The voltage function being well-known, the initial ion distribution, which is in the time domain, can then be converted to the voltage domain with the  $E_i = q_i V_T$  relation. Numerically integrating the experimental ion energy distribution in the voltage domain gives a function which, when plotted, shows  $-Rt$  as a function of  $qV_T$ . By swapping the coordinate axes, and plotting  $qV_T$  as a function of  $Rt$  instead gives us the optimum ramp voltage function. This procedure is shown in Fig. 3 assuming a Gaussian ion time distribution obtained with a linear ramp used as a reference.