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U.S. DEPARTMENT OF ENERGY

USPAS - Fundamentals of Ion Sources
4./5. Beam Quality Parameters I + II

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Introduction

- A ~1 hour overview on beam quality and how we define various quantities.
- Many topics will be repeated during other parts of the lecture.
- Mostly transversal beam quality.
- Further Reading:
  - Ian Brown – The Physics and Technology of Ion Sources
  - Martin Reiser – Theory and Design of Charged Particle Beams
  - Helmut Wiedemann – Particle Accelerator Physics
  - Many papers… (references will be given on the slides)

A beam is…

Ensemble of particles that travel mostly in the same direction (let's use z)

- Typically: $v_z \gg v_x, v_y$
- Of course, that's not quite true at the very very beginning, but more later.
- Ion sources: $k_B T_i \approx \mathcal{O}(\mathrm{eV})$ and extraction voltages $\mathcal{O}(10\mathrm{kV})$
- Can be comprised of multiple ion species:

$$q_i = Q_i \cdot e,\ m_i = A \cdot amu\,(931.5\,\mathrm{MeV/c^2})$$

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Distributions in 6D Phase Space (+t)

Particle number density:

$$n(x, y, z, p_x, p_y, p_z, t)$$

or $$n(x, y, z, v_x, v_y, v_z, t)$$

Charge density: $\rho = q \cdot n$

If the beam is in steady-state (often we extract DC beams from ion sources) one can replace t with z

Furthermore: $n(x, x', y, y', z, \Delta p/p)$ ("Trace Space")

$$x' = \frac{dx}{dz} = \frac{v_x}{v_z}, \;\; y' = \frac{dy}{dz} = \frac{v_y}{v_z}$$

4D/2D Projections / Slices

If there is no coupling between longitudinal motion and transversal motion the transversal Trace Space density is $n(x, x', y, y)$

Maybe we are even only interested in 2D projections

$$n(x, x') = \int\int dy dy' n(x, x', y, y')$$

Or slices (interesting in diagnostics and simulations)

$$n(r, r') = n(x, x', y = 0, y' = 0)$$

Because these can tell us something about our beam line transport…

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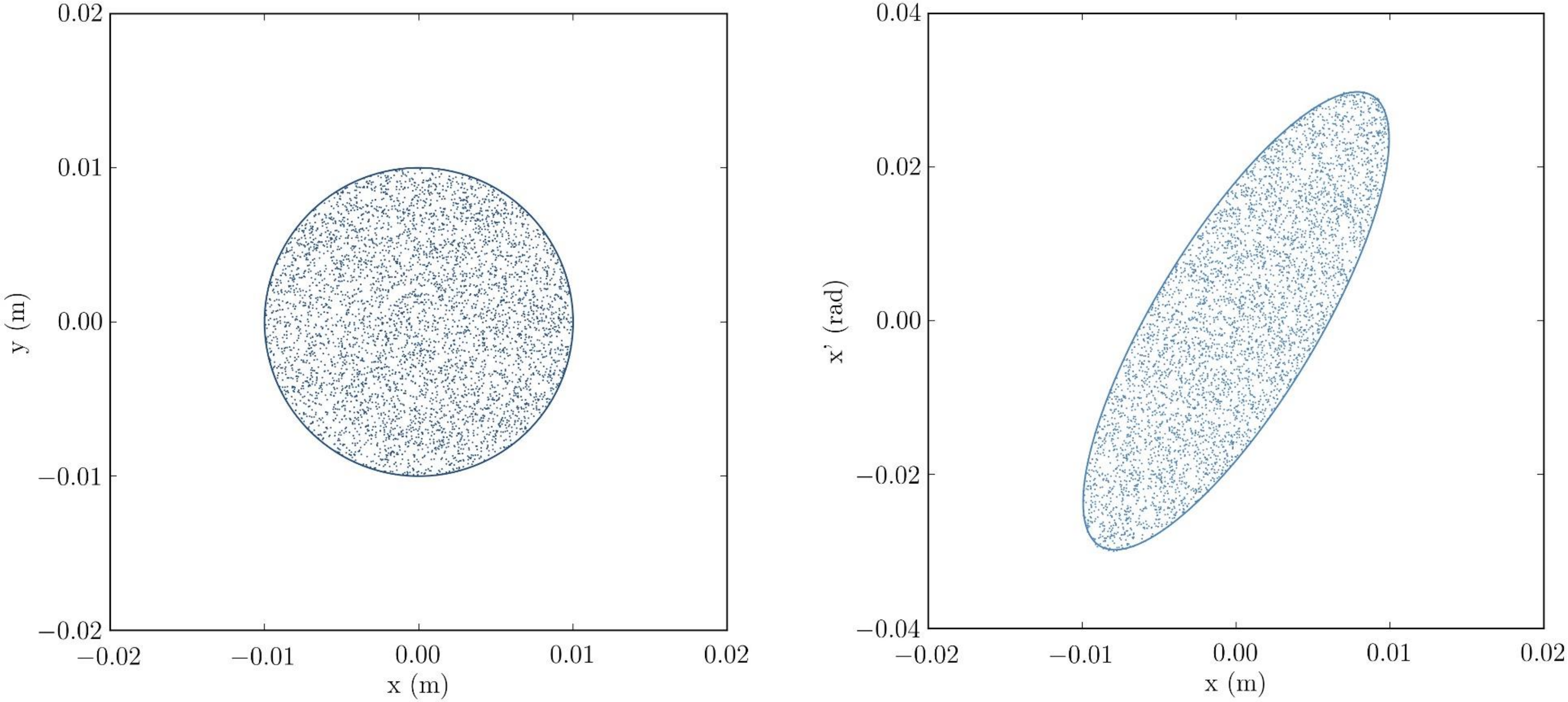
Trace Space Example



K-V Beam – Projections are uniform ellipses

Liouville's Theorem

States that for non-interacting particles in a system that can be described by a Hamiltonian, the phase space density is conserved.

$$\frac{dn}{dt} = 0, \text{ or } n = n_0 = const. \qquad \iint d^3q_i d^3P_i = const.$$

in terms of mechanical momentum

(also true for linear space-charge)

Trace space area: $A_x = \frac{1}{P}\iint dx dP_x = \frac{1}{\gamma\beta mc}\iint dx dP_x$

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Kapchinsky-Vladimirsky Distribution

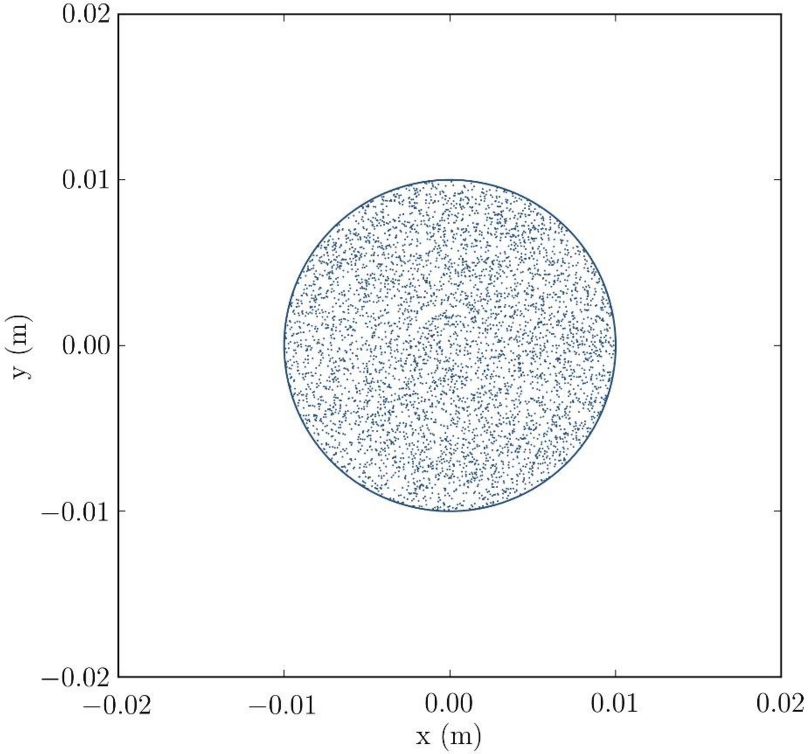
The K-V distribution is a uniformly distributed hollow ellipsoid in Trace space:

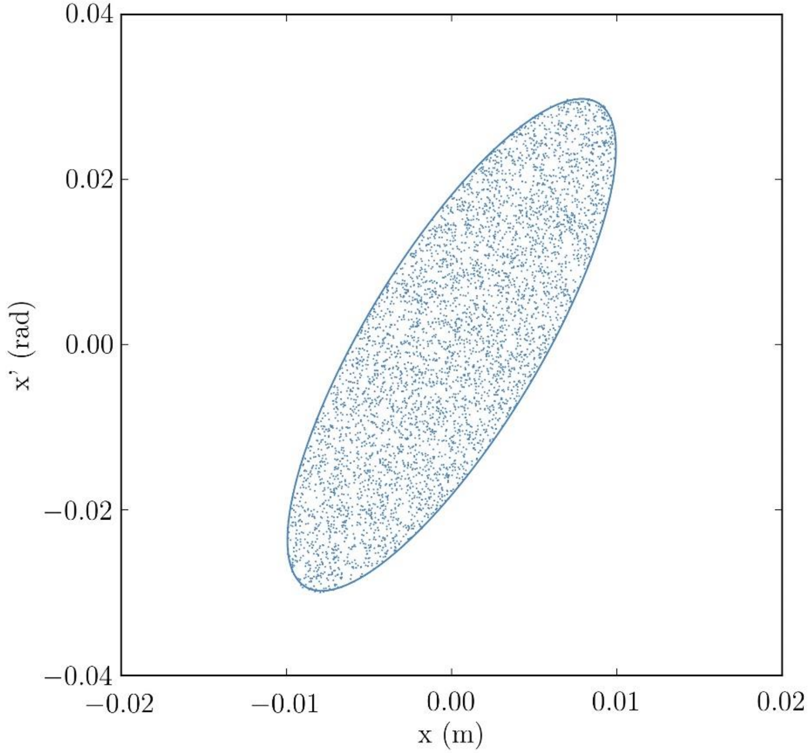
$$f(x,y,x',y') = f_0 \cdot \delta \left(\frac{x_b^2 x'^2 + \sqrt{x'^2_b x_b^2 - \epsilon_x^2}\, x x' + x'^2_b x^2}{\epsilon_x^2} + \frac{y_b^2 y'^2 + \sqrt{y'^2_b y_b^2 - \epsilon_y^2}\, y y' + y'^2_b y^2}{\epsilon_y^2} - 1 \right)$$

with x_b, y_b the maximum beam extent (b for 'beam') in x and y directions, x'_b, y'_b the maximum angles, and ϵ_x, ϵ_y the (full) beam emittances.

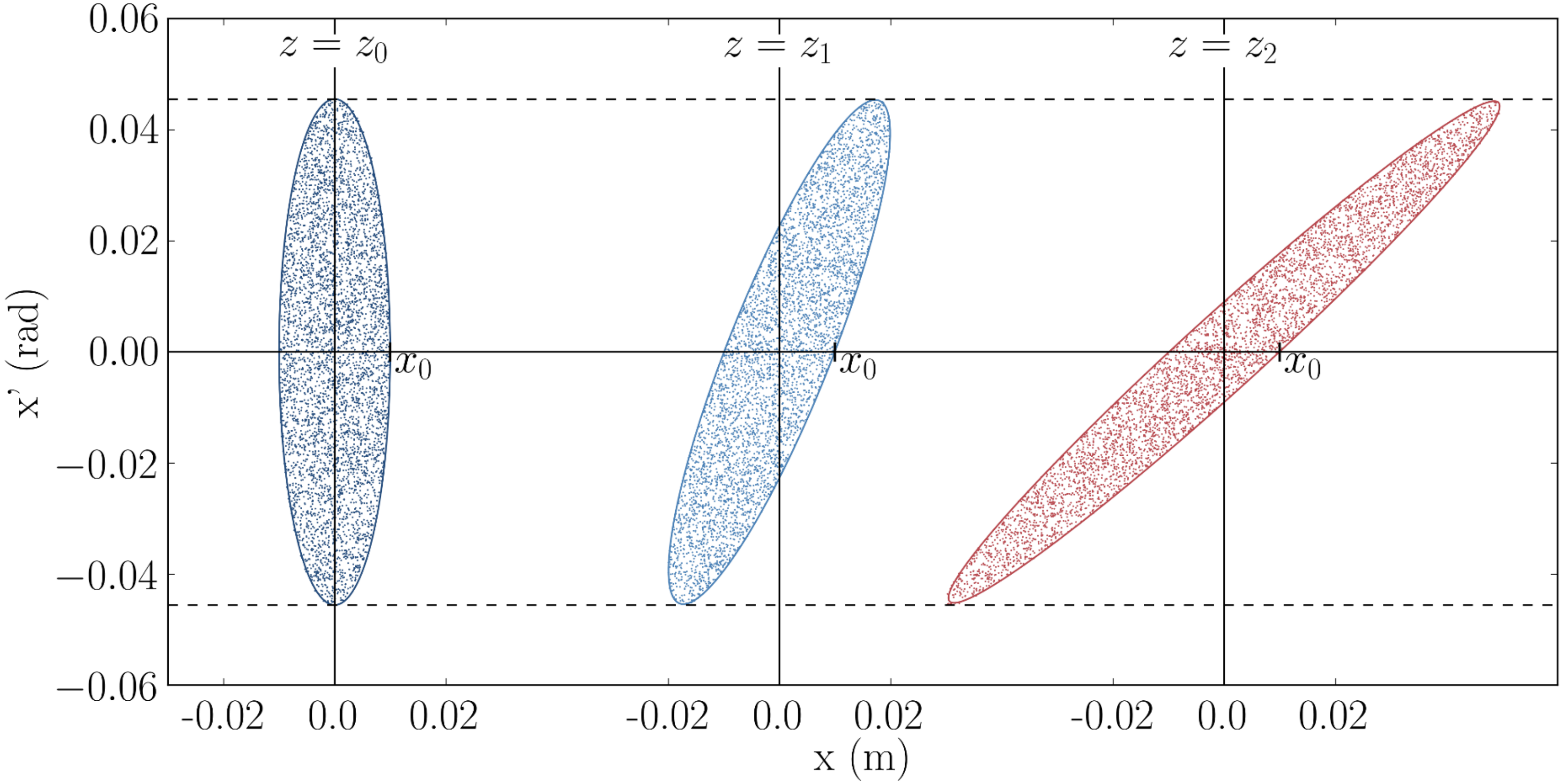
All projections in 2D subspaces are uniformly filled ellipses.

Trace Space Example





Phase Space Evolution - Drift



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Geometric Emittance

Definition from Area

$$\epsilon_x = \frac{A_x}{\pi} \quad [\pi\text{-}mm\text{-}mrad]$$

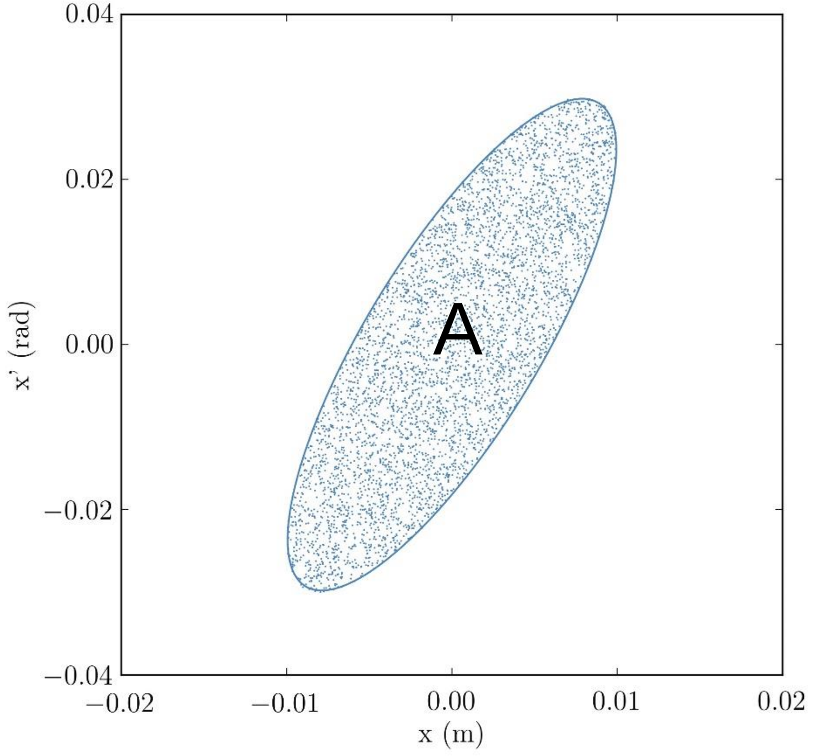
$$A_x = \frac{1}{P}\iint dx dP_x = \frac{1}{\gamma\beta mc}\iint dx dP_x$$

$$A_x = \frac{1}{\gamma\beta}\iint dx dx'$$

Normalized Emittance:

$$\epsilon_{x,norm.} = \gamma\beta\epsilon_x$$

Const. even under acceleration



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How does Emittance influence Beam Dynamics?

Paraxial equation (similar for y): $x''(s) + \kappa_x(s)x = 0$

(assuming periodic-focusing and 2 planes of symmetry)

General solution in amplitude-phase notation:

$x(s) = Aw(s)cos[\Psi(s) + \phi]$ where A and ϕ are determined by initial conditions

with $w'' + \kappa w - \frac{1}{w^3}$

Leads to definition of Courant-Snyder Invariant $\gamma x^2 + 2\alpha x x' + \beta x'^2 = A^2$

And $A^2 = \epsilon_x$

(See M. Reiser, Theory and Design of Charged Particle Beams)

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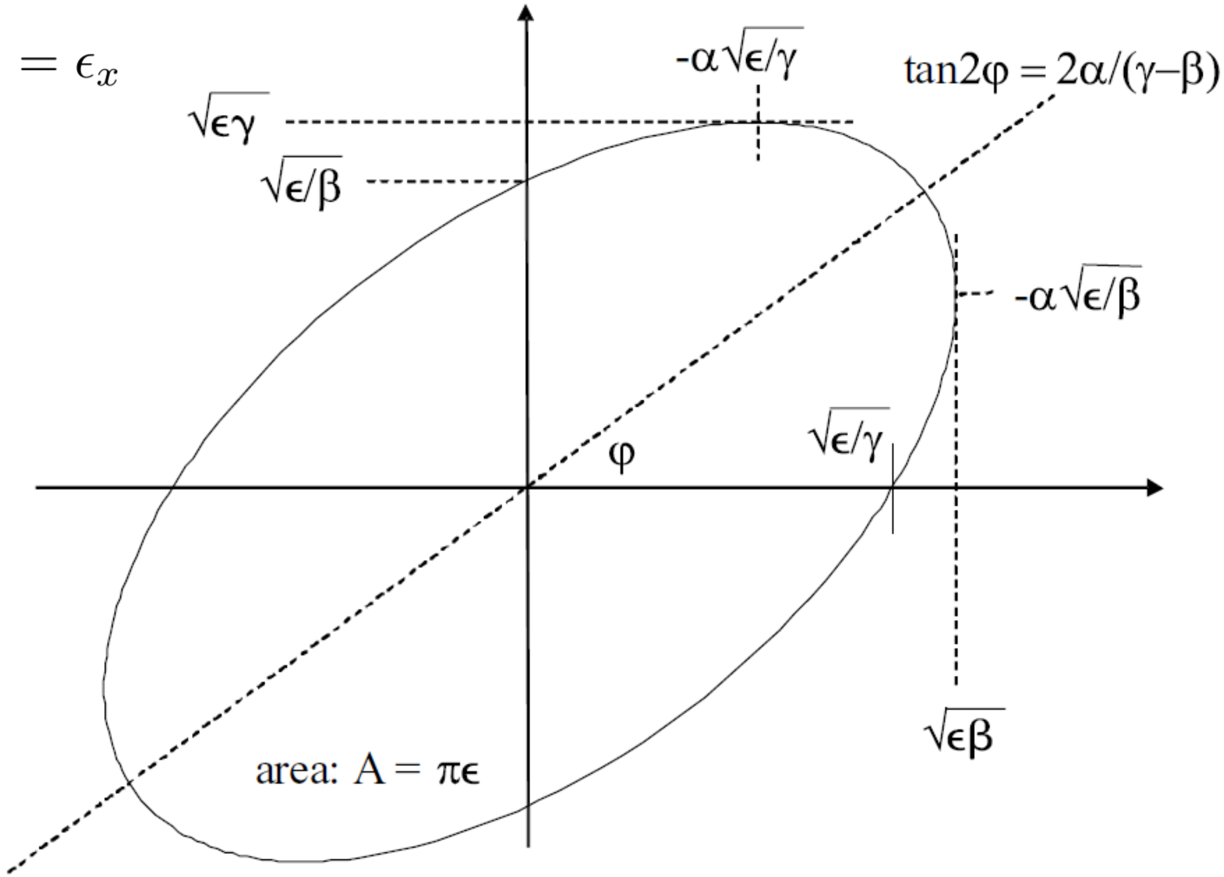
Courant-Snyder Invariant and Twiss Parameters

$$\gamma x^2 + 2\alpha x x' + \beta x'^2 = \epsilon_x$$

$$\gamma\beta - \alpha^2 = 1$$

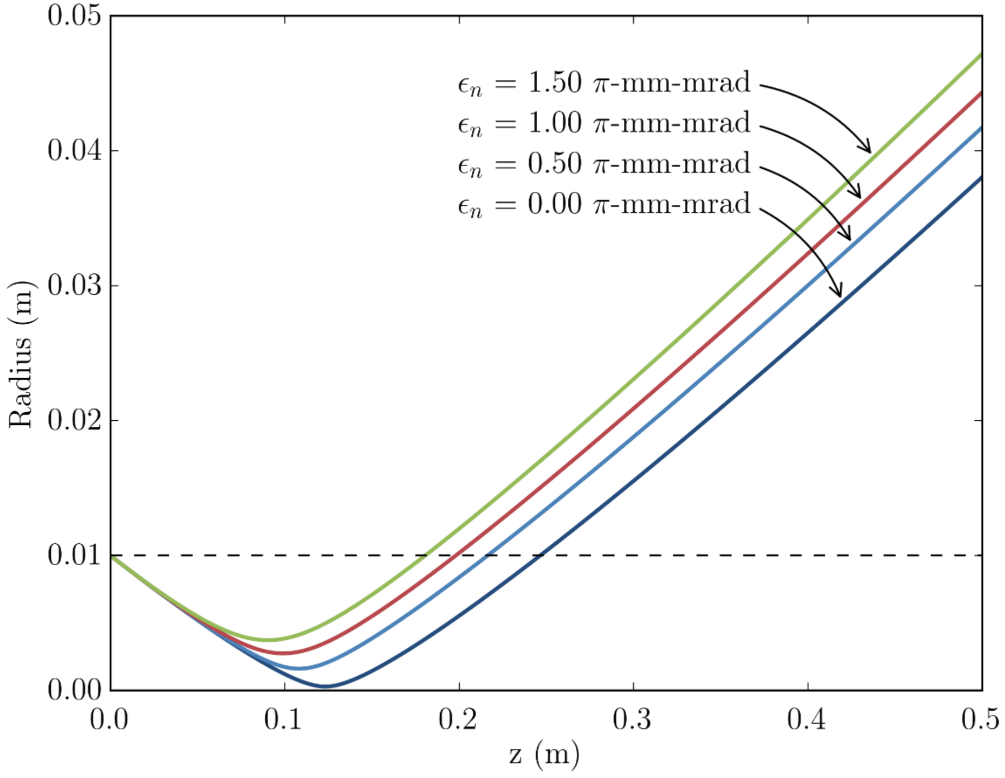


How does Emittance influence Beam Dynamics?

Courant-Snyder form of envelope equation:

$$x_m'' + \kappa x_m - \frac{\epsilon_x^2}{x_m^3} = 0$$

Emittance works against focusing…



Why preserve (reduce?) emittance?

- Kind of a no-brainer ;)
- Emittance determines the size of the final focus at a certain focal length from the focusing device.
- Emittance determines the distance beam transport elements have to have.
- Emittance determines the distance beam transport elements have to have.
- Emittance…the smaller the better…
- And we have a good definition…right?

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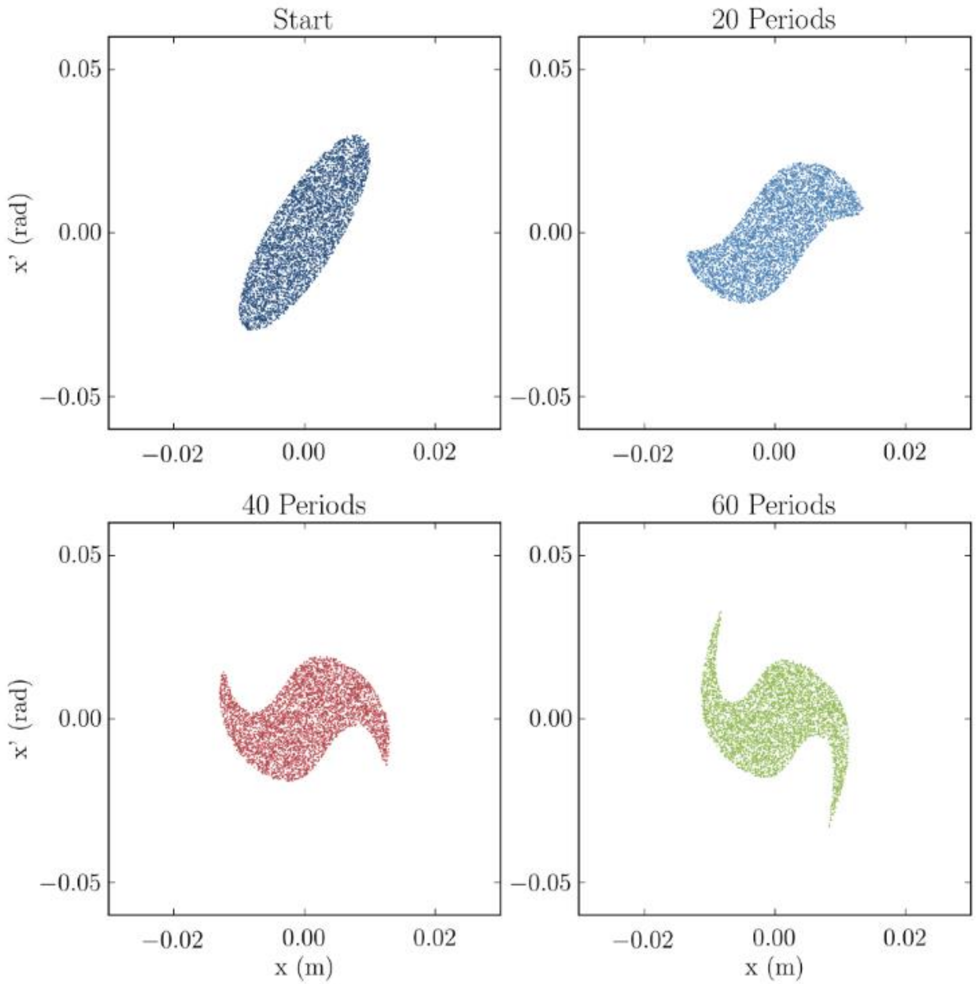
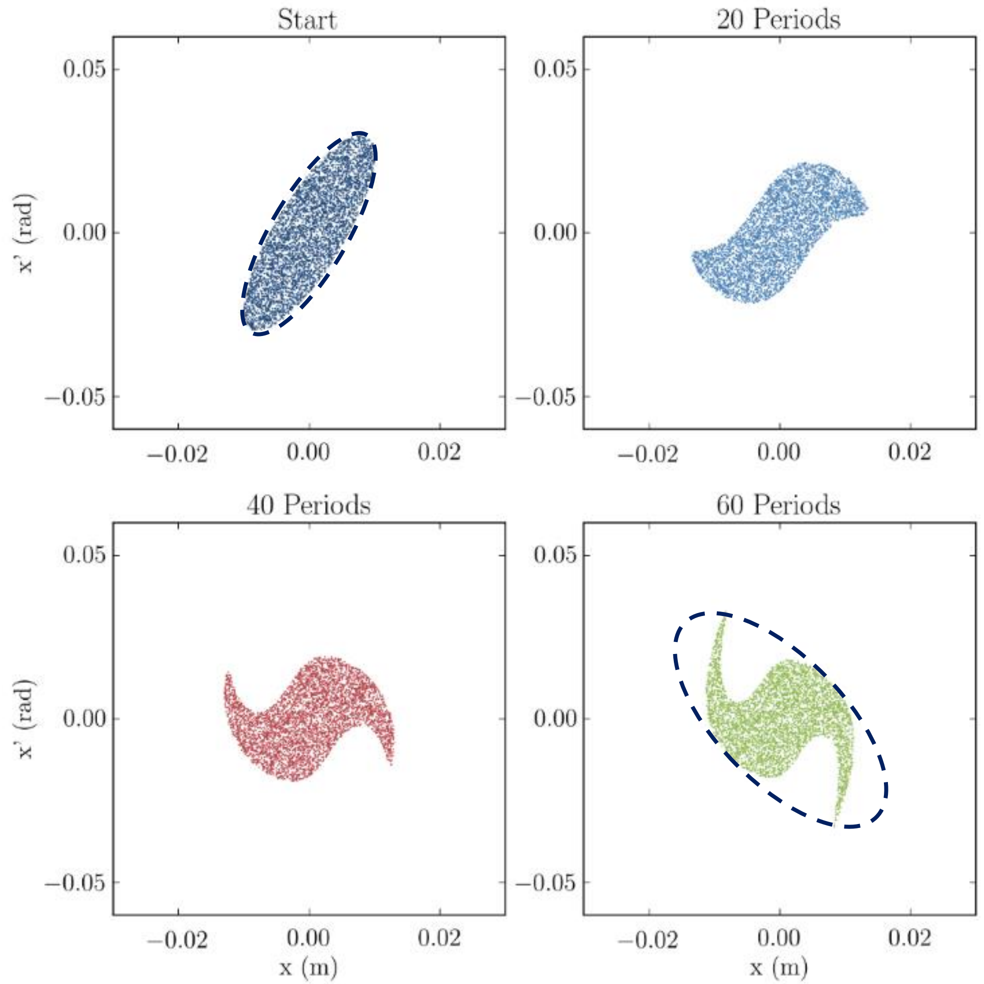
Phase Space Evolution – Aberration

- Simple envelope equation solver with spherical aberration…
- Filamentation of the trace space



Phase Space Evolution - Aberration

- Simple envelope equation solver with spherical aberration…
- Filamentation of the trace space
- Ellipse surrounding the beam is growing.
- Actual phase space volume is conserved (still Hamiltonian system)



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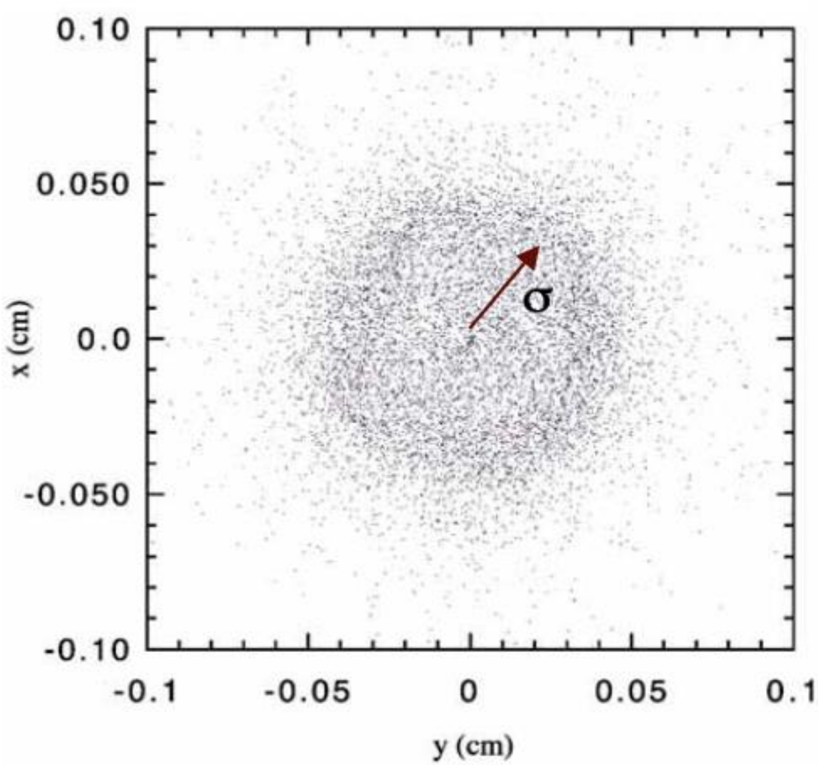
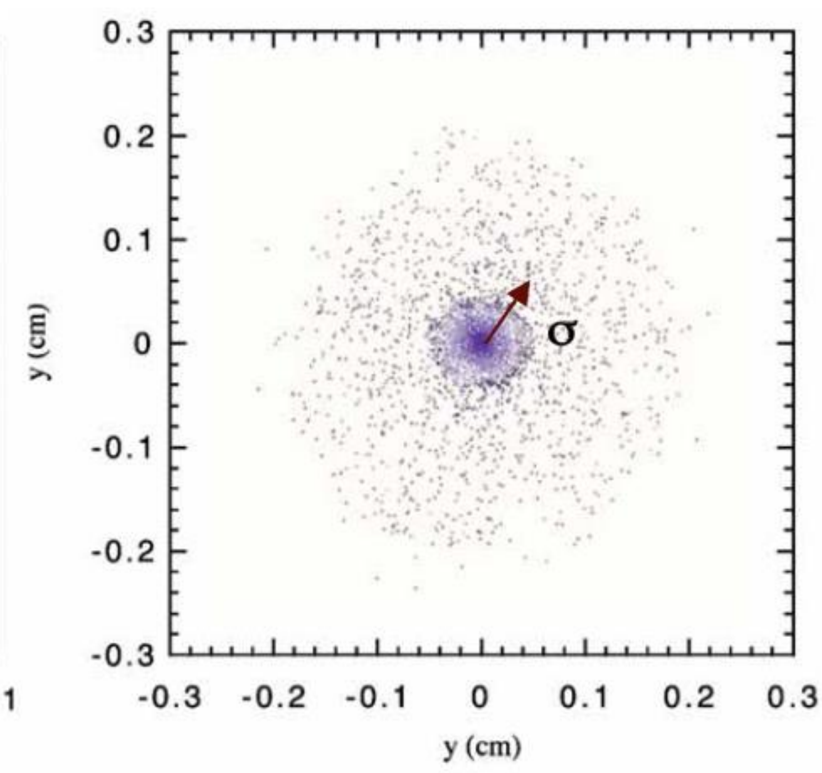
Other beam Cross-Sections

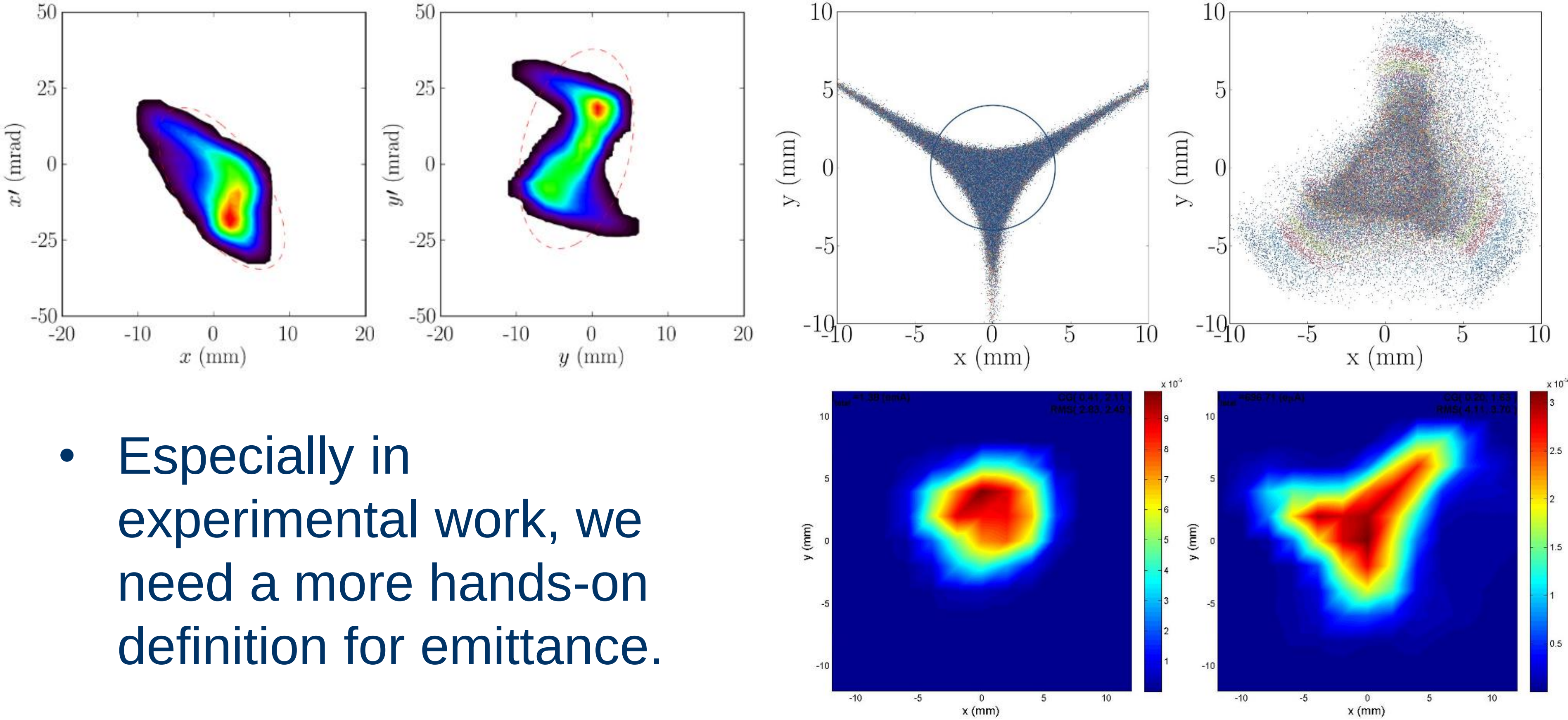


Gaussian beam



Beam with halo

Some ECRIS Cross-Sections and Phase Spaces



- Especially in experimental work, we need a more hands-on definition for emittance.

RMS Emittance

Second moments:

$$\langle x^2 \rangle = \frac{\iiiint x^2 f(x,y,x',y')dxdydx'dy'}{\iiiint f(x,y,x',y')dxdydx'dy'}$$

$$\langle x'^2 \rangle = \frac{\iiiint x'^2 f(x,y,x',y')dxdydx'dy'}{\iiiint f(x,y,x',y')dxdydx'dy'}$$

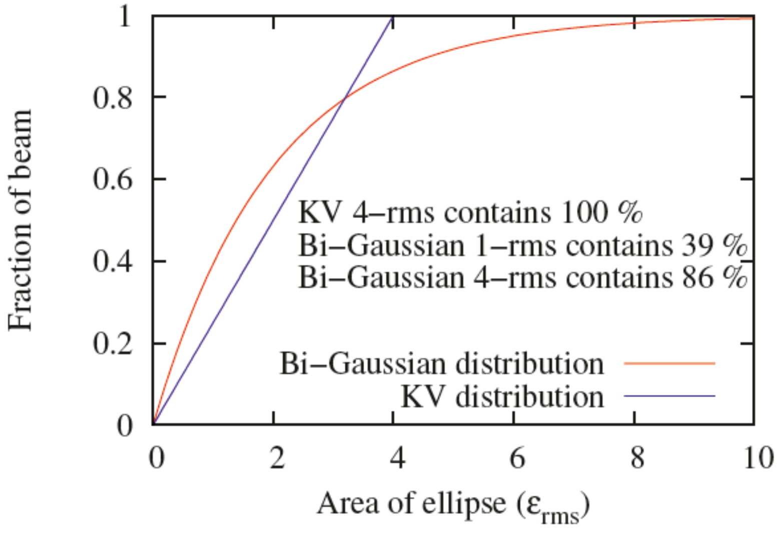
$$\langle xx' \rangle = \frac{\iiiint xx' f(x,y,x',y')dxdydx'dy'}{\iiiint f(x,y,x',y')dxdydx'dy'}$$

$$\epsilon_{rms} = \sqrt{\langle x^2 \rangle \langle x'^2 \rangle - \langle xx' \rangle^2} \qquad [mm\text{-}mrad]$$

How does this compare to full emittance?

Well, that depends…on the actual distribution.

- K-V Distribution: $\epsilon_x = 4\epsilon_{x,rms}$
- Waterbag Distribution: $\epsilon_x = 6\epsilon_{x,rms}$
- (Bi-)Gaussian Distribution: $\epsilon_x = n^2\epsilon_{x,rms}$ if truncated at $n \cdot \sigma$



Effective Emittance/Beam Size

- For analytical calculations (e.g. using the K-V distribution) It can be useful to define $\epsilon_x = 4\epsilon_{x,rms}$ as “effective emittance” as it corresponds to the K-V distribution which is often used for first order analytical investigations.
- Many authors have adopted this method.
- Similar: $X = 2\tilde{x} = s\left(\bar{x^2}\right)^{1/2}$ the “effective beam radius”

Equivalent Emittance

Often, what we are interested in is really how much beam can we transport along a beamline, through a series of apertures, etc. so it makes sense to define a 90% emittance. (or look at 90% versus maximum to determine halo)

→ The emittance contour within which 90% of the beam sits. (Often this is close to 4-rms).

$$\epsilon_{x\text{-}90\%}$$

Brightness

- The brightness is commonly defined as current density per unit solid angle.

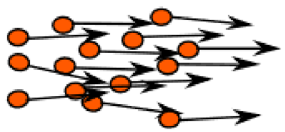
$$B = \frac{J}{d\Omega} = \frac{dI}{dSd\Omega}$$

- Or in terms of the transversal projections

$$\bar{B} = \frac{2I}{\pi^2 \epsilon_x \epsilon_y} \qquad \left[\frac{A}{m^2\text{-}rad^2}\right]$$

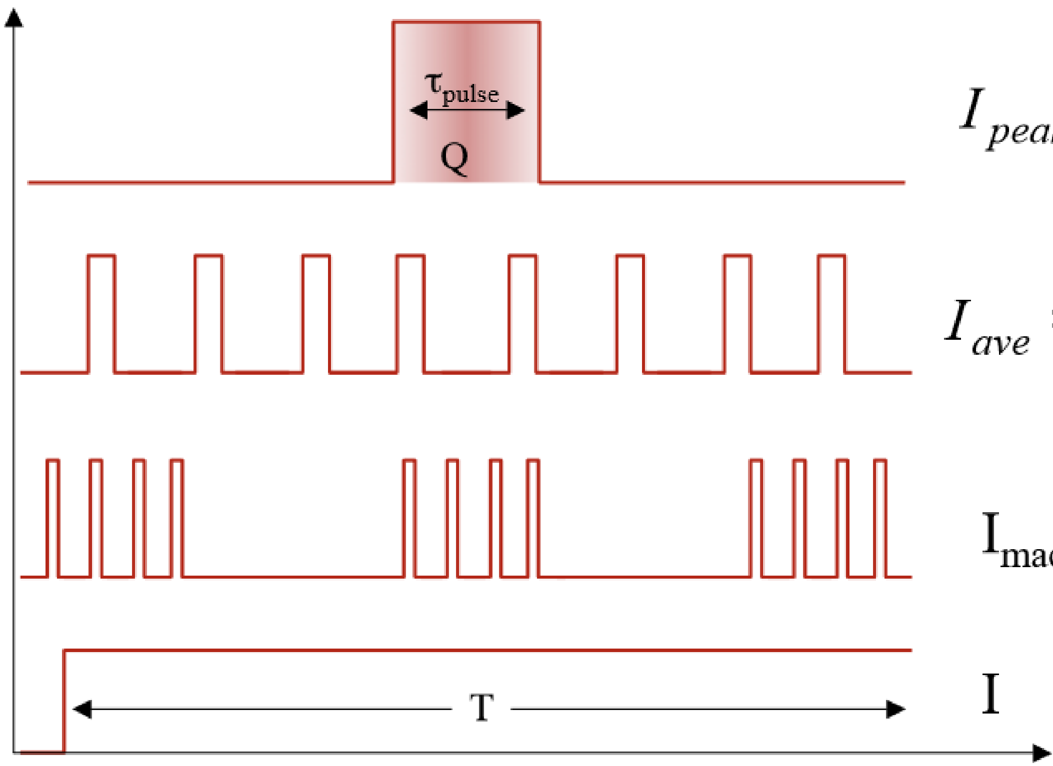
$$B_n = \frac{B}{\beta^2 \gamma^2} \qquad B_{90\%} = \frac{2 \cdot 0.9 \cdot I}{\pi^2 \epsilon_{x\text{-}90\%} \epsilon_{y\text{-}90\%}}$$

Longitudinal Beam Properties
Bunch Currents



$I \sim ne\langle v_z \rangle$

$$\text{Duty factor} = \frac{\sum \tau_{pulse}}{T}$$



$$I_{peak} = Q / \tau_{pulse}$$

$$I_{ave} = Q_{tot} / T$$

From: W. Barletta

Longitudinal Beam Properties
Phase Space Variables

Relative energy variation: $\delta = \frac{\Delta E}{E_0} = \frac{E - E_0}{E_0}$

Relative time: $\tau = t - t_0$

Momentum variation: $\frac{\Delta\gamma}{\gamma}, \quad \frac{\Delta p}{p}$

Path along beam line: z or s

Longitudinal Beam Properties Comments

Energy spread in beam comes from:

- Source HV potential stability
- Ions are created at different potentials inside the sheath
- Plasma instabilities
- Temperature kT

Measure energy spread of low energy beams? E.g. Retarding Field Analyzer.

What reduces beam quality in LEBT?

- Source performance and plasma parameters.
- Beam mismatch: Large beam going through solenoid/ Einzel lens/Dipole, will have aberrations.
- Coupling of longitudinal and transversal motion. (e.g. Energy spread going through a dipole magnet)
- Space Charge: Repulsive force of beam ions toward each other.
- Over-focusing will increase space charge density locally.
- Transporting unwanted ion species/electrons.

How can we preserve beam quality?

- Well designed beam lines.
- Stable power supplies.
- Separation of ions early on.
- Correct beam line pressure for highly charged ions, fragile ions and protons. (Space Charge Compensation)

Summary of Beam Quality Parameters

Current

Divergence

Emittance

Brightness

Energy Spread

Purity

Some have many different definitions and uses…